Optics
Image Formation by Refracting Surfaces and Lenses

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Last time

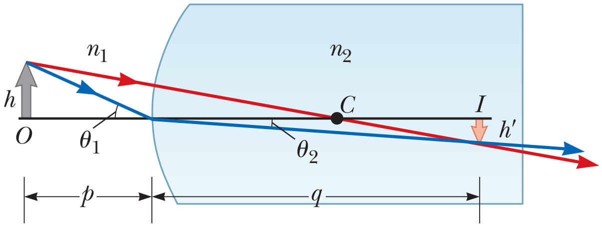
- images formed by spherical mirrors
- refracting surfaces

Overview

- refracting surfaces magnification
- lenses
- images formed by lenses

Refracting Surface Example (Problem 34)

A curved refracting surface separates a material with index of refraction n_1 from a material with index n_2. Prove that the magnification is given by $M = -\frac{n_1 q}{n_2 p}$.



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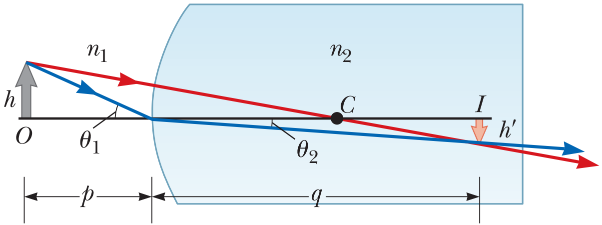

Hint: For paraxial rays, we assume θ_1 and θ_2 are small, so we may write Snell's Law as $n_1 \tan\theta_1 = n_2 \tan\theta_2$.

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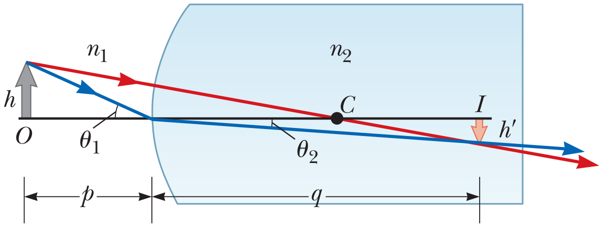


Hint: For paraxial rays, we assume θ_1 and θ_2 are small, so we may write Snell's Law as $n_1 \tan\theta_1 = n_2 \tan\theta_2$.

For small angles:

$$\begin{aligned} n_1 \tan\theta_1 &= n_2 \tan\theta_2 \\ n_1 \frac{h}{p} &= n_2 \frac{(-h')}{q} \quad \Rightarrow \quad M = \frac{h'}{h} = -\frac{n_1 q}{n_2 p} \end{aligned}$$

Images Formed by Refraction

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

See Lab 7 lecture for some more examples.

Images Formed by Thin Lenses

In lab we derived the **thin lens equation**

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

(Notice it is the same as the mirror equation!)

And the **lens maker's equation**

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

We did this by considering each side of the lens as a refracting surface.

Images Formed by Refraction

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

Reminder: Magnification with a Lens

By definition,

$$M = \frac{h'}{h}$$

And it follows from simple trigonometry that

$$M = -\frac{q}{p}$$

Same as for a mirror!

Sign Conventions for Lenses!

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Variable	is Positive	is Negative
p	object in front of surface	[virtual object][1]
q	image behind lens (real)	image in front of lens (virtual)
h' (and M)	image upright	image inverted
R_1 and R_2	object faces convex surf. (C behind surface)	object faces concave surf. (C in front of surface)
f	lens is **converging**	lens is **diverging**

[1]Useful in derivations.

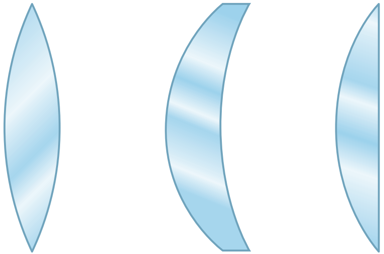
Understanding the Sign of f

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Always $n \geqslant 1$, so $(n-1)$ is positive (if zero, there's no lens!).

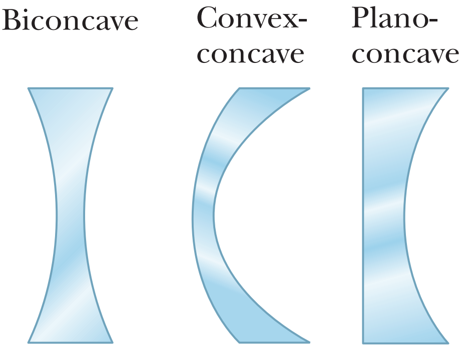
$\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ could be positive or negative, depending on the signs and magnitudes of R_1 and R_2.

