# Physics 4C Spring 2018 Final Exam

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Please show your work! Answers are not complete without clear reasoning. When asked for an expression, you must give your answer in terms of the variables given in the question and/or fundamental constants.

Answer as many questions as you can, in any order. Calculators are allowed. Books, notes, and internet connectable devices are not allowed. Use any blank space to answer questions, but please make sure it is clear which question your answer refers to.

$g = 9.8$ ms$^{-2}$

$c = 3.00 \times 10^8$ m/s

atmospheric pressure $P_0 = 1.013 \times 10^5$ Pa

density of water $\rho_w = 1000$ kg/m$^3$

$\rho_{\text{air}} = 1.20$ kg m$^{-3}$ (sea level, 20°C)

Fahrenheit to Celsius:

([°F] − 32) ÷ 1.8 =[°C]

Celsius to Fahrenheit:

([°C] × 1.8) + 32 =[°F]

$k_B = 1.38 \times 10^{-23}$ J/K

$N_A = 6.022 \times 10^{23}$

$R = 8.314$ J mol$^{-1}$ K$^{-1}$

$m_p = 1.67 \times 10^{-27}$ kg

$\sigma = 5.6696 \times 10^{-8}$ W m$^{-2}$ K$^{-4}$

1 cal = 4.186 J

$I_0 = 1.00 \times 10^{-12}$ W m$^{-2}$

## Trigonometric Identities

$\sin^2\theta + \cos^2\theta = 1$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

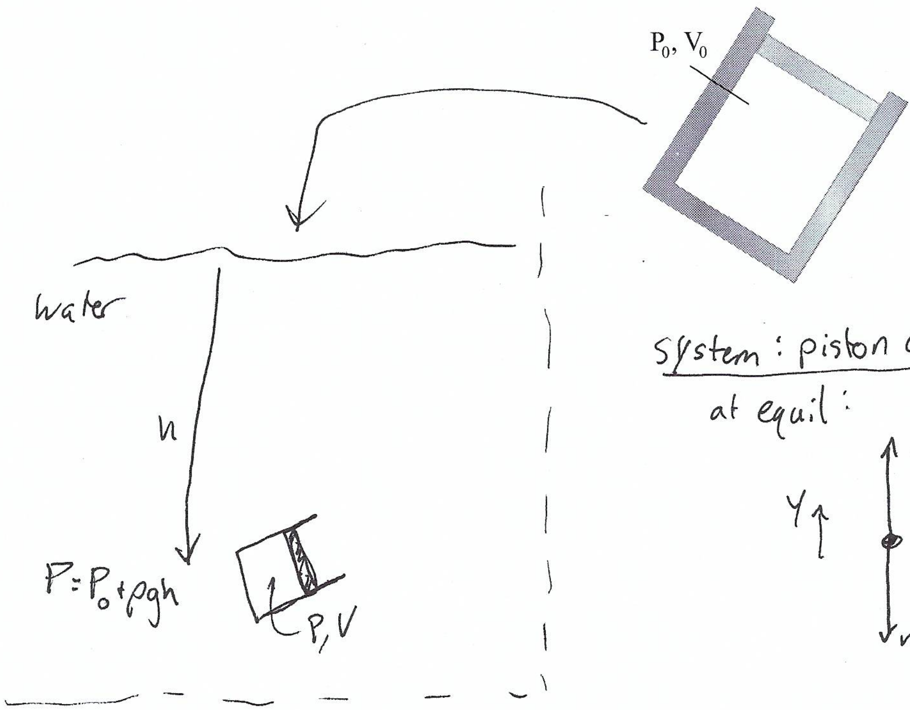
$\sin\alpha + \sin\beta = 2\cos\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)$

$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$

$\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$

$\cos\left(\theta + \frac{\pi}{2}\right) = -\sin\theta$

1. A piston cylinder of maximum volume $V_0$ is completely filled with air at atmospheric pressure, $P_0$, as shown. The total mass of the piston cylinder and air inside is $m$. The cylinder is lowered into a tank of water, and the piston is free to move without friction. Assume that both the temperature, and density of the water, $\rho$, do not change with depth. At what depth, $h$, is the cylinder neutrally buoyant? (That is, if it is released, it will be in equilibrium.) [7 pts]




system : piston cylinder & air inside

at equil :

$F_B$

y

mg

$F_{net,y} = 0$

$F_B = mg$

$\rho V \cancel{g} = m \cancel{g}$

air as an ideal gas

$PV = n\underbrace{RT}_{const}$

$\therefore PV = P_0 V_0$

$V = \dfrac{P_0 V_0}{P}$

Piston moves w/out friction
$\therefore$ P (pressure inside cylinder) is equal to the Pressure at depth h.

