Waves
Transverse Speed and Acceleration
Power of a Wave

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Last time

- solutions to the wave equation
- sine waves (covered in lab)

Question

Quick Quiz 16.2[1] A sinusoidal wave of frequency f is traveling along a stretched string. The string is brought to rest, and a second traveling wave of frequency $2f$ is established on the string.

What is the wave speed of the second wave?

(A) twice that of the first wave

(B) half that of the first wave

(C) the same as that of the first wave

(D) impossible to determine

[1] Serway & Jewett, page 489.

Question

Quick Quiz 16.2[1] A sinusoidal wave of frequency f is traveling along a stretched string. The string is brought to rest, and a second traveling wave of frequency $2f$ is established on the string.

What is the wavelength of the second wave?

(A) twice that of the first wave

(B) half that of the first wave

(C) the same as that of the first wave

(D) impossible to determine

[1] Serway & Jewett, page 489.

Question

Quick Quiz 16.2[1] A sinusoidal wave of frequency f is traveling along a stretched string. The string is brought to rest, and a second traveling wave of frequency $2f$ is established on the string.

What is the amplitude of the second wave?

(A) twice that of the first wave
(B) half that of the first wave
(C) the same as that of the first wave
(D) impossible to determine

[1] Serway & Jewett, page 489.

Overview

- transverse speed of an element of the medium
- energy transfer by a sine wave

Sine waves

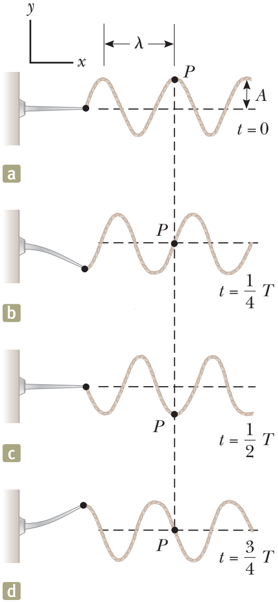
Consider a point, P, on a string carrying a sine wave.

Suppose that point is at a fixed horizontal position $x = 5\lambda/4$, a constant.

The y coordinate of P varies as:

$$\begin{aligned} y\left(\frac{5\lambda}{4}, t\right) &= A\sin(-\omega t + 5\pi/2) \\ &= A\cos(\omega t) \end{aligned}$$

The point is in simple harmonic motion!



Sine waves: Transverse Speed and Transverse Acceleration

The transverse speed v_y is the speed at which a single point on the medium (string) travels perpendicular to the propagation direction of the wave.

We can find this from the wave function

$$y(x,t) = A\sin(kx - \omega t)$$

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$$y(x,t) = A\sin(kx - \omega t)$$

$$v_y = \frac{\partial y}{\partial t} = -\omega A\cos(kx - \omega t)$$

For the transverse acceleration, we just take the derivative again:

$$a_y = \frac{\partial^2 y}{\partial t^2} = -\omega^2 A\sin(kx - \omega t)$$

Sine waves: Transverse Speed and Transverse Acceleration

$$v_y = -\omega A \cos(kx - \omega t)$$

$$a_y = -\omega^2 A \sin(kx - \omega t) = -\omega^2 y$$

If we fix $x =$const. these are exactly the equations we had for SHM!

The maximum transverse speed of a point P on the string is when it passes through its equilibrium position.

$$v_{y,\max} = \omega A$$

The maximum magnitude of acceleration occurs when $y = A$ (or max value, including sign when $y = -A$).

$$a_y = \omega^2 A$$

Questions

Can a wave on a string move with a wave speed that is greater than the maximum transverse speed $v_{y,\text{max}}$ of an element of the string?

(A) yes

(B) no

Questions

Can the wave speed be much greater than the maximum element speed?

(A) yes

(B) no

Questions

Can the wave speed be equal to the maximum element speed?

(A) yes

(B) no

Questions

Can the wave speed be less than $v_{y,\mathrm{max}}$?

(A) yes

(B) no

Sine waves: Transverse Speed and Transverse Acceleration

$$v_y = -\omega A \cos(kx - \omega t)$$

$$a_y = -\omega^2 A \sin(kx - \omega t) = -\omega^2 y$$

Rate of Energy Transfer in Sine Wave

Waves do transmit energy.

A wave pulse causes the mass at each point of the string to displace from its equilibrium point.

At what rate does this transfer happen? (Find $\frac{dE}{dt}$)

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Consider the kinetic and potential energies in a small length of string.

Kinetic:

$$\mathrm{dK} = \frac{1}{2}(\mathrm{dm})v_y^2$$

Replacing v_y:

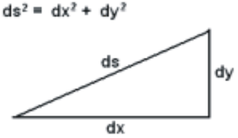
$$\mathrm{dK} = \frac{1}{2}(\mathrm{dm})A^2\omega^2\cos^2(kx - \omega t)$$

Rate of Energy Transfer in Sine Wave

Potential:

$$dU = F\,d\ell = T(ds - dx)$$

where $d\ell = ds - dx$ is the amount by which a small element of the string is stretched, ds is the stretched length and dx is the unstretched length.



$$ds = \sqrt{dx^2 + dy^2} = \sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2}\,dx \approx \left[1 + \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2\right] dx$$

[1] Diagram from solitaryroad .com, James Miller.

Rate of Energy Transfer in Sine Wave

$$\mathrm{ds} - \mathrm{dx} = \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2 \mathrm{dx}$$

$$\begin{aligned}
\mathrm{dU} &= \frac{1}{2} T \left(\frac{\partial y}{\partial x}\right)^2 \mathrm{dx} \\
&= \frac{1}{2} T \left(Ak\cos(kx - \omega t)\right)^2 \mathrm{dx} \\
&= \frac{1}{2} \mu\omega^2 A^2 \cos^2(kx - \omega t)\, \mathrm{dx}
\end{aligned}$$

having used $v = \omega/k$ and $v = \sqrt{T/\mu}$ in the last line.

Rate of Energy Transfer in Sine Wave

$$dK = \frac{1}{2}\mu\, dx\, A^2\omega^2\cos^2(kx - \omega t)$$

$$dU = \frac{1}{2}\mu A^2\omega^2\cos^2(kx - \omega t)\, dx$$

Adding $dU + dK$ gives

$$dE = \mu\omega^2 A^2\cos^2(kx - \omega t)\, dx$$

Integrating over one wavelength gives the energy per wavelength:

$$\begin{aligned} E_\lambda &= \mu\omega^2 A^2 \int_0^\lambda \cos^2(kx - \omega t)\, dx \\ &= \mu\omega^2 A^2 \frac{\lambda}{2} \end{aligned}$$

Rate of Energy Transfer in Sine Wave

For one wavelength:

$$E_\lambda = \frac{1}{2}\mu\omega^2 A^2 \lambda$$

Power averaged over one wavelength:

$$P = \frac{E_\lambda}{T} = \frac{1}{2}\mu\omega^2 A^2 \frac{\lambda}{T}$$

Average power of a wave on a string:

$$P = \frac{1}{2}\mu\omega^2 A^2 v$$

Question

Quick Quiz 16.5[2] Which of the following, taken by itself, would be most effective in increasing the rate at which energy is transferred by a wave traveling along a string?

(A) reducing the linear mass density of the string by one half
(B) doubling the wavelength of the wave
(C) doubling the tension in the string
(D) doubling the amplitude of the wave

[2] Serway & Jewett, page 496.

Summary

- energy transfer by a sine wave

Homework Serway & Jewett:

- Ch 16, onward from page 499. Probs: 33, 35, 61