Waves
Sound and Interference

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Last time

- sound
- displacement and pressure variation
- started speed of sound

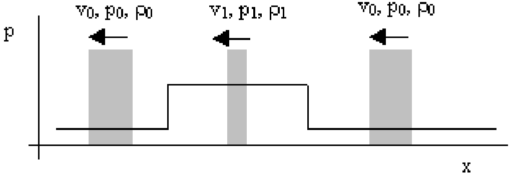
Overview

- finish finding speed of sound
- interference and sound
- standing waves and sound (?)

Speed of Sound waves

As we did for waves on a string, imagine a pulse moving to the right.

Now let us choose a reference frame where we **move with the pulse** and the air moves back to the left.



call $\leftarrow$ +ve

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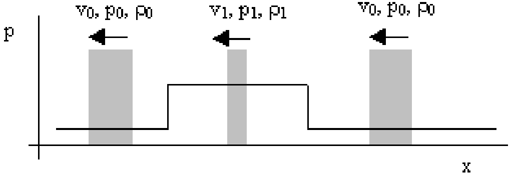


call ← +ve

The speed of the air outside the pulse is v. (This is the speed of sound relative to the air.)

Speed of Sound waves

Think about how the speed of the air thin packet changes as it moves into the higher-pressure pulse and is compressed.

It goes $v \to v + \Delta v$, where Δv is a *negative* number (it slows).

The relative volume change can be related to the speed change:

$$\frac{\Delta V}{V} = \frac{A \Delta v \, \Delta t}{A v \, \Delta t} = \frac{\Delta v}{v}$$

Now use Newton's 2nd law:

$$\begin{aligned}
F_{net} &= (\Delta m) a \\
PA - (P + \Delta P)A &= (\rho A \, \Delta x) \left(\frac{\Delta v}{\Delta t} \right) \\
\Delta P \cancel{A} &= -\rho \cancel{A} \, v \, \cancel{\Delta t} \left(\frac{\Delta v}{\cancel{\Delta t}} \right) \\
\Delta P &= -\rho v^2 \, \frac{\Delta v}{v}
\end{aligned}$$

Speed of Sound waves

Rearranging:

$$\rho v^2 = -\frac{\Delta P}{\Delta v/v}$$

Using $\frac{\Delta V}{V} = \frac{\Delta v}{v}$:

$$\rho v^2 = -\frac{\Delta P}{\Delta V/V}$$

And noticing that the LHS is the definition of B:

$$\rho v^2 = B$$

The speed of sound

$$v = \sqrt{\frac{B}{\rho}}$$

Speed of Sound waves

$$v = \sqrt{\frac{B}{\rho}}$$

Compare this expression to the speed of a pulse on a string.

$$v = \sqrt{\frac{T}{\mu}}$$

Both of these expressions can be thought of as:

$$v = \sqrt{\frac{\text{elastic quantity}}{\text{inertial quantity}}}$$

These expressions are the same in spirit, but the precise quantities are the ones that represent elasticity and inertia in each case.

Speed of Sound in Air

For air the adiabatic bulk modulus

$$B = 1.42 \times 10^5 \text{ Pa}$$

and

$$\rho = 1.2041 \text{ kg/m}^3$$

at 20°C.

This gives a speed of sound in air at 20°C of

$$v = 343 \text{ m/s}$$

This is approximately 1/3 km/s or 1/5 mi/s.

Speed of Sound in Air

The speed of sound in air at 20°C

$$v = 343 \text{ m/s}$$

Since the density of air vary with temperature, the speed of sound varies also.

For temperatures near room temperature:

$$v = (331 \text{ m/s})\sqrt{1 + \frac{T_{\text{Cel}}}{273}}$$

where T_{Cel} is the temperature in **Celsius**.

[1]The bulk modulus depends on the pressure.