$P = P_0 + \rho g h$ —(1)

using (1) :

$V = \dfrac{P_0 V_0}{P_0 + \rho g h}$

$\rho\left(\dfrac{P_0 V_0}{P_0 + \rho g h}\right) = m$

$P_0 + \rho g h = \dfrac{\rho P_0 V_0}{m}$

$\rho g h = P_0\left(\dfrac{\rho V_0}{m} - 1\right)$

$h = \dfrac{P_0}{g}\left(\dfrac{V_0}{m} - \dfrac{1}{\rho}\right)$

| | Latent heat of fusion | Latent heat of vaporization |
|---|---|---|
| water | 80 cal/g | 540 cal/g |

2. A mass of ice $m_i = 20.0$ g at $T_c = -10.0°\text{C}$ is in a perfectly thermally insulating container. When water with a mass $m_w = 35.0$ g at temperature $T_h = 80.0°\text{C}$ is poured into the container and the system reaches a final equilibrium temperature $T_E = 20.0°\text{C}$. Find the specific heat capacity of ice in calories per gram per degree Celsius as determined in this experiment. (You may need some of the information in the table above.) [8 pts]

Thermally isolated:

$$Q_{net} = 0$$

$$Q_{ice} + Q_{water} = 0$$

$$Q_{ice} = -Q_{water}$$

(ice warms) (ice melts) (melted ice warms) (water cools)

$$m_i C_i \Delta T_i + m_i L_f + m_i C_w \Delta T_{i\to w} = -m_w C_w \Delta T_w$$

$$m_i C_i (T_f - T_c) + m_i L_f + m_i C_w (T_E - T_f) = -m_w C_w (T_E - T_h)$$

Let $T_f = 0°\text{C}$ be the freezing point of water.

$$C_i = \frac{m_w C_w (T_h - T_E) - m_i L_f - m_i C_w (T_E - T_f)}{m_i (T_f - T_c)}$$

$$C_i = \frac{(35\,\text{g})(1\,\text{cal/g°C})(80-20)°\text{C} - (20\,\text{g})(80\,\text{cal/g}) - (20\,\text{g})(1\,\text{cal/g})(20-0)°\text{C}}{(20\,\text{g})(0-(-10))°\text{C}}$$

$$\underline{C_i = 0.50\ \text{cal/g°C}}$$

[def. of calorie: 1 cal. is the heat required to raise the temperature of 1g of water by 1°C $\Rightarrow$ $C_w = 1\,\text{cal/g°C}$]

3. **[18pts total]** An ideal diatomic gas initially at pressure $P_i$, and volume $V_i$ is taken through a cycle as shown.

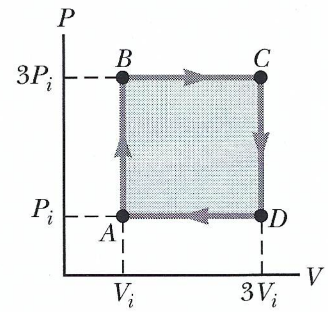
   (a) What is the change in the internal energy of the gas over the cycle $A \to A$? [1 pt]

   (b) Find the net work done on the gas per cycle. If the process is used as an engine, what is the work done by the engine per cycle? [5 pts]

   (c) In which steps is heat transferred to the gas? [2 pts]

   (d) What is the total heat transferred to the gas in one cycle, $Q_h$? [7 pts]

