Waves
Interference
Reflections and Boundaries

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Last time

- power of a wave on a string
- interference

Overview

- interference of sine waves with same freq, different amplitudes
- boundary conditions
- reflection and transmission

Question

Here are four possible phase differences between two identical waves, expressed in wavelengths:

0.20, 0.45, 0.60, and 0.80.

Rank them according to the amplitude of the resultant wave, greatest first.

(A) 0.20, 0.45, 0.60, 0.80
(B) 0.80, 0.60, 0.45, 0.20
(C) (0.20 and 0.80), 0.60, 0.45
(D) 0.45, 0.60, (0.20 and 0.80)

[1] Halliday, Resnick, Walker, page 427.

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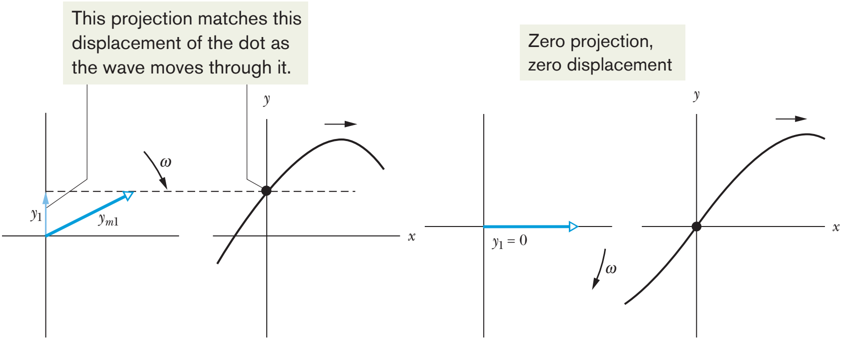
Phasors

We can represent sine waves and their addition with a **phasor diagram**.

This works for sine waves with **equal wavelengths**, even if they have different amplitudes.

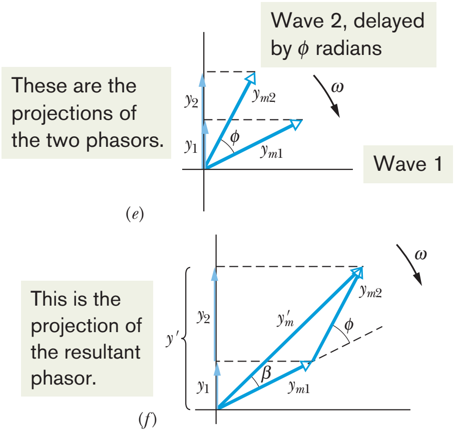
Each wave function at point (x, t) is represented by a vector.



[1] Figures from Halliday, Resnick, & Walker, 9th ed, page 429.

Phasors

Add the vectors to find the sum.



In the diagram $A' = y'_m$ is the amplitude of the resulting wave.

Example

Two sinusoidal waves $y_1(x, t)$ and $y_2(x, t)$ have the same wavelength and travel together in the same direction along a string. Their amplitudes are $A_1 = 4.0$ mm and $A_2 = 3.0$ mm, and their phase constants are 0 and $\pi/3$ rad, respectively.

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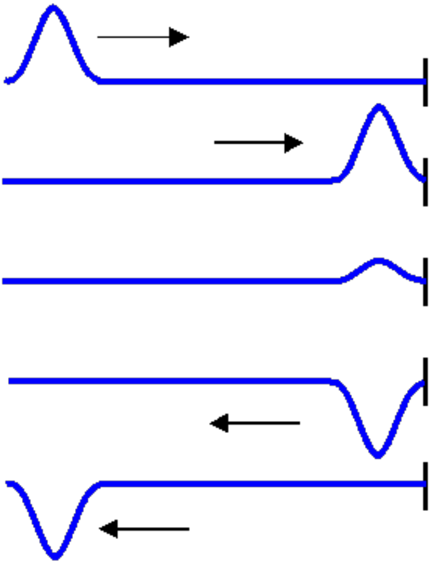
$$A' = 6.1 \text{ mm} \; ; \quad \phi' = 0.44 \text{ rad}$$

$$y'(x,t) = (6.1 \text{ mm}) \sin(kx - \omega t + 0.44)$$

Wave Reflection



Boundaries and Wave Reflection and Transmission

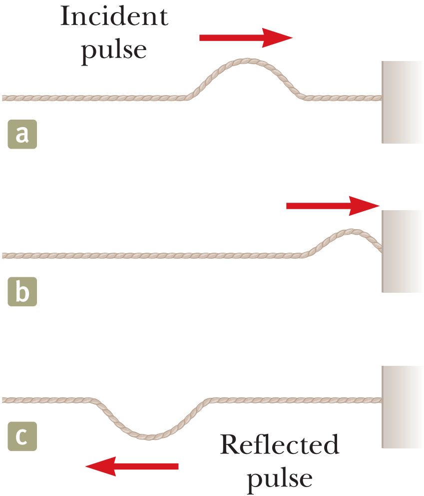
When waves reach the end of their medium, or move from one medium to another, they can be reflected.

