Physics 4C Spring 2018 Final Exam

Name: ____________________________

June 26, 2018

Please show your work! Answers are not complete without clear reasoning. When asked for an expression, you must give your answer in terms of the variables given in the question and/or fundamental constants.

Answer as many questions as you can, in any order. Calculators are allowed. Books, notes, and internet connectable devices are not allowed. Use any blank space to answer questions, but please make sure it is clear which question your answer refers to.

$g = 9.8\ \text{ms}^{-2}$

$c = 3.00 \times 10^8$ m/s

atmospheric pressure $P_0 = 1.013 \times 10^5$ Pa

density of water $\rho_w = 1000$ kg/m^3

$\rho_{\text{air}} = 1.20$ kg m^{-3} (sea level, 20°C)

Fahrenheit to Celsius:

([°F] − 32) ÷ 1.8 =[°C]

Celsius to Fahrenheit:

([°C] × 1.8) + 32 =[°F]

$k_B = 1.38 \times 10^{-23}$ J/K

$N_A = 6.022 \times 10^{23}$

$R = 8.314$ J mol^{-1} K^{-1}

$m_p = 1.67 \times 10^{-27}$ kg

$\sigma = 5.6696 \times 10^{-8}$ W m^{-2} K^{-4}

1 cal = 4.186 J

$I_0 = 1.00 \times 10^{-12}$ W m^{-2}

Trigonometric Identities

$\sin^2\theta + \cos^2\theta = 1$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

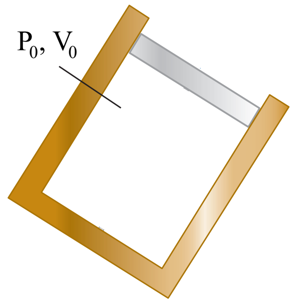
$\sin\alpha + \sin\beta = 2\cos\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)$

$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$

$\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$

$\cos\left(\theta + \frac{\pi}{2}\right) = -\sin\theta$

1. A piston cylinder of maximum volume V_0 is completely filled with air at atmospheric pressure, P_0, as shown. The total mass of the piston cylinder and air inside is m. The cylinder is lowered into a tank of water, and the piston is free to move without friction. Assume that both the temperature, and density of the water, ρ, do not change with depth. At what depth, h, is the cylinder neutrally buoyant? (That is, if it is released, it will be in equilibrium.) [7 pts]



	Latent heat of fusion	Latent heat of vaporization
water	80 cal/g	540 cal/g

2. A mass of ice $m_i = 20.0$ g at $T_c = -10.0°$C is in a perfectly thermally insulating container. When water with a mass $m_w = 35.0$ g at temperature $T_h = 80.0°$C is poured into the container and the system reaches a final equilibrium temperature $T_E = 20.0°$C. Find the specific heat capacity of ice in calories per gram per degree Celsius as determined in this experiment. (You may need some of the information in the table above.) [8 pts]

3. **[18pts total]** An ideal diatomic gas initially at pressure P_i, and volume V_i is taken through a cycle as shown.

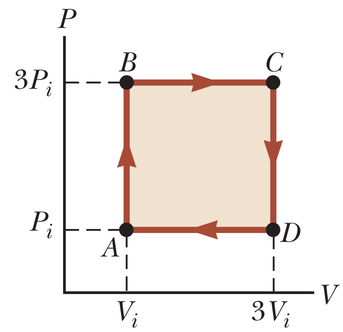
   (a) What is the change in the internal energy of the gas over the cycle $A \to A$? [1 pt]

   (b) Find the net work done on the gas per cycle. If the process is used as an engine, what is the work done by the engine per cycle? [5 pts]

   (c) In which steps is heat transferred to the gas? [2 pts]

   (d) What is the total heat transferred to the gas in one cycle, Q_h? [7 pts]

   (e) If the process is used as an engine, what is its efficiency? [3 pts]



4. Show that the wave function $y = Ae^{-(kx-\omega t)^2}$ specifically is a solution of the linear wave equation, where A is a constant, k is a wavenumber, and ω is an angular frequency. [10 pts]