   (e) If the process is used as an engine, what is its efficiency? [3 pts]




a) $\Delta E_{int,\,cycle} = 0$    [$E_{int}$ is a state variable]

b) isovolumetric

$$W_{cycle} = \cancel{W_{AB}}^{\,0} + W_{BC} + \cancel{W_{CD}}^{\,0} + W_{DA}$$

$$= -\int_B^C P\,dV - \int_D^A P\,dV \quad \text{P is const.}$$

$$= -(3P_i)(3V_i - V_i) - (P_i)(V_i - 3V_i)$$

$$= -6P_iV_i + 2P_iV_i$$

$$W_{cycle\ on\ gas} = -4P_iV_i$$

$$W_{by} = -W_{on}$$

$$W_{eng} = 4P_iV_i$$

c) Heat transferred to the gas in $A \to B$ and $B \to C$

[Temperature increases in both processes
$\Downarrow$
$\Delta E_{int}$ increases: $W_{AB} = 0 \Rightarrow Q$ is +ve
(1st law: $\Delta E_{int} = Q + W$) $W_{BC}$ is -ve $\Rightarrow Q$ is -ve]

d)

$$Q_h = Q_{AB} + Q_{BC}$$

$$= nC_v\Delta T_{AB} + nC_p\Delta T_{BC}$$

$$= n\left(\tfrac{5}{2}R\right)(T_B - T_A) + n\left(\tfrac{7}{2}R\right)(T_C - T_B)$$

$$PV = nRT$$
$$\Downarrow$$
$$nRT_A = P_iV_i$$
$$nRT_B = 3P_iV_i$$
$$nRT_C = 9P_iV_i$$

$$= \tfrac{5}{2}(3P_iV_i - P_iV_i) + \tfrac{7}{2}(9P_iV_i - 3P_iV_i)$$

$$= 5P_iV_i + 21P_iV_i$$

$$Q_h = 26P_iV_i$$

e) $$e = \frac{W_{eng}}{Q_h} = \frac{4P_iV_i}{26P_iV_i} = \frac{2}{13} = 15.4\%$$

4. Show that the wave function $y = Ae^{-(kx-\omega t)^2}$ specifically is a solution of the linear wave equation, where $A$ is a constant, $k$ is a wavenumber, and $\omega$ is an angular frequency. [10 pts]

wave equation: $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$ and $v = \frac{\omega}{k}$ —— (3)

$$\frac{\partial y}{\partial x} = \left(-2k(kx-\omega t)\right) A e^{-(kx-\omega t)^2}$$

$$\frac{\partial^2 y}{\partial x^2} = -2k^2 A e^{-(kx-\omega t)^2} + \left(-2k(kx-\omega t)\right)^2 A e^{-(kx-\omega t)^2}$$ (using product rule)

$$= \left(4(kx-\omega t)^2 - 2\right) k^2 A e^{-(kx-\omega t)^2} \quad \text{—— (1)}$$

$$\frac{\partial y}{\partial t} = \left(2\omega(kx-\omega t)\right) A e^{-(kx-\omega t)^2}$$

$$\frac{\partial^2 y}{\partial t^2} = -2\omega^2 A e^{-(kx-\omega t)^2} + \left(2\omega(kx-\omega t)\right)^2 A e^{-(kx-\omega t)^2}$$

$$= \left(4(kx-\omega t)^2 - 2\right)\omega^2 A e^{-(kx-\omega t)^2} \quad \text{—— (2)}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

using (1), (2), & (3):

$$\cancel{\left(4(kx-\omega t)^2 - 2\right)} k^2 \cancel{A e^{-(kx-\omega t)^2}} = \frac{k^2}{\omega^2}\cancel{\left(4(kx-\omega t)^2 - 2\right)}\omega^2 \cancel{A e^{-(kx-\omega t)^2}}$$

$$k^2 = \frac{k^2}{\cancel{\omega^2}}\cancel{\omega^2}$$

$1 = 1$ ✓ $\Rightarrow$ $y = Ae^{-(kx-\omega t)^2}$ is a solution of the wave equation.