The behavior is different in difference circumstances. (You saw this in lab!)

We can describe the different circumstances mathematically using **boundary conditions** on our wave function.

These will help us to correctly predict how a wave will reflect or be transmitted.

Wave Reflection from a fixed end point



The reflected pulse is inverted. How does this happen?

Wave Reflection from a fixed end point

The boundary condition for a fixed end point at position $x = 0$ is:

$$y(x = 0, t) = 0$$

At any time, the point of the string at $x = 0$ cannot have any vertical displacement. It is tied to a wall!

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This pulse will continue in the $+x$ direction forever, past the end of the string. Makes no sense.

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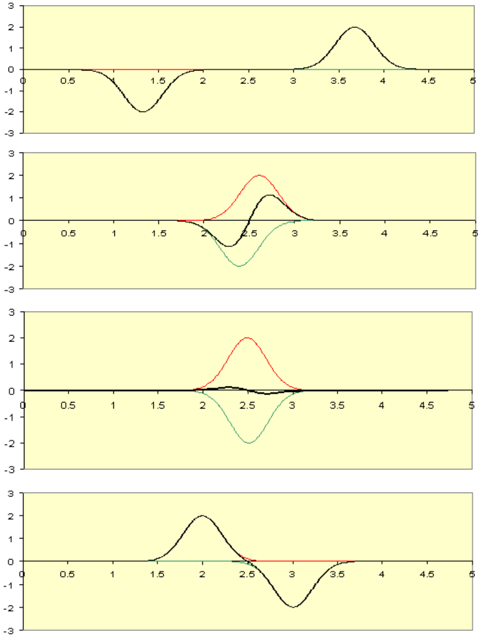
What if we imagine the string continues inside the wall, and there is a pulse traveling behind the wall in the $-x$ direction?



[1]Wall at $x = 2.5$. Digrams by Michal Fowler http://galileo.phys.virginia.edu

Wave Reflection from a fixed end point

If we allow another wave function:

$$y_2(x,t) = -f(-x - vt)$$

the total wave function will satisfy the boundary condition!

$$\begin{aligned} y(x,t) &= y_1(x,t) + y_2(x,t) \\ y(x,t) &= f(x - vt) + [-f(-x - vt)] \\ y(x=0,t) &= 0 \end{aligned}$$

$-f(-x - vt)$ corresponds to an inverted wave pulse that is also flipped left-to-right.

The reflected pulse is inverted.

Wave Reflection from a fixed end point

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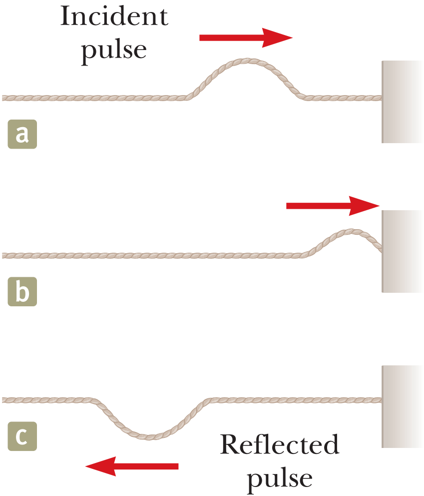


Wave Reflection from a freely movable end point

Now we have a different boundary condition.

The *slope* of the string at the boundary must be zero.

