Waves
Nonsinusoidal Periodic Waves
Intensity

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Last time

- standing waves in rods and membranes
- beats

Overview

- nonsinusoidal waves & Fourier components
- intensity of a wave
- sound level
- the Doppler effect

Nonsinusoidal Periodic Waves

Not all periodic wave functions are pure, single-frequency sinusoidal functions.

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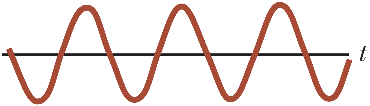
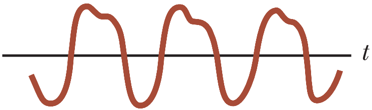
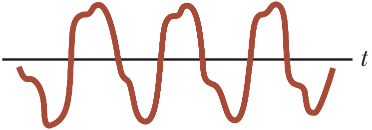


Tuning fork



Flute



Clarinet

For example this is why a flute and a clarinet playing the same note still sound a bit different.

Other harmonics in addition to the fundamental are sounded.

Nonsinusoidal Periodic Waves

How do these patterns come about physically?

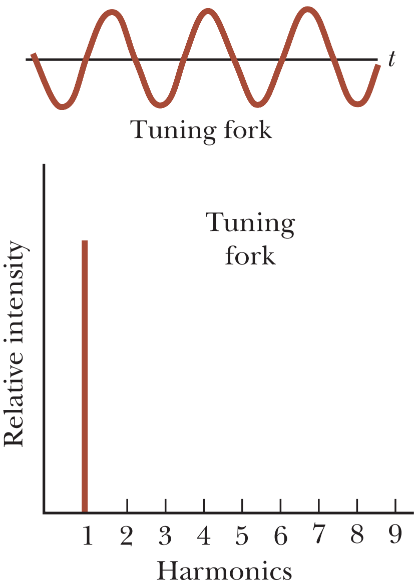
They are made up of standing sound waves in the columns of the instruments.

The first harmonic dominates, but the second, third, fourth, and higher harmonics are also permitted.

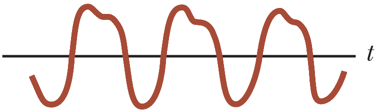
Interference between these higher harmonics and the first harmonic creates these more elaborate patterns.

$$y(t) = A_1 \cos(2\pi f_1 t) + A_2 \cos(2\pi f_2 t) + A_3 \cos(2\pi f_3 t) + \ldots$$

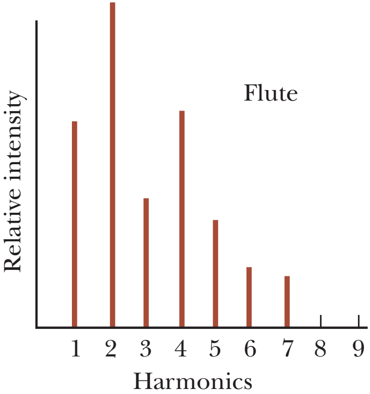
Nonsinusoidal Periodic Waves



Nonsinusoidal Periodic Waves

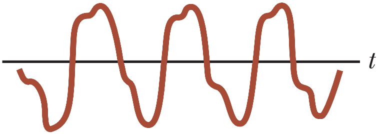


Flute

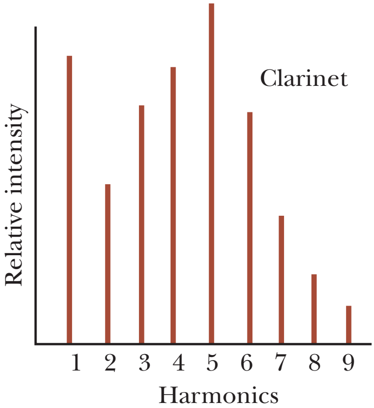


(Supposedly)

Nonsinusoidal Periodic Waves



Clarinet



Fourier's Theorem

These particular periodic functions created by instruments can be expressed as sums of harmonics. What about other periodic functions?

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Any periodic function (that is piecewise continuous) can be represented as a **discrete sum** of sine and cosine functions of the form:

$$y(t) = \frac{1}{2}A_0 + \sum_{n=1}^{\infty}\left(A_n \cos(2\pi nft) + B_n \sin(2\pi nft)\right)$$

Some of the A's and/or B's may be zero.

This is called a **Fourier series**.

Fourier's Theorem

Why does this work?

Sine and cosine functions of the form $\sin(nx)$ and $\cos(nx)$ where n is any positive integer form a complete *othonogonal set* of functions.

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If $n \neq m$ (n and m are integers):

$$\int_{-\pi}^{\pi} \sin(mx)\cos(nx)\,\mathrm{dx} = \int_{-\pi}^{\pi} \sin(mx)\sin(nx)\,\mathrm{dx} = \\ = \int_{-\pi}^{\pi} \cos(mx)\cos(nx)\,\mathrm{dx} = 0$$

and

$$\int_{-\pi}^{\pi} \sin(nx)\cos(nx)\,\mathrm{dx} = 0$$

In this sense they are orthogonal.

