Waves
Standing Waves
Sound Waves

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Last time

- boundaries, reflection and transmission
- standing waves

Overview

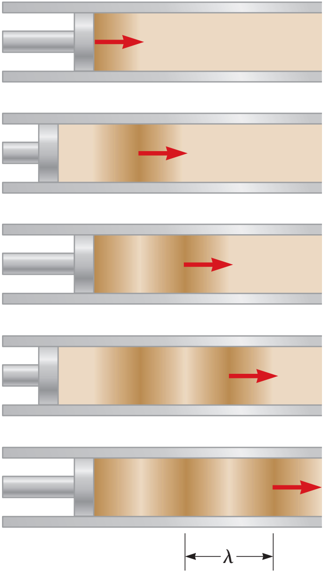
- sound
- displacement and pressure
- speed of sound
- interference of sound waves (?)

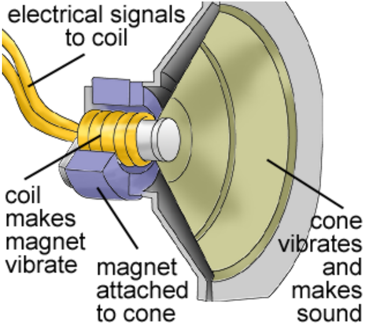
Sound Waves

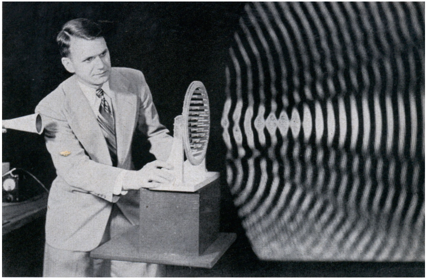
An important application of standing waves is the creation of musical instruments.

Before looking into that, we will understand sound as a longitudinal wave that causes pressure variations in air or other substances. (Ch 17.)

Pressure Variations







Sound Waves

Sound wave are longitudinal, so we imagine thin slices of air being displaced left and right along the direction of propagation of the wave (the x-axis).

This is similar to what we did to derive the wave equation considering a chain of masses connected by springs.

Now let s be the magnitude of the left-right displacement of a thin slice of air from its equilibrium position.

For a pulse wave function:

$$s(x,t) = f(x - vt)$$

For a sine-type wave function:

$$s(x,t) = s_{\max} \cos(kx - \omega t)$$

Sound Waves: Displacement and Pressure Variation

We wish to relate the displacement of slices of air to the pressure variations in the air that they cause.

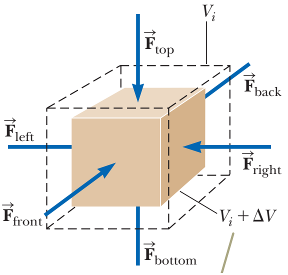
The relation between pressure and volume changes in air (at constant temperature) is characterized by the *bulk modulus*.

Bulk Modulus: Volume Elasticity

Bulk modulus, B (or sometimes K)

The ratio of the pressure change over the outside of a material to its fractional change in volume.

$$B = -\frac{\Delta P}{\Delta V / V_i}$$



The negative sign ensures B will be a positive number. Units are Pascals, Pa.

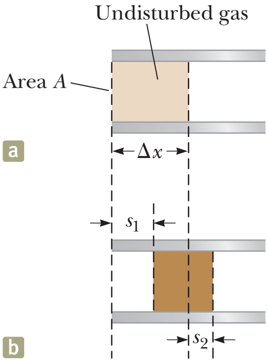
The reciprocal of the bulk modulus, $1/B$, is the **compressibility** of the material.

[1] See Serway & Jewett, Chapter 12 section 4.

Pressure and Sound

From the definition

$$\Delta P = -B \frac{\Delta V}{V_i}$$



For a column of air of cross sectional area A:

$$V_i = A\,\Delta x \quad , \quad \Delta V = A\,\Delta s$$

$$\Delta s = s_2 - s_1$$

Then letting $\Delta x \to 0$

$$\Delta P = -B \frac{\partial s}{\partial x}$$

Pressure and Sound

$$\Delta P = -B \frac{\partial s}{\partial x}$$

Recalling for a sine-type wave: $s(x, t) = s_{\text{max}} \cos(kx - \omega t)$,

$$\Delta P = B\, s_{\text{max}} k \sin(kx - \omega t)$$

Look at $B\, s_{\text{max}} k$. The units are:

$$[\text{Pa}]\ [\text{m}]\ [\text{m}^{-1}] = [\text{Pa}]$$

So, $B\, s_{\text{max}} k$ is a pressure.

