Optics
Wave Behavior in Optics
Interference from Multiple Slits
Diffraction Gratings

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June 18, 2020

Last time

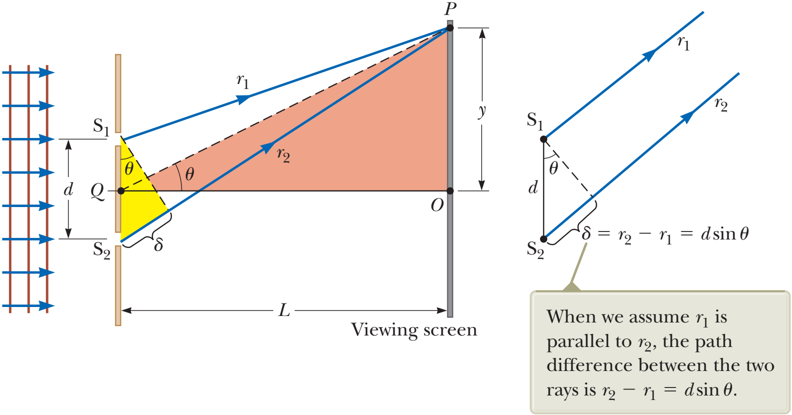
- images formed by lenses
- images formed by lens combinations

Overview

- Interference of light: the Double-Slit experiment
- multiple slit interference
- diffraction gratings

Young's Experiment: Finding the Maxima

Effectively, the two rays are parallel.



Looking at the right triangle with hypotenuse d (the slit separation distance): $\delta = d \sin\theta$.

Young's Experiment: Finding the Angles of the Maxima

Maxima (bright fringes) occur when

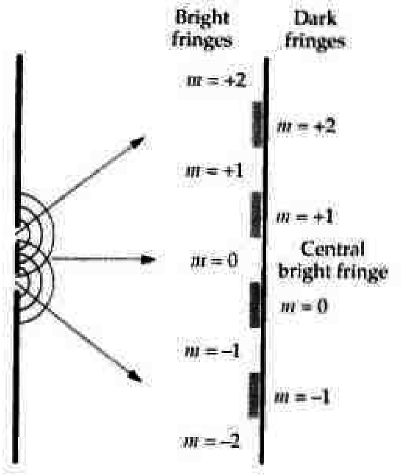
$$d \sin \theta_{\text{max}} = m\lambda \qquad \text{where } m \in \mathbb{Z}$$

Minima (dark fringes) occur when

$$d \sin \theta_{\text{min}} = \left(m + \frac{1}{2}\right)\lambda \qquad \text{where } m \in \mathbb{Z}$$

These expressions give us the angles (measured outward from the axis that passes through the midpoint of the slits) where the bright and dark fringes occur.

Order Number

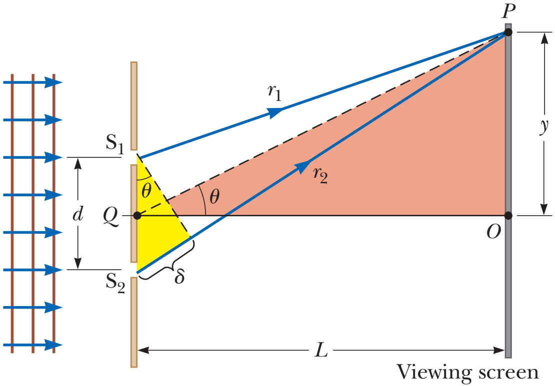


m is the order number. The central bright fringe is the "0th order fringe", the neighboring ones are the "1st order fringes", etc.

[1] Figure from Quantum Mechanics and the Multiverse by Thomas D. Le.

Young's Experiment: Finding the Position of the Maxima

We can also predict the distance from the center of the screen, y, in terms of the distance from the slits to the screen, L.



$$\tan \theta = \frac{y}{L}$$

Young's Experiment: Finding the Position of the Maxima

Maxima (bright fringes) occur at

$$y_{\text{max}} = L \tan \theta_{\text{max}}$$

Minima (dark fringes) occur at

$$y_{\text{min}} = L \tan \theta_{\text{min}}$$

Young's Experiment: Finding the Position of the Maxima

When θ is also small, $\sin\theta \approx \tan\theta$, and we can use our earlier expressions for the fringe angles.