5. A wire made of a metal with coefficient of thermal expansion $\alpha$ is held between two clamps under zero tension. Reducing the ambient temperature by a temperature change of magnitude $\Delta T$, results in a decrease in the wire's equilibrium length, and increases the tension in the wire. Taking the density (mass per unit volume) of the wire to be $\rho$, and its Young's modulus to be $Y$, find an expression for the wave speed of transverse waves on this wire. [8 pts]

$$v = \sqrt{\frac{T}{\mu}}$$

$T$ — Tension

$\mu$ — mass per unit length

$$Y = \frac{T/A}{\Delta L / L}$$

Let $L$ be the equilibrium length of the wire at the lower (final) temperature.

$$\Delta L = \alpha L \Delta T$$

$$\frac{\Delta L}{L} = \alpha \Delta T$$

$$\frac{T}{A} = Y\left(\frac{\Delta L}{L}\right)$$

$$\frac{T}{A} = Y \alpha \Delta T$$

$$T = Y \alpha \Delta T A$$

$$V = A\ell$$

$\ell$ — length of wire

$$\rho = \frac{m}{V} = \frac{m}{A\ell} = \frac{\mu}{A}$$

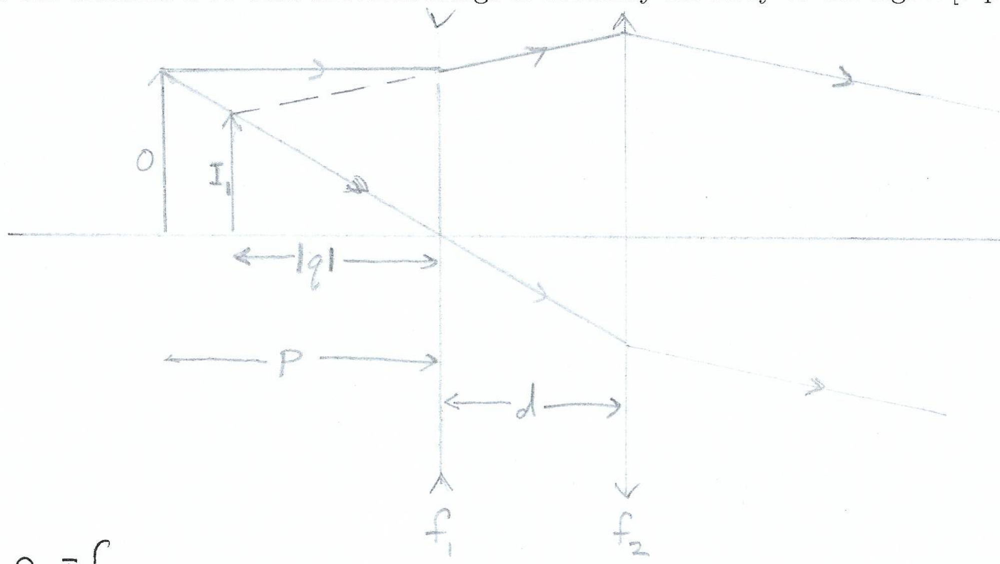
$$\mu = \rho A$$

$$v = \sqrt{\frac{T}{\mu}}$$

$$= \sqrt{\frac{Y\alpha\Delta T \cancel{A}}{\rho \cancel{A}}}$$

$$v = \sqrt{\frac{Y\alpha\Delta T}{\rho}}$$

6. An object is placed a distance $p$ to the left of a diverging lens of focal length $f_1$. A converging lens of focal length $f_2$ is placed a distance d to the right of the diverging lens. Find the distance $d$ so that the final image is infinitely far away to the right. [7 pts]




$I_2$ @ $\infty \Rightarrow p_2 = f_2$

$$\left(\frac{1}{f_2} = \frac{1}{p_2} + \cancel{\frac{1}{q_2}}^{\,0}\right)$$

The object for lens 2 / image from lens 1, $I_1$, is at the focal point of $f_2$ lens.

