Waves
Standing Waves in Rods and Membranes
Beats

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Last time

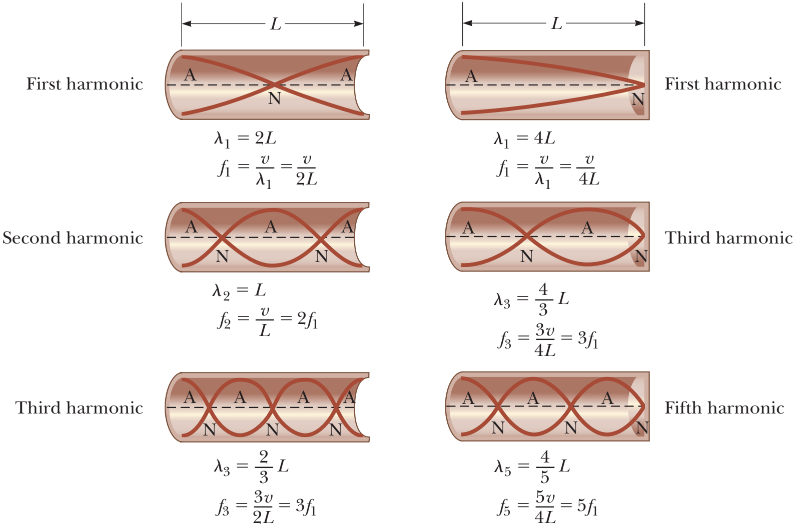
- interference example
- standing waves and sound

Overview

- musical instruments
- standing waves in rods and membranes
- beats
- nonsinusoidal waves

Standing Sound Waves in air columns - Displ. wave

Two open ends | One closed end



[1]Figure from Serway & Jewett, page 547.

Reminder: Standing Sound Waves in air columns

For double open ended tubes:

The wavelengths of the normal modes are given by the constraint $|\cos(0)| = |\cos(kL)| = 1$:

$$\lambda_n = \frac{2L}{n}$$

where n is a positive natural number (1, 2, 3...).

The **natural frequencies**:

$$f_n = \frac{nv}{2L} = n\, f_1$$

where n is a positive natural number.

The **fundamental frequency**, also called the **first harmonic** is the lowest frequency sound produced in the column. It is

$$f_1 = \frac{v}{2L}$$

Reminder: Standing Sound Waves in air columns

For tubes with one closed end: **fundamental frequency** $f_1 = \frac{v}{4L}$

The wavelengths of the normal modes are given by the constraint $\cos(0) = 1$, $|\cos(kL)| = 0$:

$$\lambda_{2n-1} = \frac{4L}{(2n-1)}$$

where n is a positive natural number (1, 2, 3...).

The **natural frequencies**:

$$f_{2n-1} = \frac{(2n-1)v}{4L} = (2n-1)f_1$$

where n is a positive natural number (1, 2, 3...).

(For this case the # of nodes = # of antinodes = n.)

Musical Instruments

Didgeridoo:

Longer didgeridoos have lower pitch, but tubes that flare outward have higher pitches this can also change the spacing of the resonant frequencies.

[1]Matt Roberts via Getty Images.

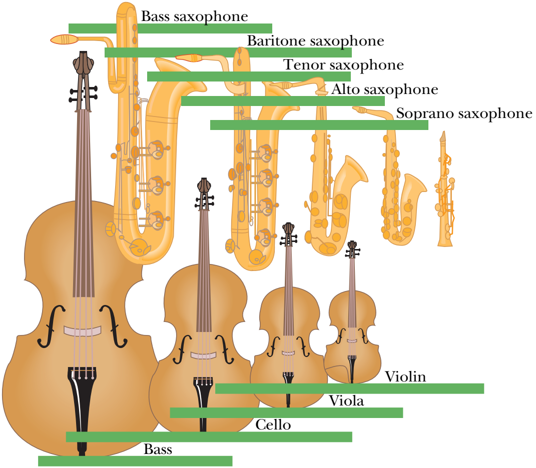
Musical Instruments, Pipe Organ

The longest pipes made for organs are open-ended 64-foot stops (tube is effectively 64 feet+ long). There are two of them in the world. The fundamental frequency associated with such a pipe is 8 Hz.

32' stops give 16 Hz sound, 16' stops give 32 Hz, 8' stops give 64 Hz, *etc*.

[0]Picture of Sydney Town Hall Grand Organ from Wikipedia, user Jason7825.

Musical Instruments



In general, larger instruments can create lower tones, whether string instruments or tube instruments.