Sign Examples: Converging Lenses



Biconvex	Convex-concave	Plano-convex
R_1 R_2	R_1 R_2	R_1 R_2
R_1 +ve	R_1 +ve	R_1 +ve
R_2 −ve	R_2 +ve	R_2 ∞
	$R_1 < R_2$	
f +ve	f +ve	f +ve

Sign Examples: Diverging Lenses



R_1 −ve	R_1 +ve	R_1 ∞
R_2 +ve	R_2 +ve	R_2 +ve
	$R_1 > R_2$	
f −ve	f −ve	f −ve

Focal Length Question

Quick Quiz 36.6 What is the focal length of a pane of window glass?

(A) zero
(B) infinity
(C) the thickness of the glass
(D) impossible to determine

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Ray Diagrams for Converging Lenses

Again, draw rays whose behavior we know.

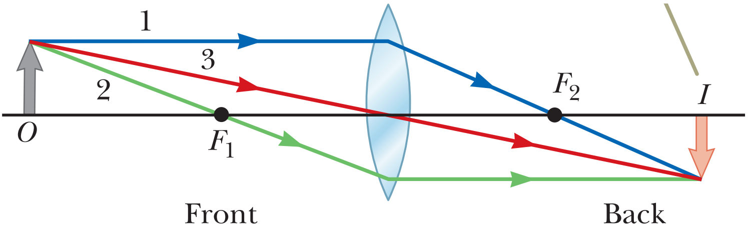
Rays parallel to the principle axis are refracted through the focal point.

Rays that travel through the center of the lens are (effectively) not refracted.

For Converging Lenses:

1. Draw a ray from the top of the object parallel to the principle axis refracted through the focal point F.
2. Draw a ray from the top of the object through the focal point (or back to the focal point) and refracted parallel to the principal axis.
3. Draw a ray from the top of the object through the center of the lens, and continuing in a straight line.

Ray Diagrams for Converging Lenses



Ray Diagrams for Converging Lenses

There are two cases of interest:

Object is beyond focal point

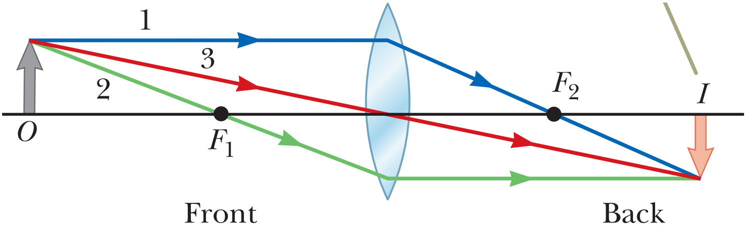


Image is real and inverted.

If $p > 2f$, the image is diminished. If $f < p < 2f$ the image is enlarged.

Object Beyond Focal Point

The object is the Sun.

[1]Photo from
http://www.mahalo.com/how-to-start-a-fire-with-a-magnifying-glass

Ray Diagrams for Converging Lenses

Object is closer than the focal point

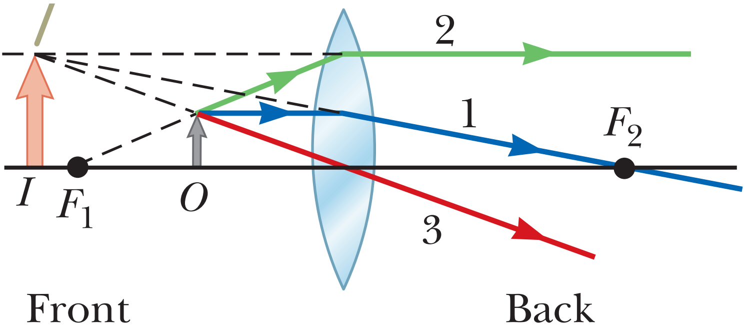


Image is virtual, upright, and magnified.

Object Closer Than the Focal Point

The object is the stamp.

Summary

- refracting surfaces magnification
- lenses
- images formed by lenses

Final Exam 9:15-11:15am, Tuesday, June 23.