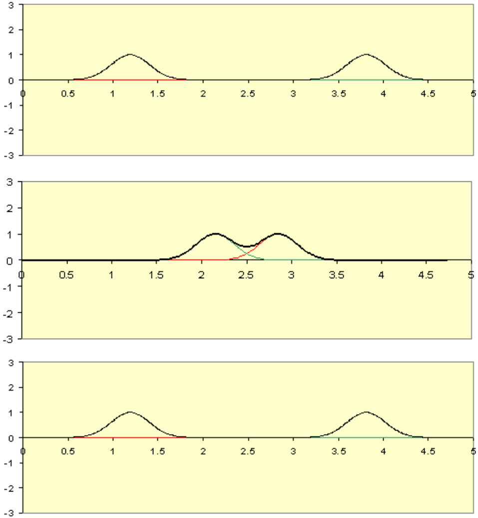
$$\left.\frac{\partial y}{\partial x}\right|_{x=0} = 0$$

This ensures that the string will stay attached to the wall and there will not be an infinite force on the last tiny bit of string.

To satisfy this boundary condition, imagine there is another pulse that is upright but moving in the $-x$ direction.

Wave Reflection from a freely movable end point

Imagine the free end of the string at $x = 2.5$. The slope there is zero at all times.



Wave Reflection from a freely movable end point

The new boundary condition is satisfied if $y_2 = f(-x - vt)$:

Let $u_1 = x - vt$ and $u_2 = -x - vt$.

$$
\begin{aligned}
y(x,t) &= f(x - vt) + f(-x - vt) \\
\frac{\partial y(x,t)}{\partial x} &= \frac{\partial f(u_1)}{\partial x} + \frac{\partial f(u_2)}{\partial x} \\
&= f'(u_1) + (-1)f'(u_2)
\end{aligned}
$$

The terms cancel when $u_1 = u_2$, that is, at $x = 0$.

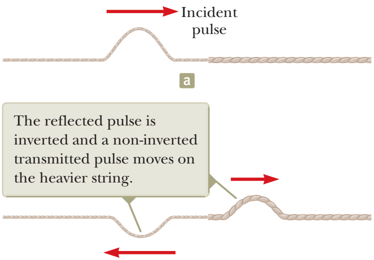
$$
\left.\frac{\partial y}{\partial x}\right|_{x=0} = 0
$$

The pulse $f(-x - vt)$ is not inverted, but is reflected left-to-right.

Transmitted and Reflected Waves at a Boundary

If two ropes of different linear mass densities, μ_1 and μ_2 are attached together (under the same tension), an incoming pulse will be partially transmitted and partially reflected.

$\mu_1 < \mu_2$



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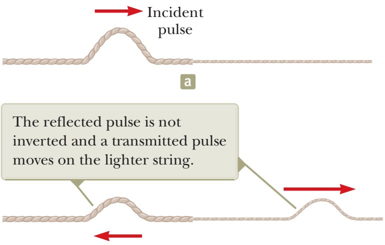

[1]Serway & Jewett, page 495.

Transmitted and Reflected Waves at a Boundary

The boundary conditions in this case are different again:

Now the y, the displacement, and $\frac{\partial y}{\partial x}$, slope of the string, must be continuous at the boundary.

Transmitted and Reflected Waves at a Boundary

Suppose $y_i(x,t) = f(x - v_1 t)$.

Then the reflected wave is

$$y_r = a_r\, f(-x - v_1 t)$$

and the transmitted wave is

$$y_t = a_t\, f\left(\frac{v_1}{v_2}(x - v_2 t)\right) \,.$$

a_r is the *reflection coefficient* (which is negative if the wave function is inverted) and a_t is the *transmission coefficient*.

To apply the boundary condition on the displacements:

$$\begin{aligned} y_i|_{x=0} + y_r|_{x=0} &= y_t|_{x=0} \\ f(-v_1 t) + a_r\, f(-v_1 t) &= a_t\, f(-v_1 t) \\ 1 + a_r &= a_t \end{aligned}$$

Wave Reflection from a freely movable end point

And using the boundary condition:

$$\left.\frac{\partial y_i(x,t)}{\partial x}\right|_{x=0} + \left.\frac{\partial y_r(x,t)}{\partial x}\right|_{x=0} = \left.\frac{\partial y_t(x,t)}{\partial x}\right|_{x=0}$$

The reflection and transmission coefficients can be found:

$$a_r = \frac{v_2 - v_1}{v_1 + v_2}$$

$$a_t = \frac{2v_2}{v_1 + v_2}$$

The height and width of the reflected and transmitted pulses are determined by the waves speeds (or equivalently, the string mass densities) on either side of the boundary.

Summary

- interference of sine waves with same freq, different amplitudes
- boundary conditions
- reflection and transmission

Test this Friday (TBC).

Homework

- WebAssign due tonight
- more WebAssigns posted