$$d + |q| = f_2$$

and $q$ is $-ve$, so $q = -|q|$

$$d - q = f_2$$

$$q = d - f_2$$

For lens 1:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f_1}$$

$$\frac{1}{p} + \frac{1}{d-f_2} = \frac{1}{f_1}$$

$$\frac{1}{d-f_2} = \frac{1}{f_1} - \frac{1}{p}$$

$$d - f_2 = \frac{pf_1}{p-f_1}$$

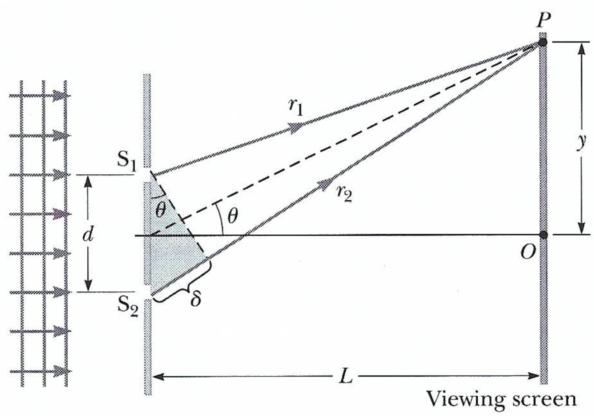
$$d = \frac{pf_1}{p-f_1} + f_2$$

$f_1$ is $-ve$, so this term is $-ve$.

7. **[11pts total]** In the double-slit arrangement shown, a pair of very narrow slits have a separation $d = 0.250$ mm, and are illuminated with light at two wavelengths $\lambda_1 = 400.0$ nm and $\lambda_2 = 600.0$ nm.

(a) If the distance to the screen is $L = 1.00$ m, find the shortest distance away from the center of the screen to the point $P$ ($P \neq O$) where the two wavelengths both have a bright peak. What is the order number of this fringe for $\lambda_1 = 400.0$ nm? [7 pts]

(b) If the entire apparatus were submerged in water, how would the pattern change? Assuming that $n_{\text{water}} = 4/3$, what would be the answer to part (a) in this case? [4 pts]




(✱) b) cont'd.
y will change to y'
The entire pattern will narrow and the bright fringes move closer together.

$$y' = \frac{m_1 \lambda_1' L}{d} = \frac{m_1 \lambda_1 L}{nd}$$

$$y' = \frac{y}{n}$$

$$y' = \frac{3}{4}(4.80\text{ mm})$$

$$\underline{y' = 3.60\text{ mm}}$$

a) maxima / bright spot when

$\delta = d\sin\theta = m\lambda \qquad m \in \mathbb{Z}$

Both $\lambda$ have a bright peak in the same place:

$$d\sin\theta = m_1\lambda_1 = m_2\lambda_2$$

$$\frac{m_1}{m_2} = \frac{\lambda_2}{\lambda_1} \quad \text{—(1)}$$

$$\frac{m_1}{m_2} = \frac{600\text{ nm}}{400\text{ nm}}$$

$$\frac{m_1}{m_2} = \frac{3}{2}$$ ← This is a fully simplified fraction.

$\therefore$ $\underline{m_1 = 3}$ and $m_2 = 2$ are the smallest integers that satify (1).

For $\lambda_1$ this point is the $\underline{\text{3rd order maximum}}$.

$$\sin\theta = \frac{m_1\lambda_1}{d} = \frac{(3)(400\times10^{-9}\text{m})}{(0.250\times10^{-3}\text{m})} = 0.0048$$

$\therefore$ $\sin\theta \approx \theta$ since $\theta$ is very small.

$$\tan\theta = \frac{y}{L}$$

$\theta$ small $\Rightarrow \sin\theta \approx \tan\theta$

$$\frac{y}{L} = \frac{m_1\lambda_1}{d}$$

$$y = \frac{m_1\lambda_1 L}{d}$$

$$= \frac{(3)(400\times10^{-9}\text{m})(1\text{m})}{(0.250\times10^{-3}\text{m})}$$

$$\underline{y = 4.80\text{ mm}}$$

b) when submerged in water:

$$\lambda_1' = \frac{\lambda_1}{n} \qquad n = \frac{4}{3}$$

$$\lambda_2' = \frac{\lambda_2}{n}$$

The second & 3rd fringes still coincide:

$$m_1'\lambda_1' = m_2'\lambda_2'$$

$$m_1'\frac{\lambda_1}{\cancel{n}} = m_2'\frac{\lambda_2}{\cancel{n}}$$

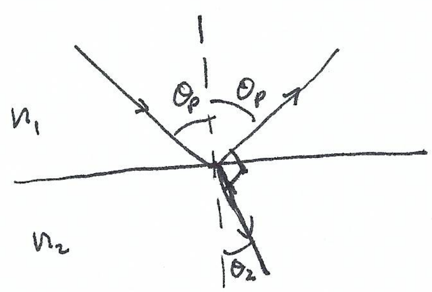
$$\Rightarrow \frac{m_1'}{m_2'} = \frac{3}{2} \Rightarrow \underline{m_1' = 3} \quad (✱)$$

8. **[9pts total]** Polarizing angle.

