Waves
Interference

Lana Sheridan

De Anza College

May 22, 2020

Last time

- transverse speed of an element of the medium
- energy transfer by a sine wave

Overview

- power of a sine wave
- interference

Rate of Energy Transfer in Sine Wave

$$\mathrm{dK} = \frac{1}{2}\mu\,\mathrm{dx}\,A^2\omega^2\cos^2(kx-\omega t)$$

$$\mathrm{dU} = \frac{1}{2}\mu A^2\omega^2\cos^2(kx-\omega t)\,\mathrm{dx}$$

Adding $\mathrm{dU}+\mathrm{dK}$ gives

$$\mathrm{dE} = \mu\omega^2 A^2\cos^2(kx-\omega t)\,\mathrm{dx}$$

Integrating over one wavelength gives the energy per wavelength:

Rate of Energy Transfer in Sine Wave

$$dK = \frac{1}{2}\mu\, dx\, A^2\omega^2 \cos^2(kx - \omega t)$$

$$dU = \frac{1}{2}\mu A^2\omega^2 \cos^2(kx - \omega t)\, dx$$

Adding $dU + dK$ gives

$$dE = \mu\omega^2 A^2 \cos^2(kx - \omega t)\, dx$$

Integrating over one wavelength gives the energy per wavelength:

$$\begin{aligned} E_\lambda &= \mu\omega^2 A^2 \int_0^\lambda \cos^2(kx - \omega t)\, dx \\ &= \mu\omega^2 A^2 \frac{\lambda}{2} \end{aligned}$$

Rate of Energy Transfer in Sine Wave

For one wavelength:

$$E_\lambda = \frac{1}{2}\mu\omega^2 A^2 \lambda$$

Power averaged over one wavelength:

$$P = \frac{E_\lambda}{T} = \frac{1}{2}\mu\omega^2 A^2 \frac{\lambda}{T}$$

Average power of a wave on a string:

$$P = \frac{1}{2}\mu\omega^2 A^2 v$$

Question

Quick Quiz 16.5[1] Which of the following, taken by itself, would be most effective in increasing the rate at which energy is transferred by a wave traveling along a string?

(A) reducing the linear mass density of the string by one half
(B) doubling the wavelength of the wave
(C) doubling the tension in the string
(D) doubling the amplitude of the wave

[1] Serway & Jewett, page 496.

Question

Quick Quiz 16.5[1] Which of the following, taken by itself, would be most effective in increasing the rate at which energy is transferred by a wave traveling along a string?

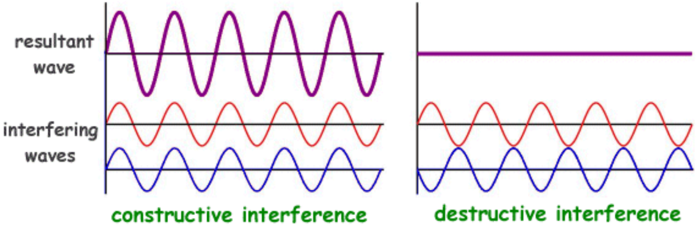
(A) reducing the linear mass density of the string by one half
(B) doubling the wavelength of the wave
(C) doubling the tension in the string
(D) doubling the amplitude of the wave $\longleftarrow$

[1] Serway & Jewett, page 496.

Interference of Waves (Reminder from Lab)

When two wave disturbances interact with one another they can amplify or cancel out.

Waves of the same frequency that are "in phase" will reinforce, amplitude will increase; waves that are "out of phase" will cancel out.



Interference of Waves (Reminder from Lab)

Waves that exist at the same time in the same position in space add together.

superposition principle

If two or more traveling waves are moving through a medium, the resultant value of the wave function at any point is the algebraic sum of the values of the wave functions of the individual waves.

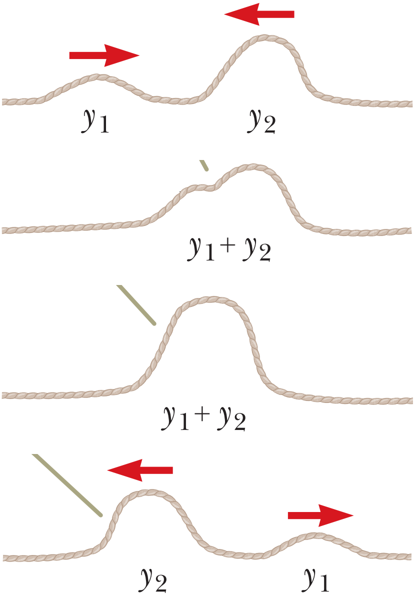
This works because the wave equation we are studying is *linear*.