Maxima (bright fringes) occur at

$$y_{\text{max}} = L\,\frac{m\lambda}{d} \qquad \text{(small } \theta\text{)}$$

Minima (dark fringes) occur at

$$y_{\text{min}} = L\,\frac{\left(m + \frac{1}{2}\right)\lambda}{d} \qquad \text{(small } \theta\text{)}$$

Young's Experiment Question

Quick Quiz 37.1[1] Which of the following causes the fringes in a two-slit interference pattern to move farther apart?

(A) decreasing the wavelength of the light
(B) decreasing the screen distance L
(C) decreasing the slit spacing d
(D) immersing the entire apparatus in water

[1] Serway & Jewett, page 1138.

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Double-Slit Fringes of Two Wavelengths, Ex 37.2

A light source emits visible light of two wavelengths: $\lambda = 430$ nm and $\lambda' = 510$ nm. The source is used in a double-slit interference experiment in which $L = 1.50$ m and $d = 0.0250$ mm.

Find the separation distance between the third-order bright fringes for the two wavelengths.

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$$\begin{aligned}\Delta y &= y'_{\text{max}} - y_{\text{max}} \\ &= L\,\frac{m\lambda'}{d} - L\,\frac{m\lambda}{d}\end{aligned}$$

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$$\begin{aligned}
\Delta y &= y'_{\text{max}} - y_{\text{max}} \\
&= L\,\frac{m\lambda'}{d} - L\,\frac{m\lambda}{d} \\
&= L\,\frac{m}{d}(\lambda' - \lambda) \\
&= \underline{1.44 \text{ cm}}
\end{aligned}$$

Young's Experiment: Intensity Distribution

Consider the electric field at a certain point P on the screen from each slit:

$$E_1 = E_0 \sin(\omega t)$$

$$E_2 = E_0 \sin(\omega t + \phi)$$

Young's Experiment: Intensity Distribution

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Using

$$\sin\alpha + \sin\beta = 2\cos\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)$$

We see that the net E-field at that point P is:

$$\begin{aligned} E_P &= E_1 + E_2 \\ &= \underbrace{\left[2E_0\cos\left(\frac{\phi}{2}\right)\right]}_{\textbf{Amplitude}} \underbrace{\sin\left(\omega t + \frac{\phi}{2}\right)}_{\textbf{Time-fluctuation}} \end{aligned}$$

Young's Experiment: Intensity Distribution

$$E_{P,\max} = 2E_0 \cos\left(\frac{\phi}{2}\right)$$

Relating E-field to intensity, recall (Ch. 34):

$$I \propto E_{P,\max}^2$$

So,

$$I = I_{\max} \cos^2\left(\frac{\phi}{2}\right)$$

The phase difference is related to δ, the path difference by:

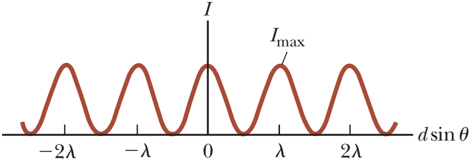
$$\frac{\phi}{2\pi} = \frac{\delta}{\lambda}$$

So, using $\delta = d\sin\theta$ (and $\phi = 2\pi d \sin\theta/\lambda$)

$$I = I_{\max} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

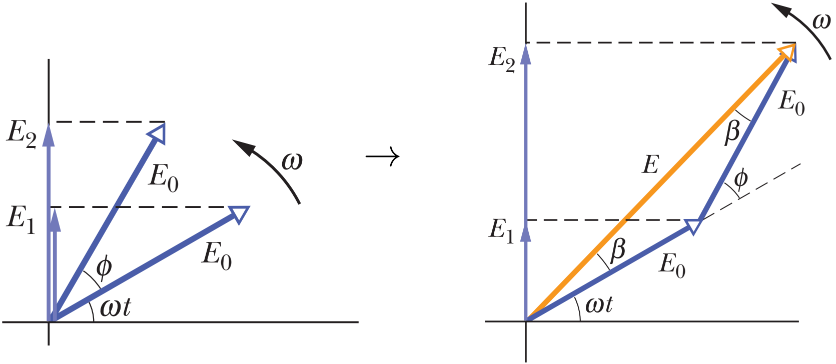
Young's Experiment: Intensity Distribution

$$I = I_{\mathrm{max}} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$



Young's Experiment: Intensity Distribution

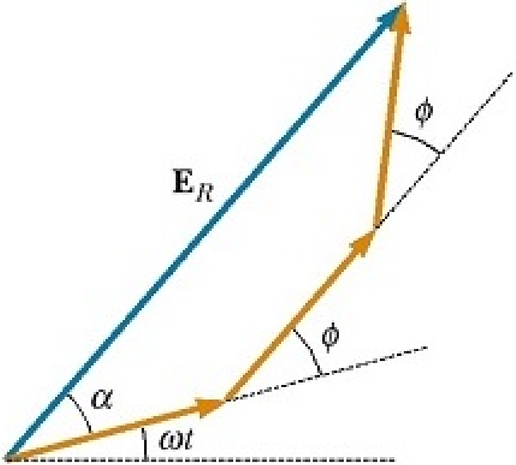
$$E = 2E_0 \cos\left(\frac{\phi}{2}\right)$$