[0] Halliday, Resnick, Walker, 9th ed, page 458.

Reminder: Speed of Sound in Air

The speed of sound in air at 20°C

$$v = 343 \text{ m/s}$$

Since the density of air varies a lot with temperature, the speed of sound varies also.

For temperatures near room temperature:

$$v = (331 \text{ m/s})\sqrt{1 + \frac{T_{\text{Cel}}}{273}}$$

where T_{Cel} is the temperature in **Celsius**.

Question

Quick Quiz 18.5[1] Balboa Park in San Diego has an outdoor organ. When the air temperature increases, the fundamental frequency of one of the organ pipes

(A) stays the same,

(B) goes down,

(C) goes up,

(D) is impossible to determine.

[1] Serway & Jewett, page 548.

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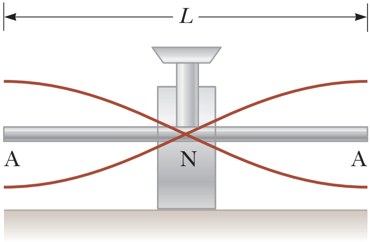
Standing waves in rods

Both longitudinal and transverse standing waves can be created in rods.

Illustration of a longitudinal standing oscillation in a rod, free at the ends, and clamped in the middle:



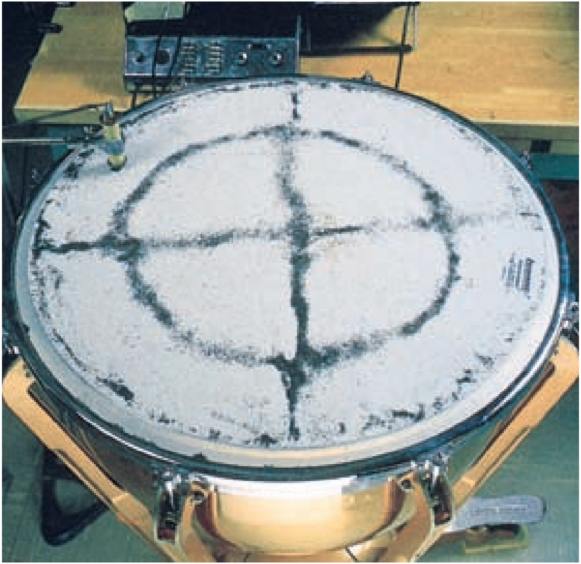
$$\lambda_1 = 2L$$

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

The brown curve represents left-right displacement of the particles in the rod.

Some musical instruments make use of transverse standing waves, *eg.* triangle, glockenspiel, chimes.

Standing waves in membranes

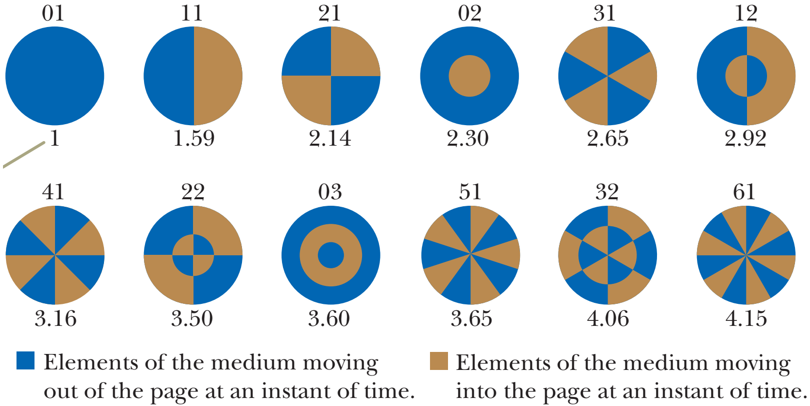


[1]Dust on a kettledrum, Halliday, Resnick, Walker, page 434.

Standing waves in membranes

The standing solutions are called Bessel functions, specifically, cylindrical functions.

Standing waves in membranes



[1] Serway & Jewett, page 550.

Beats

We already considered interference of sine waves when both waves had the same frequency. But what if they do not?

Consider two waves with the same amplitude but different frequencies, f, and therefore different angular frequencies, ω:

$$\begin{aligned} y_1(x,t) &= A\sin(k_1 x - \omega_1 t + \phi_1) \\ y_2(x,t) &= A\sin(k_2 x - \omega_2 t + \phi_2) \end{aligned}$$

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Now let's consider the effect of these waves at the point $x = 0$, and suppose $\phi_1 = \phi_2 = \frac{\pi}{2}$.