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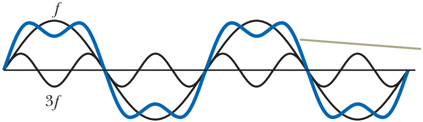
Meanwhile:

$$\int_{-\pi}^{\pi} \sin(nx)\sin(nx)\,\mathrm{dx} = \int_{-\pi}^{\pi} \cos(nx)\cos(nx)\,\mathrm{dx} = \pi$$

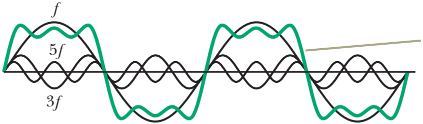
This makes them work like a set of independent directions. Just like *any* vector in 3-dimensional space can be represented as a sum of 3 components, *any* periodic function can be represented by a sum of components of these functions.

Example: Square Wave

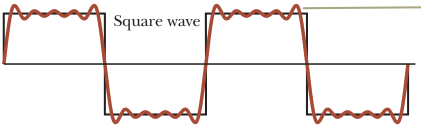
$$\sin(2\pi \boldsymbol{f}\ t) + \frac{1}{3}\sin(2\pi \boldsymbol{3f}\ t)$$



$$\sin(2\pi \boldsymbol{f}\ t) + \frac{1}{3}\sin(2\pi \boldsymbol{3f}\ t) + \frac{1}{5}\sin(2\pi \boldsymbol{5f}\ t)$$



Sum of all terms up to $9f$.



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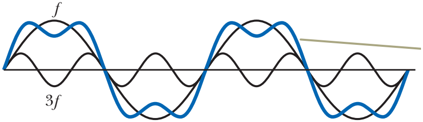

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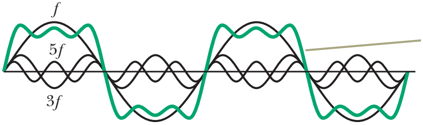

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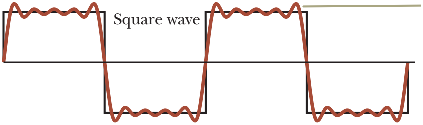

For a square wave (amplitude $\frac{1}{2}$):

$$y(t) = \frac{2}{\pi}\left(\sin(2\pi f\ t) + \frac{1}{3}\sin(2\pi 3f\ t) + \frac{1}{5}\sin(2\pi 5f\ t) + \ldots\right)$$

Fourier's Theorem

What about non-periodic functions? Wave pulses, for example?

The idea of a Fourier series can be extended, but now it is not enough to consider just terms like $\sin(nx)$ where n is a positive integer.

We need to "sum" over a continuous range of values for n.

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This becomes a **Fourier transform**.

$$y(t) = \int_{-\infty}^{\infty} g(f) e^{2\pi i f t} \, \mathrm{df}$$

$g(f)$ gives "amplitudes" as a complex-valued function of frequency.

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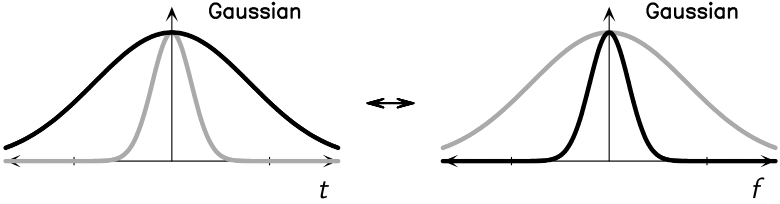
$$e^{inx} = \cos nx + i \sin nx$$

Fourier's Theorem



Narrow $\rightarrow$ broad
Broad $\rightarrow$ narrow

[1]Figure from the National Radio Astronomy Observatory, Charlottesville, website.

Intensity of a Wave

Intensity

the average power of a wave per unit area

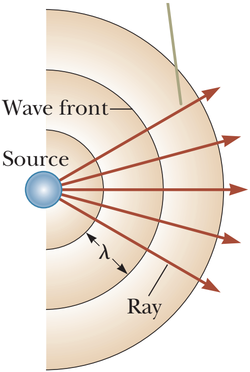
$$I = \frac{P_{\text{avg}}}{A}$$

Intensity is used for waves that move on 3 dimensional media, such as sound or light.

The waves travel in one direction, and the area A is arranged perpendicular to the direction of the wave travel.

Intensity of a Waves from Point Sources

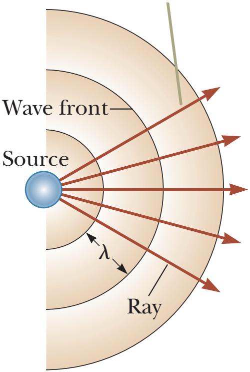
When a point source emits waves the waves propagate outward with spherical wave fronts.



Rays are directed lines that trace out the direction of travel of the wave.