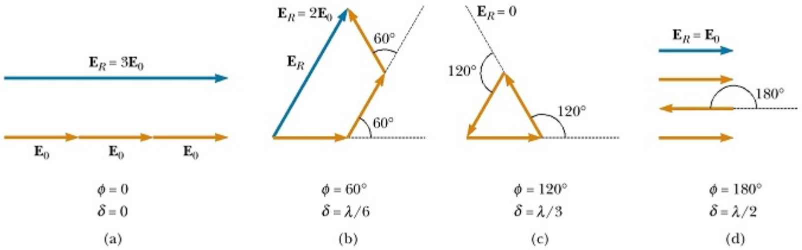
$$I = I_{\text{max}} \cos^2\left(\frac{\phi}{2}\right)$$

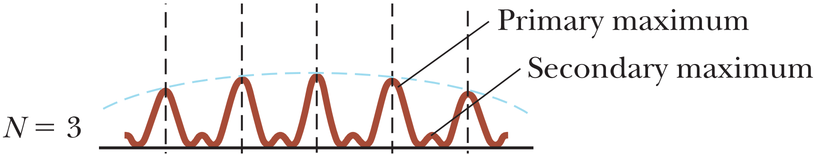
Interference with Three Slits

$$E_1 = E_0 \sin(\omega t) \ , \quad E_2 = E_0 \sin(\omega t + \phi) \ , \quad E_3 = E_0 \sin(\omega t + 2\phi)$$

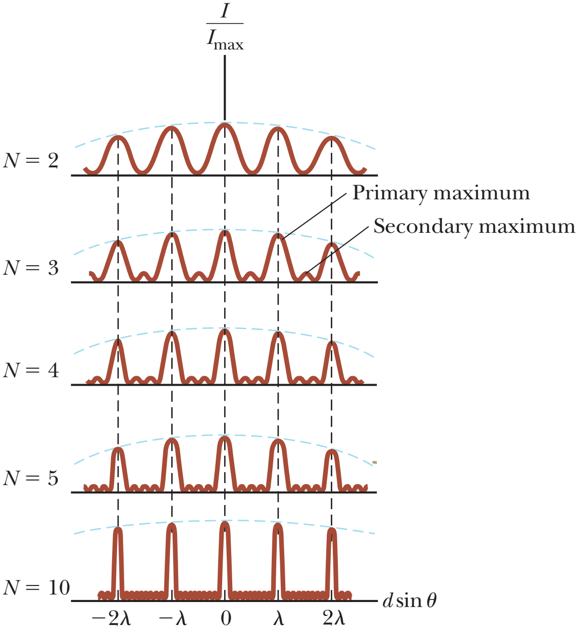


Interference with Three Slits





Interference Patterns from Many Slits



Diffraction Grating

A diffraction grating is a device that works in a similar way to Young's two slits, but produces a brighter set of fringes for the same source, and the bright fringes are narrower.

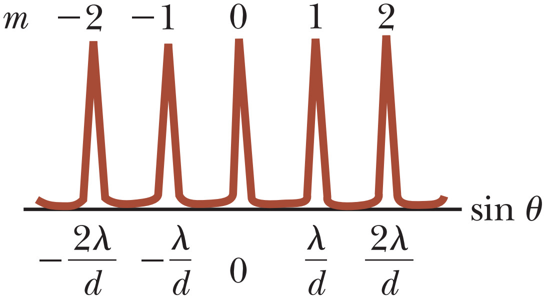
It breaks the light from a source up into very, very many coherent sources. (Young's slit does the same, but only breaks the light into 2 sources.)

It is used mainly for spectroscopy (determining the spectrum of a type of atom or molecule) and in monochromators (devices that select a particular frequency of light).

Interference Pattern from a Diffraction Grating

A diffraction grating has so many slits that effectively $N \to \infty$.

