Physics 4C Spring 2018 Test 1

Name: Key

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Please show your work! Answers are not complete without clear reasoning. When asked for an expression, you must give your answer in terms of the variables given in the question and/or fundamental constants, including g.

Answer as many questions as you can, in any order. Books, notes, and internet-connectable devices are not allowed. Use any blank space to answer questions, but please make sure it is clear which question your answer refers to.

$g = 9.8\ \mathrm{m/s^2}$

atmospheric pressure $P_0 = 1.013 \times 10^5$ Pa

density of water $\rho_w = 1000\ \mathrm{kg/m^3}$

Trigonometric Identities

$\sin^2\theta + \cos^2\theta = 1$

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$

$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

$\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$

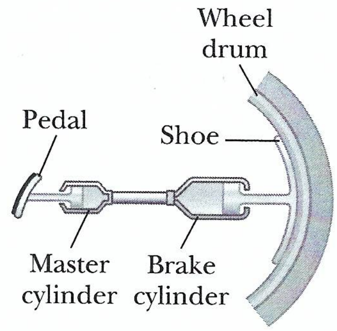
$\cos\left(\theta + \frac{\pi}{2}\right) = -\sin\theta$

$\sec\theta := \frac{1}{\cos\theta}$

$\csc\theta := \frac{1}{\sin\theta}$

$\cot\theta := \frac{1}{\tan\theta}$

1. The diagram shows the essential parts of a hydraulic brake system. The area of the piston in the master cylinder is A_1 and that of the piston in the brake cylinder is a larger area A_2. The coefficient of friction between shoe and wheel drum is μ_k. The wheel has a radius of R, a mass m, and a moment of inertia $\frac{1}{2}mR^2$.



(a) Why is the braking system designed so that the piston in the brake cylinder has a larger area than the piston in the master cylinder attached to the break petal? What principle is used in this design? [3 pts]

(b) Find an expression for the magnitude of the frictional torque about the axle when a force F is exerted on the brake pedal. (Assume the radius of the wheel drum is approximately the same as the radius of the entire wheel.) [4 pts]

(c) If the wheel is initially rotating with an angular speed ω_0 and the force F exerted on the brake pedal is constant, how long does it take for the wheel to come to a stop? [5 pts]

a) When the piston in the brake cylinder has a larger area than the master cylinder piston, this design magnifies the force put onto the petal into a larger force on the wheel drum and therefore a greater friction force slowing the wheel. The principle used is Pascal's principle.

b) $P_1 = P_2$

$\frac{F}{A_1} = \frac{F_2}{A_2}$

$F_2 = \frac{A_2}{A_1} F$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\tau = R f_k$ $\quad f_k = \mu_k N$

F_2 is the normal force

$\tau = R\left(\mu_k \frac{A_2}{A_1} F\right)$

$\underline{\tau = \mu_k \frac{A_2}{A_1} R F}$

c) $\tau = I\alpha \Rightarrow$ const. force $\Rightarrow$ const torque $\Rightarrow$ const α

$\cancel{\omega}^{0} = \omega_0 - \alpha t$

$\omega_0 = \alpha t$

$t = \frac{\omega_0}{\alpha}$ — (1)

$\tau = I\alpha$

$\alpha = \frac{\tau}{I}$

$= \frac{\mu_k \frac{A_2}{A_1} \cancel{R} F}{\frac{1}{2} m R^{\cancel{2}}}$

$\alpha = \frac{2\mu_k A_2 F}{A_1 m R}$

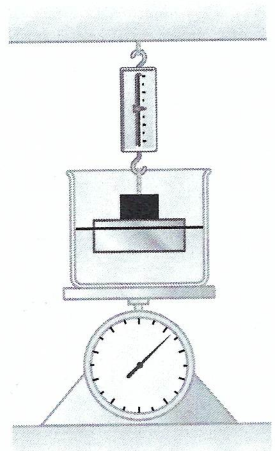
into (1):

$\underline{t = \frac{\omega_0 A_1 m R}{2\mu_k A_2 F}}$

2. A beaker of mass m_b containing oil of mass m_o and density ρ_o rests on a scale. A block of iron of mass m (shown in black in the diagram) is suspended from a spring scale and rests on a foam block of density $\rho_o/2$ that floats on the oil. The foam block is a rectangular prism of height d. Three-quarters of the volume of the foam block is submerged in the oil. The hanging scale shows that the tension in the rope is $\frac{1}{3}mg$.