Each surface moving out has larger area than the last: $A = 4\pi r^2$

Intensity of a Waves from Point Sources



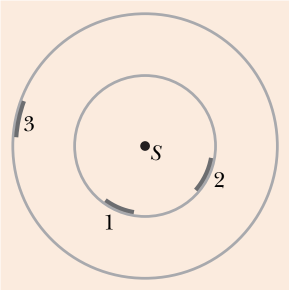
At a distance r the intensity is

$$I = \frac{P_{\text{avg}}}{4\pi r^2}$$

[1]Figure from Serway & Jewett, page 513.

Intensity Question

The figure indicates the location of three small patches 1, 2, and 3 that lie on the surfaces of two imaginary spheres; the spheres are centered on an isotropic point source S of sound. **The rates at which energy is transmitted through the three patches by the sound waves are equal.** Rank the patches according to the intensity of the sound on them, greatest first.



(A) 1, 2, 3

(B) (1 and 2), 3

(C) 3, (1 and 2)

(D) all the same

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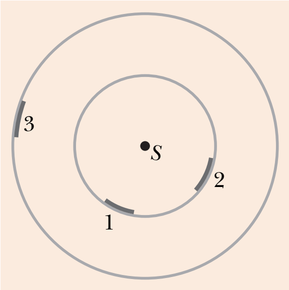

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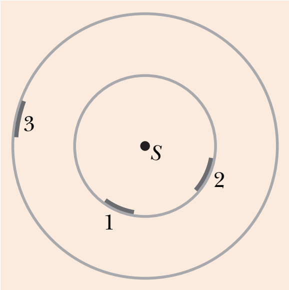

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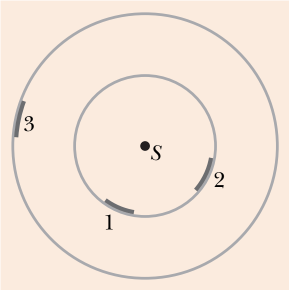

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Power and Intensity of Sound Waves

$$\text{Power} = \mathbf{F} \cdot \mathbf{v}$$

Consider a sound wave traveling in the x direction. $\mathcal{A}$ **is area.**

$\mathbf{F} = (\Delta P)\mathcal{A}\mathbf{i}$ and $\mathbf{v} = \frac{\partial s}{\partial t}\mathbf{i}$

$$\begin{aligned}
\text{Power} &= (\Delta P)\mathcal{A}\frac{\partial}{\partial t}\left(s_{\text{max}}\cos(kx - \omega t)\right) \\
&= \rho v \omega \mathcal{A} s_{\text{max}} \sin(kx - \omega t)\left(\omega s_{\text{max}} \sin(kx - \omega t)\right) \\
&= \rho v \omega^2 \mathcal{A}\, s_{\text{max}}^2 \sin^2(kx - \omega t)
\end{aligned}$$

Power and Intensity of Sound Waves

$$\text{Power} = \rho v \omega^2 \mathcal{A}\, s_{\text{max}}^2 \sin^2(kx - \omega t)$$

To find the average power, we need to average this power arriving at a point over a full cycle, time period T.

Consider a fixed position so that x is a constant.

$$\begin{aligned}
\text{Power}_{\text{avg}} &= \frac{1}{T}\int_0^T \left(\rho v \omega^2 \mathcal{A}\, s_{\text{max}}^2 \sin^2(kx - \omega t)\right) \text{dt} \\
&= \rho v \omega^2 \mathcal{A}\, s_{\text{max}}^2 \, \frac{1}{T}\int_0^T \sin^2(kx - \omega t)\, \text{dt} \\
&= \rho v \omega^2 \mathcal{A}\, s_{\text{max}}^2 \, \frac{1}{T}\int_0^T \frac{1}{2}(1 - \cos(2kx - 2\omega t))\, \text{dt}
\end{aligned}$$

Power of a sound wave:

$$\text{Power}_{\text{avg}} = \frac{1}{2}\rho v \omega^2 \mathcal{A}\, s_{\text{max}}^2$$

Power and Intensity of Sound Waves

Power of a sound wave:

$$\text{Power}_{\text{avg}} = \frac{1}{2}\rho\,\mathcal{A}\,\omega^2\,s_{\text{max}}^2 v$$

Dividing this by the area gives the intensity of a sound arriving on that area:

$$I = \frac{1}{2}\rho v(\omega s_{\text{max}})^2$$

This can be written in terms of the pressure variation amplitude, $\Delta P_{\text{max}} = \rho v \omega s_{\text{max}}$:

$$I = \frac{(\Delta P_{\text{max}})^2}{2\rho v}$$

Summary

- Fourier components and nonsine waveforms
- intensity
- sound level

5th Test Tuesday, June 9.

Homework Serway & Jewett (suggested, same as yesterday):

- Ch 17, onward from page 523. Probs: 30
- Ch 18, onward from page 555. Probs: 60