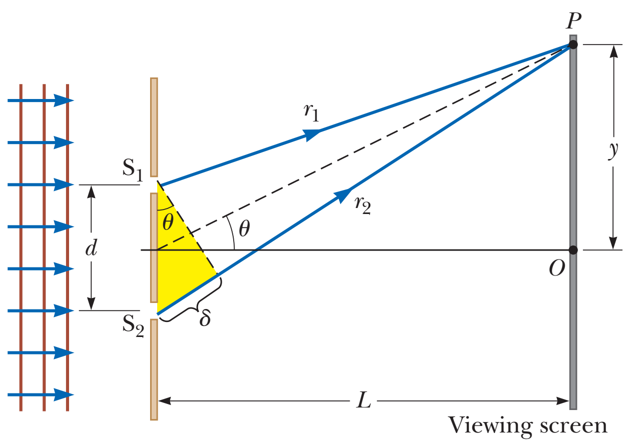
5. A wire made of a metal with coefficient of thermal expansion α is held between two clamps under zero tension. Reducing the ambient temperature by a temperature change of magnitude ΔT, results in a decrease in the wire's equilibrium length, and increases the tension in the wire. Taking the density (mass per unit volume) of the wire to be ρ, and its Young's modulus to be Y, find an expression for the wave speed of transverse waves on this wire. [8 pts]

6. An object is placed a distance p to the left of a diverging lens of focal length f_1. A converging lens of focal length f_2 is placed a distance d to the right of the diverging lens. Find the distance d so that the final image is infinitely far away to the right. [7 pts]

7. **[11pts total]** In the double-slit arrangement shown, a pair of very narrow slits have a separation $d = 0.250$ mm, and are illuminated with light at two wavelengths $\lambda_1 = 400.0$ nm and $\lambda_2 = 600.0$ nm.

(a) If the distance to the screen is $L = 1.00$ m, find the shortest distance away from the center of the screen to the point P ($P \neq O$) where the two wavelengths both have a bright peak. What is the order number of this fringe for $\lambda_1 = 400.0$ nm? [7 pts]

(b) If the entire apparatus were submerged in water, how would the pattern change? Assuming that $n_{\text{water}} = 4/3$, what would be the answer to part (a) in this case? [4 pts]



8. **[9pts total]** Polarizing angle.

   (a) Explain in a sentence or two what the polarizing angle (Brewster's angle) is and what happens at this angle. Also, derive an expression for it, in terms of refractive indices at a boundary between materials. [5 pts]

   (b) For a particular transparent medium surrounded by air, find the polarizing angle θ_p for light incident on the medium in terms of the critical angle for total internal reflection θ_{crit} in that medium. [4 pts]

–Extra Workspace–

$Y = \frac{F/A}{\Delta L/L_i}$

$S = \frac{F/A}{\Delta x/h}$

$B = -\frac{\Delta P}{\Delta V/V_i}$

$P = \frac{F}{A}$

$P = P_0 + \rho g h$

$B = \rho g V$

$A_1 v_1 = A_2 v_2$

$P + \frac{1}{2}\rho v^2 + \rho g y =$const.

$\Delta L = \alpha L_i \, \Delta T$

$\Delta V = \beta V_i \, \Delta T$

$PV = nRT$

$PV = N k_B T \ (nR = N k_B)$

$n = \frac{m}{M}$

$Q = C \, \Delta T$

$Q = cm \, \Delta T$

$Q = mL$

$\Delta E_{\text{int}} = W + Q$

$W_{\text{on gas}} = -\int_{V_i}^{V_f} P \, \text{dV}$

$W = nRT \ln\left(\frac{V_i}{V_f}\right)$

$P = kA \left|\frac{\text{dT}}{\text{dx}}\right|$

$P = kA\left(\frac{T_h - T_c}{L}\right)$

$P = \sigma A e T^4$

$P = \frac{2}{3}\frac{N}{V}\bar{K}$

$\bar{K} = \frac{1}{2} m_0 \overline{v^2} = \frac{3}{2} k_B T$

$K_{\text{tot,trans}} = \frac{3}{2} N k_B T = \frac{3}{2} nRT$

$e = 1 - \frac{1}{(V_1/V_2)^(\gamma - 1)}$

$Q = n C_V \, \Delta T$

$Q = n C_P \, \Delta T$

$\Delta E_{\text{int}} = n C_V \, \Delta T$

$\gamma = \frac{C_P}{C_V}$

$PV^\gamma = \text{const.}$

$TV^{\gamma - 1} = \text{const.}$

$n_V(E) \propto e^{-E/k_B T}$

$N_v \propto v^2 e^{-m_0 v^2/2 k_B T}$

$v_{\text{rms}} = \sqrt{\frac{3 k_B T}{m_0}}$

$\Delta S = \int_i^f \frac{\text{dQ}_\text{r}}{T}$

$\Delta S = n C_v \ln\left(\frac{T_f}{T_i}\right) + nR \ln\left(\frac{V_f}{V_i}\right)$