(This choice is arbitrary, but we must pick a point in space.)

Beats

The wave functions at this point are:

$$
\begin{aligned}
y_1(0,t) &= A\sin(\frac{\pi}{2} - \omega_1 t) = A\cos(2\pi f_1 t) \\
y_2(0,t) &= A\sin(\frac{\pi}{2} - \omega_2 t) = A\cos(2\pi f_2 t)
\end{aligned}
$$

Using the trig identity:

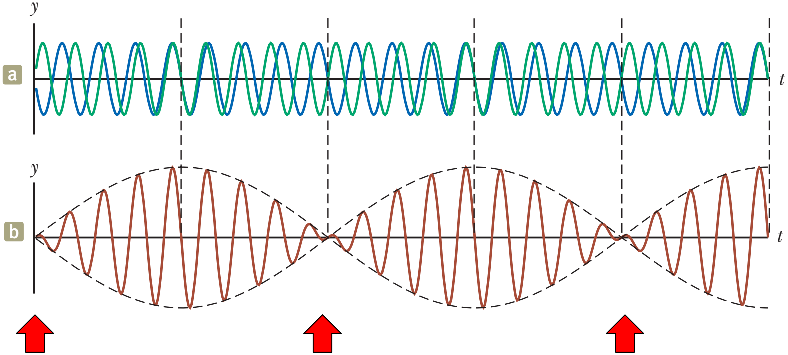
$$
\cos\theta + \cos\psi = 2\cos\left(\frac{\theta - \psi}{2}\right)\cos\left(\frac{\theta + \psi}{2}\right)
$$

$$
\begin{aligned}
y(x,t) &= y_1 + y_2 \\
&= \underbrace{\left[2A\cos\left(2\pi\,\frac{f_1 - f_2}{2}\,t\right)\right]}_{\textbf{time-varying amplitude}}\underbrace{\cos\left(2\pi\frac{f_1 + f_2}{2}t\right)}_{\textbf{fast oscillation}}
\end{aligned}
$$

Beats

$$y(x,t) = \left[2A\cos\left(2\pi\,\frac{f_1 - f_2}{2}\,t\right)\right]\cos\left(2\pi\frac{f_1 + f_2}{2}t\right)$$

y vs t (position, x fixed):



$t = \frac{1}{2(f_1 - f_2)}$ $t = \frac{3}{2(f_1 - f_2)}$ $t = \frac{5}{2(f_1 - f_2)}$

Beats

The time difference between minima is $\Delta t = \frac{1}{|f_1 - f_2|}$.

Thus the frequency of the beats is

$$f_{\text{beat}} = |f_1 - f_2|$$

If f_1 and f_2 are similar the beat frequency is much smaller than either f_1 or f_2.

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Humans cannot hear beats if $f_{\text{beat}} \gtrsim 30$ Hz.

If the two frequencies are very different we hear a chord.

If the two frequencies are very close, we hear periodic variations in the sound level.

This is used to tune musical instruments. When instruments are coming into tune with each other the beats get less and less frequent, and vanish entirely when they are perfectly in tune.

Question

A tuning fork is known to vibrate with frequency 262 Hz. When it is sounded along with a mandolin string, four beats are heard every second. Next, a bit of tape is put onto each tine of the tuning fork, and the tuning fork now produces five beats per second with the same mandolin string. What is the frequency of the string?

(A) 257 Hz

(B) 258 Hz

(C) 266 Hz

(D) 267 Hz

[1] Serway & Jewett, objective question 7.

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Nonsinusoidal Periodic Waves

Not all periodic wave functions are pure, single-frequency sinusoidal functions.

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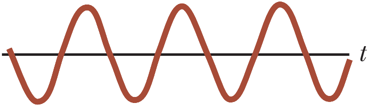
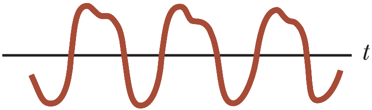
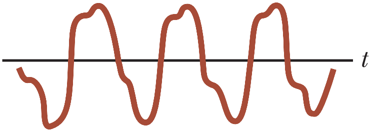


Tuning fork



Flute



Clarinet

For example this is why a flute and a clarinet playing the same note still sound a bit different.

Other harmonics in addition to the fundamental are sounded.

Summary

- musical instruments
- standing waves in rods and membranes
- beats

Test Tuesday, June 9.

Homework Serway & Jewett (suggested):

- Ch 17, onward from page 523. Probs: 30
- Ch 18, onward from page 555. Probs: 60