(a) Explain in a sentence or two what the polarizing angle (Brewster's angle) is and what happens at this angle. Also, derive an expression for it, in terms of refractive indices at a boundary between materials. [5 pts]

(b) For a particular transparent medium surrounded by air, find the polarizing angle $\theta_p$ for light incident on the medium in terms of the critical angle for total internal reflection $\theta_{\text{crit}}$ in that medium. [4 pts]

a) If unpolarized light is incident on a surface at the polarizing angle all of the light reflected off of the surface will be polarized parallel to the surface.




For complete polarization to occur the angle between the reflected and transmitted (refracted) ray is $90° / \frac{\pi}{2}$ rad.

$$\theta_p + 90° + \theta_2 = 180°$$

$$\Rightarrow \theta_p + \theta_2 = 90°$$

$$\theta_2 = 90° - \theta_p$$

Snell's law:

$$n_1 \sin\theta_p = n_2 \sin\theta_2$$

$$= n_2 \sin(90 - \theta_p)$$

$$= n_2 \cos\theta_p$$

$$\frac{\sin\theta_p}{\cos\theta_p} = \frac{n_2}{n_1}$$

$$\underline{\tan\theta_p = \frac{n_2}{n_1}} \quad \text{or} \quad \underline{\theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right)}$$

b) critical angle

$$\theta_{crit} = \sin^{-1}\left(\frac{n_1}{n_2}\right)$$

for a ray leaving a transparent medium of refractive index $n$ going to boundary w/ air:

$$\theta_{crit} = \sin^{-1}\left(\frac{1}{n}\right) \Rightarrow n = \frac{1}{\sin\theta_{crit}} \quad \text{—(1)}$$

$$\theta_p = \tan^{-1}\left(\frac{n}{1}\right)$$

$$n = \tan\theta_p \quad \text{—(2)}$$

equate (1) and (2)

$$\underline{\tan\theta_p = \frac{1}{\sin\theta_{crit}}} \quad \text{or} \quad \underline{\theta_p = \tan^{-1}\left(\frac{1}{\sin\theta_{crit}}\right)}$$

–Extra Workspace–

$Y = \frac{F/A}{\Delta L/L_i}$

$S = \frac{F/A}{\Delta x/h}$

$B = -\frac{\Delta P}{\Delta V/V_i}$

$P = \frac{F}{A}$

$P = P_0 + \rho g h$

$B = \rho g V$

$A_1 v_1 = A_2 v_2$

$P + \frac{1}{2}\rho v^2 + \rho g y = \text{const.}$

$\Delta L = \alpha L_i \, \Delta T$

$\Delta V = \beta V_i \, \Delta T$

$PV = nRT$

$PV = N k_B T \; (nR = N k_B)$

$n = \frac{m}{M}$

$Q = C \, \Delta T$

$Q = cm \, \Delta T$

$Q = mL$

$\Delta E_{\text{int}} = W + Q$

$W_{\text{on gas}} = -\int_{V_i}^{V_f} P \, \mathrm{dV}$

$W = nRT \ln\left(\frac{V_i}{V_f}\right)$

$P = kA \left|\frac{\mathrm{dT}}{\mathrm{dx}}\right|$

$P = kA \left(\frac{T_h - T_c}{L}\right)$

$P = \sigma A e T^4$

$P = \frac{2}{3}\frac{N}{V}\bar{K}$

$\bar{K} = \frac{1}{2} m_0 \overline{v^2} = \frac{3}{2} k_B T$

$K_{\text{tot,trans}} = \frac{3}{2} N k_B T = \frac{3}{2} nRT$

$e = 1 - \frac{1}{(V_1/V_2)^{(\gamma - 1)}}$

$Q = n C_V \, \Delta T$

$Q = n C_P \, \Delta T$

$\Delta E_{\text{int}} = n C_V \, \Delta T$

$\gamma = \frac{C_P}{C_V}$

$PV^\gamma = \text{const.}$

$TV^{\gamma - 1} = \text{const.}$

$n_V(E) \propto e^{-E/k_B T}$

$N_v \propto v^2 e^{-m_0 v^2/2 k_B T}$

$v_{\text{rms}} = \sqrt{\frac{3 k_B T}{m_0}}$

$\Delta S = \int_i^f \frac{\mathrm{dQ_r}}{T}$

$\Delta S = n C_v \ln\left(\frac{T_f}{T_i}\right) + nR \ln\left(\frac{V_f}{V_i}\right)$