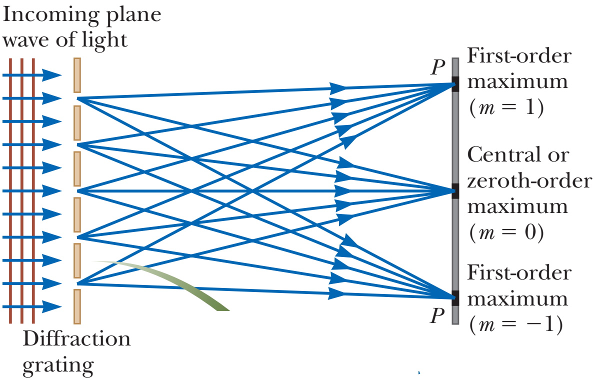
With monochromatic light, the peaks are sharp and well-separated.



For light that is composed of several frequencies, the peaks for each will be separated out.

Diffraction Gratings

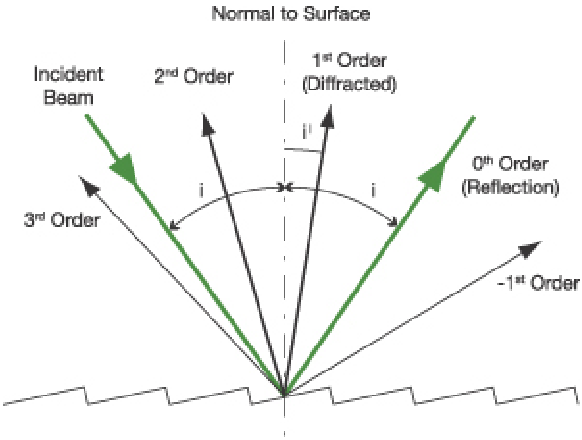
There are two types of diffraction grating.
Transmission gratings:



Many slits allow light to pass through.

Diffraction Gratings

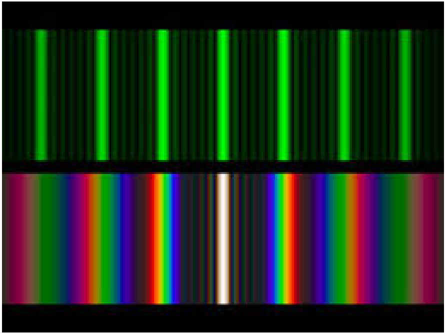
Reflection gratings:



Light reflects off of a series of mirrored surfaces.

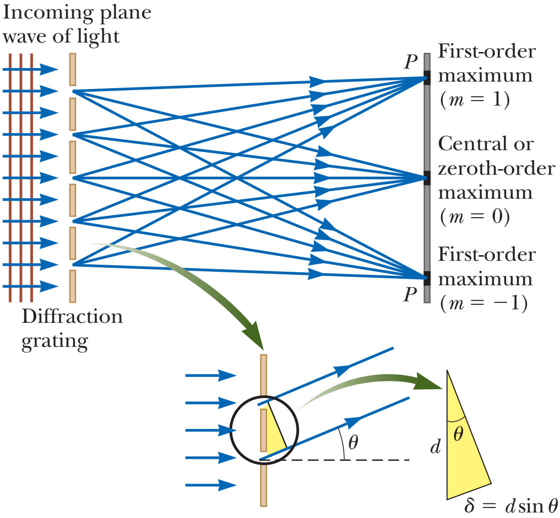
[1] http://exoplanet.as.arizona.edu/~lclose/a302/lecture14/lecture_14.html

Diffraction Grating Pattern



Diffraction Grating

We can find the maxima (bright fringes) of the pattern produced in a diffraction grating in exactly the same way we did for Young's slits.



Diffraction Grating

Once again, light from different slits interferes constructively when the path differnce $\delta = m\lambda$ (m is an integer).