(a) Find an expression for the base area A of the foam block in terms of m, ρ_o, and d. [8 pts]

(b) Find an expression for the reading on the bottom scale in terms of m_b, m_o, and m. [4 pts]



Let F_{bs} be the reading on the bottom scale.

a) System: foam block

$\vec{F}_B$, $\vec{N}$, $m_{fb}\vec{g}$

$F_{net,y} = 0$

$F_B = N + m_{fb}g$

$\rho_o g V_{sub} = N + m_{fb}g$

$V_{sub} = \frac{3}{4}V_{fb} = \frac{3}{4}Ad$

$\rho_o g\left(\frac{3}{4}Ad\right) = \frac{2}{3}mg + m_{fb}g$

$m_{fb} = \frac{\rho_o}{2}V_{fb} = \frac{\rho_o}{2}Ad$

$\frac{3}{4}\rho_o g d A = \frac{2}{3}mg + \frac{1}{2}\rho_o g d A$

$\frac{1}{4}\rho_o \cancel{g} d A = \frac{2}{3}m\cancel{g}$

$$\underline{A = \frac{8}{3}\frac{m}{\rho_o d}}$$

System: iron block

$\vec{N}$, $\vec{T}$, $m\vec{g}$

$F_{net,y} = 0$

$N + T = mg$

$N = mg - T$

$N = mg - \frac{1}{3}mg$

$N = \frac{2}{3}mg$

b) System: oil + beaker

$\vec{F}_{bs}$, $(m_o+m_b)\vec{g}$, $\vec{F}_B$

$F_{net,y} = 0$

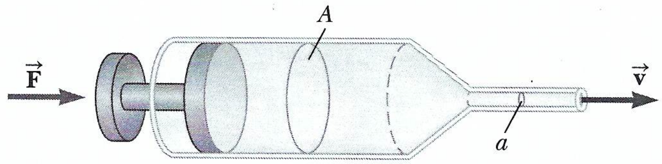
$F_{bs} = (m_o+m_b)g + F_B$

$F_{bs} = (m_o+m_b)g + \rho_o g\left(\frac{3}{4}Ad\right)$

$F_{bs} = \left(m_o+m_b+\cancel{\rho_o}\left(\frac{\cancel{3}}{4}\frac{8}{\cancel{3}}\frac{m}{\cancel{\rho_o d}}\cancel{d}\right)\right)g$

$$\underline{F_{bs} = (m_o+m_b+2m)g}$$

3. A hypodermic syringe contains a fluid with density ρ. The barrel of the syringe has a cross-sectional area A, and the needle has a cross-sectional area a. In the absence of a force on the plunger, the pressure everywhere is $P_0 = 1.00$ atm. A force acts on the plunger, making medicine squirt horizontally from the needle with a speed v.



(a) Find an expression for the volume flow rate through the surface of area A shown in the diagram. [2 pts]

(b) Find an expression for the magnitude of the force on the plunger. [6 pts]

a) Volume flow rate $= \frac{V}{t} = \overbrace{Av_1 = \underline{av}}^{\text{continuity}}$

b) $P_1 + \frac{1}{2}\rho v_1^2 + \cancel{\rho g h_1} = P_2 + \frac{1}{2}\rho v^2 + \cancel{\rho g h_2}$

$h_1 = h_2$

$P_1 = \frac{F}{A} + P_0$

$\cancel{P_0} + \frac{F}{A} + \frac{1}{2}\rho v_1^2 = \cancel{P_0} + \frac{1}{2}\rho v^2$

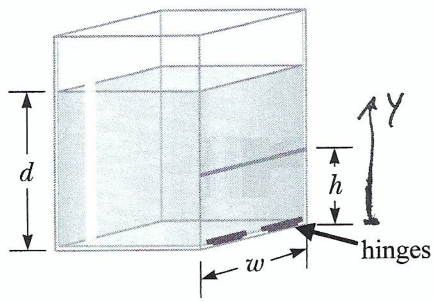
$Av_1 = av$

$v_1 = \frac{a}{A}v$

$\frac{F}{A} = \frac{1}{2}\rho v^2\left(1 - \frac{a^2}{A^2}\right)$

$\underline{F = \frac{1}{2}\rho v^2 A\left(1 - \frac{a^2}{A^2}\right)}$

4. The tank ~~in~~ shown is filled with water of depth d. At the bottom of one sidewall is a rectangular hatch of height h and width w that is hinged at the *bottom* of the hatch. Find the magnitude of the torque exerted by the water about the hinges. [8 pts]



$$d\vec{\tau} = \vec{r} \times d\vec{F}$$

$$d\tau = r\, P\, dA \qquad dA = w\,dy$$

$$d\tau = y(\rho g(d-y))\, w\, dy$$

$$\tau = \int_0^h \rho g w\, ((dy) - y^2)\, dy$$

$$= \rho g w \left[\frac{y^2}{2} d - \frac{y^3}{3}\right]_0^h$$

$$= \rho g w \left[\frac{h^2 d}{2} - \frac{h^3}{3}\right]$$

$$\underline{\tau = \rho g w h^2 \left(\frac{d}{2} - \frac{h}{3}\right)}$$