$S = k_B \ln W$

$e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{|Q_h| - |Q_c|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|}$

$\text{COP (cooling)} = \frac{|Q_c|}{W}$

$\text{COP (heating)} = \frac{|Q_h|}{W}$

$e = \frac{T_h - T_c}{T_h} = 1 - \frac{T_c}{T_h}$

$$\int_v^{v+\mathrm{dv}} N_v \, \mathrm{dv} = \int_v^{v+\mathrm{dv}} 4\pi N \left(\frac{m_0}{2\pi k_B T}\right)^{3/2} v^2 e^{-m_0 v^2/2 k_B T} \, \mathrm{dv}$$

$f = \frac{1}{T}$

$k = \frac{2\pi}{\lambda}$

$\omega = 2\pi f$

$T = 2\pi \sqrt{\frac{m}{k}}$

$T = 2\pi \sqrt{\frac{L}{g}}$

$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$

$y(x,t) = f(x \mp vt)$

$v = f\lambda = \frac{\omega}{k}$

$v = \sqrt{\frac{T}{\mu}}$

$y(x,t) = A \sin(kx - \omega t + \phi)$

$v_y = \frac{\partial y}{\partial t}$

$a_y = \frac{\partial^2 y}{\partial t^2}$

$P = \frac{1}{2}\mu\omega^2 A^2 v$

$y(x,t) = [2A\sin(kx)]\ \cos(\omega t)$

$v = (331 \text{ m/s})\sqrt{1 + \frac{T_{\text{Cel}}}{273}}$

$B = -\frac{\Delta P}{\Delta V / V_i}$

$y(t) = \sum_{n=1}^{\infty} \left(A_n \sin(2\pi n f t) + B_n \cos(2\pi n f t)\right)$

$v = \sqrt{\frac{B}{\rho}}$

$s(x,t) = s_{\max} \cos(kx - \omega t)$

$\Delta P(x,t) = (\Delta P_{\max}) \sin(kx - \omega t)$

$\Delta P_{\max} = B\, s_{\max} k = \rho v \omega s_{\max}$

$I = \frac{P_{\text{avg}}}{A}$

$P_{\text{avg}} = \frac{1}{2}\rho v \omega^2 A\, s_{\max}^2$

$\lambda_n = \frac{2L}{n}$

$f_n = \frac{nv}{2L}$

$\lambda_{2n+1} = \frac{4L}{(2n+1)}$

$f_{2n+1} = \frac{(2n+1)v}{4L}$

$|L_2 - L_1| = n\lambda$

$|L_2 - L_1| = \frac{(2n+1)\lambda}{2}$

$f_{\text{beat}} = |f_1 - f_2|$

$\beta = 10 \log_{10}\left(\frac{I}{I_0}\right)$

$f' = \left(\frac{v \pm v_0}{v \mp v_s}\right) f$

$\sin\theta = \frac{v}{v_s}$

$\text{Mach number} = \frac{v_s}{v}$

$E = hf$

$n = \frac{c}{v}$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\sin\theta_c = \frac{n_2}{n_1}$

$M = \frac{h'}{h}$

$f = \frac{R}{2}$

$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$

$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$I \propto E^2$

$d \sin\theta = m\lambda$

$d \sin\theta = (m + \frac{1}{2})\lambda$

$I = I_{\max} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$

$2nt = (m + \frac{1}{2})\lambda$

$2nt = m\lambda$

$r \approx \sqrt{\frac{m\lambda R}{n}}$

$\sin\theta = m\frac{\lambda}{a}$

$I = I_{\max}\left(\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right)^2$

$\theta_{\min} = 1.22\,\frac{\lambda}{D}$

$I = I_{\max} \cos^2\theta$

$\tan\theta_p = \frac{n_2}{n_1}$