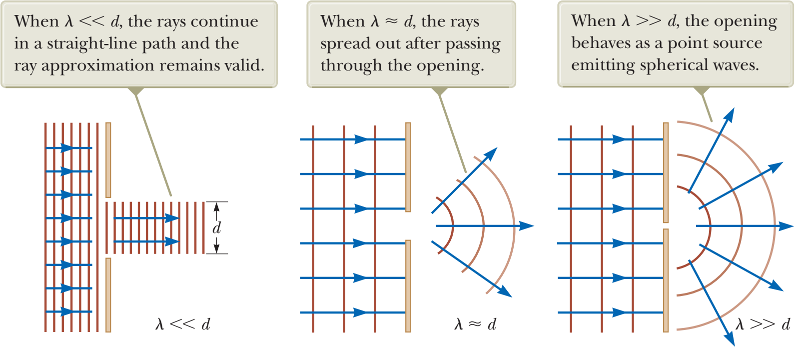
$$\delta = d \sin \theta$$

Maxima (bright fringes) occur when

$$d \sin \theta_{\text{max}} = m\lambda \qquad \text{where } m \in \mathbb{Z}$$

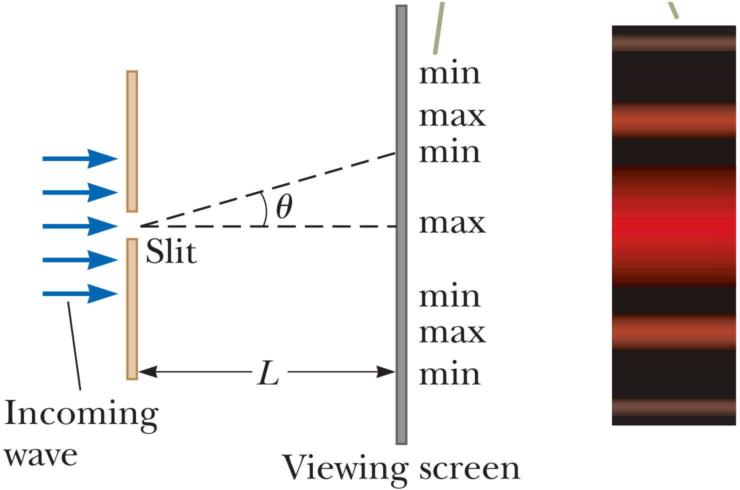
Diffraction

We already know that light and other waves that travel through a small gap ($< \lambda$) diverge, and that the smaller the gap, the more divergence.

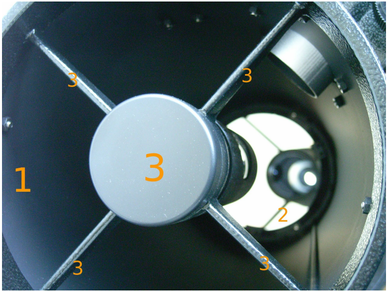


The intensity of light in each direction is not the same however.

Diffraction Patterns



Diffraction Spikes

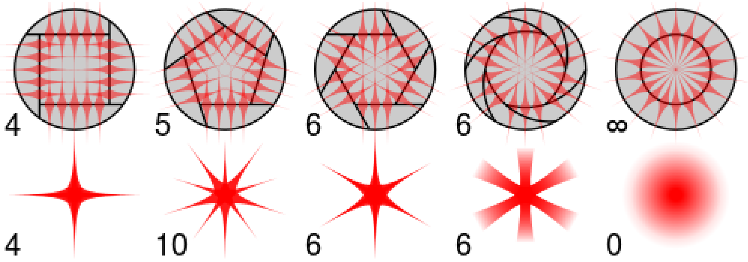


[1]NASA, ESA, and H. Richer (University of British Columbia); Svon Halenbach

Diffraction Spikes in Camera Apertures

Iris diaphragms adjust the amount of light allowed into a camera body.

They cause characteristic diffraction patterns on photos taken of bright lights.

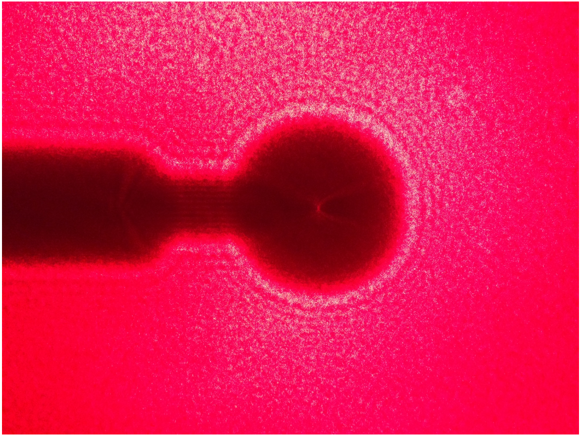


[1]Wikipedia user Cmglee

Diffraction Patterns: Arago Spot

Directly in the center of the shadow produced by a round object lit with coherent light, a spot of light can be observed!

This is called the Arago spot, Fresnel bright spot, or Poisson spot.



[1]Photo taken at Exploratorium in SF, own work.

Summary

- two-slit interference
- multiple slit interference
- diffraction gratings

Final Exam 9:15-11:15am, Tuesday, June 23.

Homework Serway & Jewett:

- new: Ch 38, onward from page 1182. CQs: 5; Probs: 60