This means solutions to the wave equations can be added:

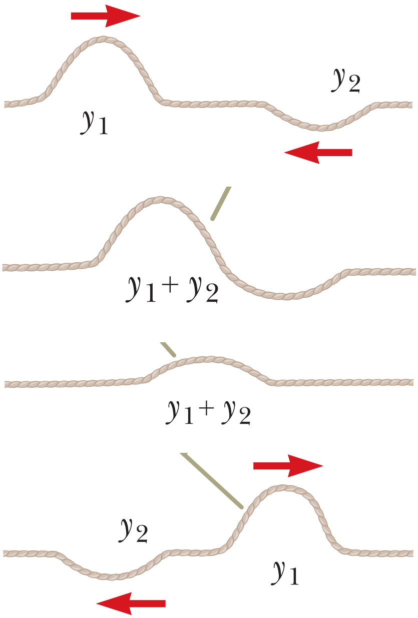
$$y(x,t) = y_1(x,t) + y_2(x,t)$$

y is the resultant wave function.

Interference of Waves: Constructive Interference



Interference of Waves: Destructive Interference



Superposition of Sine Waves

Consider two sine waves with the same wavelength and amplitude, but different phases, that interfere.

$$y_1(x,t) = A\sin(kx - \omega t) \quad y_2(x,t) = A\sin(kx - \omega t + \phi)$$

Add them together to find the resultant wave function, using the identity:

$$\sin\theta + \sin\psi = 2\cos\left(\frac{\theta - \psi}{2}\right)\sin\left(\frac{\theta + \psi}{2}\right)$$

Then

$$y(x,t) = \left[2A\cos\left(\frac{\phi}{2}\right)\right]\sin(kx - \omega t + \frac{\phi}{2})$$

New amplitude **Sine oscillation**

Dependence on Phase Difference

The amplitude of the resultant wave is $A' = 2A\cos\left(\frac{\phi}{2}\right)$, where ϕ is the phase difference.

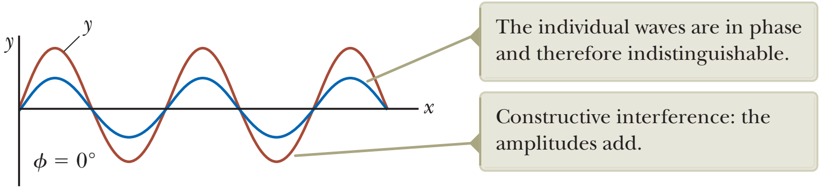
For what value of ϕ is A' maximized?

Dependence on Phase Difference

The amplitude of the resultant wave is $A' = 2A\cos\left(\frac{\phi}{2}\right)$, where ϕ is the phase difference.

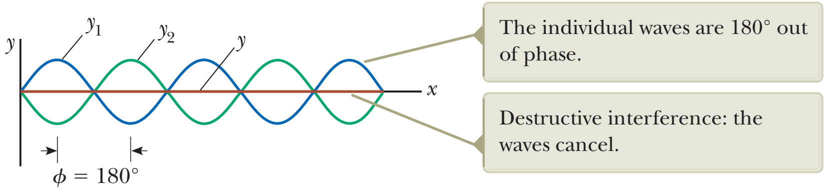
For what value of ϕ is A' maximized? $\phi = 0$ or $\phi = 2\pi, -2\pi,\ 4\pi,$ etc.

The waves are "in phase" and constructively interfere.



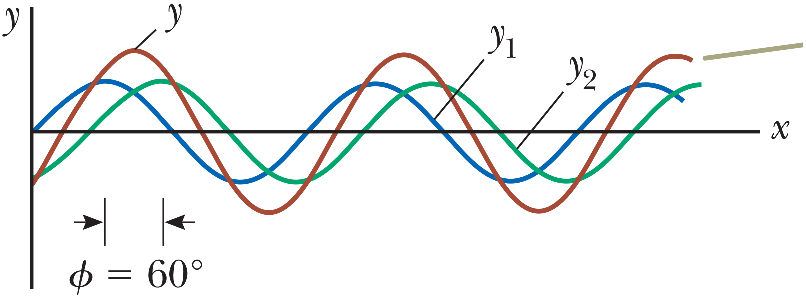
Dependence on Phase Difference

If $\phi = \pi, -\pi,\ 3\pi, -3\pi$, etc. $A' = 0$. Destructive interference.



Interference of Two Sine Waves (equal wavelength)

$$y(x,t) = \left[2A\cos\left(\frac{\phi}{2}\right)\right]\sin(kx - \omega t + \frac{\phi}{2})$$



Phase Differences

We can count phase differences in terms of wavelengths also.

If two waves have a phase difference of 1 wavelength then $\phi = 2\pi$. Constructive interference.

If two waves have a phase difference of half a wavelength then $\phi = \pi$. Destructive interference.

Summary

- energy transfer by a sine wave
- interference

Homework

- WebAssign due Tuesday night