$S = k_B \ln W$

$e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{|Q_h| - |Q_c|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|}$

$\text{COP (cooling)} = \frac{|Q_c|}{W}$

$\text{COP (heating)} = \frac{|Q_h|}{W}$

$e = \frac{T_h - T_c}{T_h} = 1 - \frac{T_c}{T_h}$

$$\int_v^{v+\text{dv}} N_v \, \text{dv} = \int_v^{v+\text{dv}} 4\pi N \left(\frac{m_0}{2\pi k_B T}\right)^{3/2} v^2 e^{-m_0 v^2/2 k_B T} \, \text{dv}$$

$f = \frac{1}{T}$

$k = \frac{2\pi}{\lambda}$

$\omega = 2\pi f$

$T = 2\pi \sqrt{\frac{m}{k}}$

$T = 2\pi \sqrt{\frac{L}{g}}$

$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$

$y(x,t) = f(x \mp vt)$

$v = f\lambda = \frac{\omega}{k}$

$v = \sqrt{\frac{T}{\mu}}$

$y(x,t) = A \sin(kx - \omega t + \phi)$

$v_y = \frac{\partial y}{\partial t}$

$a_y = \frac{\partial^2 y}{\partial t^2}$

$P = \frac{1}{2}\mu\omega^2 A^2 v$

$y(x,t) = [2A\sin(kx)] \ \cos(\omega t)$

$v = (331 \text{ m/s})\sqrt{1 + \frac{T_{\text{Cel}}}{273}}$

$B = -\frac{\Delta P}{\Delta V / V_i}$

$y(t) = \sum_{n=1}^{\infty} \left(A_n \sin(2\pi n f t) + B_n \cos(2\pi n f t)\right)$

$v = \sqrt{\frac{B}{\rho}}$

$s(x,t) = s_{\max}\cos(kx - \omega t)$

$\Delta P(x,t) = (\Delta P_{\max})\sin(kx - \omega t)$

$\Delta P_{\max} = B\, s_{\max} k = \rho v \omega s_{\max}$

$I = \frac{P_{\text{avg}}}{A}$

$P_{\text{avg}} = \frac{1}{2}\rho v \omega^2 A\, s_{\max}^2$

$\lambda_n = \frac{2L}{n}$

$f_n = \frac{nv}{2L}$

$\lambda_{2n+1} = \frac{4L}{(2n+1)}$

$f_{2n+1} = \frac{(2n+1)v}{4L}$

$|L_2 - L_1| = n\lambda$

$|L_2 - L_1| = \frac{(2n+1)\lambda}{2}$

$f_{\text{beat}} = |f_1 - f_2|$

$\beta = 10\log_{10}\left(\frac{I}{I_0}\right)$

$f' = \left(\frac{v \pm v_0}{v \mp v_s}\right) f$

$\sin\theta = \frac{v}{v_s}$

$\text{Mach number} = \frac{v_s}{v}$

$E = hf$

$n = \frac{c}{v}$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\sin\theta_c = \frac{n_2}{n_1}$

$M = \frac{h'}{h}$

$f = \frac{R}{2}$

$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$

$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$I \propto E^2$

$d\sin\theta = m\lambda$

$d\sin\theta = (m + \frac{1}{2})\lambda$

$I = I_{\max}\cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$

$2nt = (m + \frac{1}{2})\lambda$

$2nt = m\lambda$

$r \approx \sqrt{\frac{m\lambda R}{n}}$

$\sin\theta = m\frac{\lambda}{a}$

$I = I_{\max}\left(\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right)^2$

$\theta_{\min} = 1.22\,\frac{\lambda}{D}$

$I = I_{\max}\cos^2\theta$

$\tan\theta_p = \frac{n_2}{n_1}$