Pressure Waves

$$\Delta P(x,t) = \Delta P_{\text{max}} \sin(kx - \omega t)$$

where

$$\Delta P_{\text{max}} = B\, s_{\text{max}} k$$

It is easier to express the amplitude in terms of the wave speed, since it is usually easier to look up the wave speed than the bulk modulus:

$$\Delta P_{\text{max}} = (\rho v^2)\, s_{\text{max}} \frac{\omega}{v}$$

Then

$$\Delta P_{\text{max}} = \rho v \omega s_{\text{max}}$$

Sound Waves

Displacement:

$$s(x,t) = s_{\text{max}} \cos(kx - \omega t)$$

Pressure:

$$\Delta P(x,t) = \Delta P_{\text{max}} \sin(kx - \omega t)$$

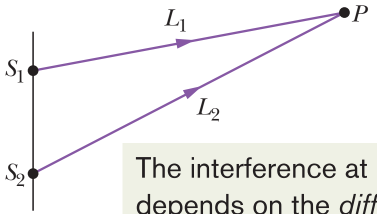
where

$$\Delta P_{\text{max}} = B\, s_{\text{max}} k = \rho v \omega s_{\text{max}}$$

Interference of Sound Waves

Imagine two point sources of sinusoidal sound waves that emit identical signals: same amplitude, wavelength, and phase.

$$\Delta P_1(x,t) = \Delta P_2(x,t) = \Delta P_{\text{max}} \sin(kx - \omega t)$$

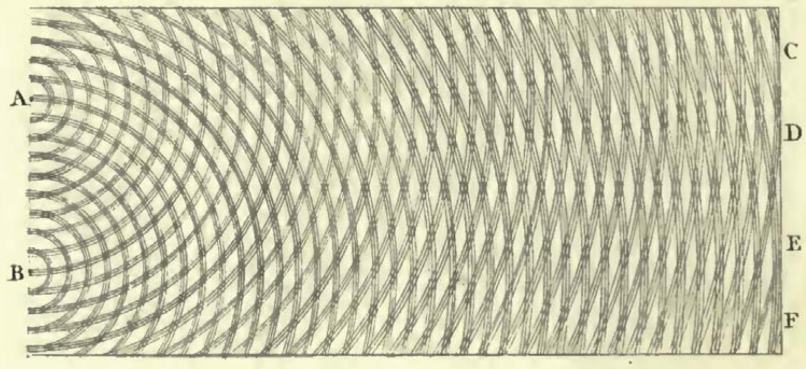


The interference at P depends on the *difference* in the path lengths to reach P.

The sound will be louder at different points, depending on the difference in the path lengths that the sound waves take.

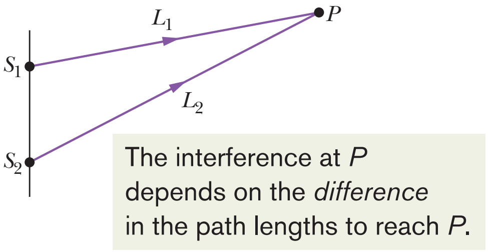
Interference of Sound Waves

Interference pattern from two sources, with equal wavelength and in phase:



[1] Thomas Young, On the Nature of Light and Colours, Lecture 39, Course of Lectures on Natural Philosophy and Mechanical Arts (London, 1897)

Interference of Sound Waves



This is because the path difference will correspond to a phase offset of the arriving waves at P:

$$\begin{aligned}
\Delta P &= \Delta P_1 + \Delta P_2 \\
&= \Delta P_{\text{max}}(\sin(kL_1 - \omega t) + \sin(kL_2 - \omega t)) \\
&= \underbrace{\left[2\Delta P_{\text{max}} \cos\left(\frac{k(L_2 - L_1)}{2}\right)\right]}_{\textbf{new amplitude}} \sin\left(\frac{k(L_2 + L_1)}{2} - \omega t\right)
\end{aligned}$$

Interference of Sound Waves

The new amplitude could be written as:

$$2\Delta P_{\max}\cos\left(\frac{\pi(L_2 - L_1)}{\lambda}\right)$$

When $|L_2 - L_1| = n\lambda$ and $n = 0, 1, 2, \ldots$ the sound from the two speakers is loudest (a maximum).

When $|L_2 - L_1| = \frac{(2n+1)\lambda}{2}$ and $n = 0, 1, 2, \ldots$ the sound from the two speakers is cancelled out (a minimum).

Summary

- speed of sound
- interference with sound
- standing waves and sound (?)