Let

$$\Delta P_{\text{max}} = B\, s_{\text{max}} k$$

Pressure and Sound

We can now express sound as a pressure wave:

$$\Delta P(x,t) = (\Delta P_{\text{max}}) \sin(kx - \omega t)$$

ΔP is the variation of the pressure from the ambient (background) pressure.

If the sound wave is in air at sea level, the background pressure is $P_0 = 1.013 \times 10^5$ Pa. ΔP will be much smaller than this!

Example

A sound wave in air has a sinusoidal pressure variation, measured in Pascals:

$$\Delta P(x, t) = 0.900 \sin(10\pi x - 3430\pi t)$$

with the wavenumber in m^{-1} and the angular frequency in rad/s.

What is the wavelength and frequency of this wave?

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Write the corresponding displacement wave expression for this sound wave. ($B_{\mathrm{air}} = 1.42 \times 10^5$ Pa)

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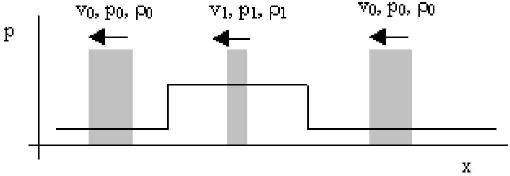
$$s(x, t) = (202 \text{ nm}) \cos(kx - \omega t)$$

Speed of Sound waves

As we did for waves on a string, imagine a pulse moving to the right.

Now let us choose a reference frame where we **move with the pulse** and the air moves back to the left.



call $\leftarrow$ +ve

The speed of the air outside the pulse is v. (This is the speed of sound relative to the air.)

Speed of Sound waves

Think about how the speed of the air thin packet changes as it moves into the higher-pressure pulse and is compressed.

It goes $v \rightarrow v + \Delta v$, where Δv is a *negative* number (it slows).

The relative volume change can be related to the speed change:

$$\frac{\Delta V}{V} = \frac{A \Delta v \, \Delta t}{A v \, \Delta t} = \frac{\Delta v}{v}$$

Now use Newton's 2nd law:

$$\begin{aligned}
F_{net} &= (\Delta m) a \\
PA - (P + \Delta P)A &= (\rho A \, \Delta x) \left(\frac{\Delta v}{\Delta t} \right) \\
\Delta P \cancel{A} &= -\rho \cancel{A} \, v \, \cancel{\Delta t} \left(\frac{\Delta v}{\cancel{\Delta t}} \right) \\
\Delta P &= -\rho v^2 \, \frac{\Delta v}{v}
\end{aligned}$$

Speed of Sound waves

Rearranging:

$$\rho v^2 = -\frac{\Delta P}{\Delta v / v}$$

Using $\frac{\Delta V}{V} = \frac{\Delta v}{v}$:

$$\rho v^2 = -\frac{\Delta P}{\Delta V / V}$$

And noticing that the LHS is the definition of B:

$$\rho v^2 = B$$

The speed of sound

$$v = \sqrt{\frac{B}{\rho}}$$

Speed of Sound waves

$$v = \sqrt{\frac{B}{\rho}}$$

Compare this expression to the speed of a pulse on a string.

$$v = \sqrt{\frac{T}{\mu}}$$

Both of these expressions can be thought of as:

$$v = \sqrt{\frac{\text{elastic quantity}}{\text{inertial quantity}}}$$

These expressions are the same in spirit, but the precise quantities are the ones that represent elasticity and inertia in each case.

Speed of Sound in Air

For air the adiabatic bulk modulus

$$B = 1.42 \times 10^5 \text{ Pa}$$

and

$$\rho = 1.2041 \text{ kg/m}^3$$

at 20°C.

Speed of Sound in Air

For air the adiabatic bulk modulus

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and

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at 20°C.

This gives a speed of sound in air at 20°C of

$$v = 343 \text{ m/s}$$

This is approximately 1/3 km/s or 1/5 mi/s.

Speed of Sound in Air

The speed of sound in air at 20°C

$$v = 343 \text{ m/s}$$

Since the density of air varies a lot with temperature, the speed of sound varies also.

For temperatures near room temperature:

$$v = (331 \text{ m/s})\sqrt{1 + \frac{T_{\text{Cel}}}{273}}$$

where T_{Cel} is the temperature in **Celsius**.

Summary

- sound
- displacement and pressure
- speed of sound
- interference of sound waves (?)

Homework Serway & Jewett (suggested):

- Ch 17, onward from page 523. OQs: 1, 7; CQs: 5; Probs: 16
- Ch 21, page 650, problem 56. (sound and adiabatic processes)