Physics 4C Spring 2018 Test 3

Name: Key

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Please show your work! Answers are not complete without clear reasoning. When asked for an expression, you must give your answer in terms of the variables given in the question and/or fundamental constants.

Answer as many questions as you can, in any order. Calculators are allowed. Books, notes, and internet connectable devices are not allowed. Use any blank space to answer questions, but please make sure it is clear which question your answer refers to.

$g = 9.8\ \mathrm{ms}^{-2}$

$c = 3.00 \times 10^8\ \mathrm{m/s}$

$\rho_{\mathrm{air}} = 1.20\ \mathrm{kg\ m}^{-3}$ (sea level, 20°C)

$I_0 = 1.00 \times 10^{-12}\ \mathrm{W\ m}^{-2}$

$v = (331\ \mathrm{m/s})\sqrt{1 + \frac{T_{\mathrm{Cel}}}{273}}$

$B = -\frac{\Delta P}{\Delta V / V_i}$

$y(t) = \sum_{n=1}^{\infty} \left(A_n \sin(2\pi n f t) + B_n \cos(2\pi n f t)\right)$

Trigonometric Identities

$\sin^2\theta + \cos^2\theta = 1$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\sin\alpha + \sin\beta = 2\cos\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)$

$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$

$\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$

$\cos\left(\theta + \frac{\pi}{2}\right) = -\sin\theta$

1. A standing-wave pattern is observed in a thin wire with a length of 1.20 m. Let the point $x = 0$ be the left end of the wire. The wave function is

$$y = 3.00 \sin\left(\frac{10\pi}{3}x\right)\cos(180\pi t)$$

where x is in meters, y is in millimeters, and t is in seconds.

(a) What is the wavenumber of this wavefunction? [1 pt]

(b) What is the wave speed on this wire? [2 pts]

(c) How many loops does this pattern exhibit? [3 pt]

(d) Consider an element of the wire at a point $x = 0.05$ m. What is the maximum transverse displacement of this element? [3 pts]

(e) Show that the transverse motion of the element at $x = 0.05$ m is simple harmonic motion (SHM), by referring to the definition of SHM. [5 pts]

a) wavenumber has the symbol k.

$$\underline{k = \frac{10\pi}{3}\,m^{-1}}$$

b) $v = \frac{\omega}{k}$

$$= \frac{180\cancel{\pi}}{\left(\frac{10\cancel{\pi}}{3}\right)}$$

$$\underline{v = 54\,m/s}$$

c) # loops = # of half-λs

$$= \frac{L}{(\lambda/2)} \qquad \lambda = \frac{2\pi}{k}$$

$$= \frac{Lk}{\pi}$$

$$= \frac{(1.20m)\left(\frac{10\cancel{\pi}}{3}\right)}{\cancel{\pi}}$$

$$= \underline{4 \text{ loops}}$$

d) Transverse displacement is y.

@ $x = 0.05$ m

$$y_{max}(x=0.05) = \overset{mm}{3.00}\sin\Big(\underbrace{\tfrac{10\pi}{3}(0.05m)}_{\pi/6}\Big)$$

$$= \underline{1.50\,mm}$$

e) Def. of SHM

$$\frac{\partial^2 y}{\partial t^2} = -\underbrace{c}_{\text{some constant}}\, y$$

i.e. the acceleration of the oscillator is directly proportional to its displacement from equilibrium, but in the opposite direction. $\left(a_y = \frac{\partial^2 y}{\partial t^2}\right)$

at $x = 0.05$ m

$$y = 1.50\cos(180\pi t)$$

we need to show this satisfies the definition.

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial t}\left((-180\pi)\,1.50\sin(180\pi t)\right)$$

$$= -(180\pi)^2\,1.50\cos(180\pi t)$$

$$= \underbrace{-(180\pi)^2}_{\text{a const.}}\,y$$

$\therefore$ satifies definition.

2. Two hollow pipes have the same length. At 20°C, pipe 1 has two adjacent resonant frequencies at 400 Hz and 600 Hz and pipe 2 has two adjacent resonances at 300 Hz and 500 Hz.

   (a) Which tube has one closed end and which tube has two open ends? [2 pts]

   (b) What is the fundamental frequency of each tube? [6 pts]

   (c) What is the length of the tubes? [3 pts]

   (d) How many displacement nodes are present in the 500 Hz resonance in pipe 2? (Include any that may be located at the ends of the tube.) [3 pts]

a) Pipe 1 has two open ends
Pipe 2 has one closed end.

b) Pipe 1:
400 & 600 Hz are adjacent frequencies:

$f_n - f_{n-1} =$

$\cancel{n}f_1 - (\cancel{n}-1)f_1 = 600 - 400 \text{ Hz}$

$\underline{f_1 = 200 \text{ Hz}}$

(Pipe 1)

Pipe 2:
300 & 500 Hz are adjacent frequencies:

$f_{2n+1} - f_{2n-1} = 500 - 300 \text{ Hz}$

$(\cancel{2n}+1)f_1 - (\cancel{2n}-1)f_1 = 200 \text{ Hz}$

$f_1 + f_1 = 200 \text{ Hz}$

$\Rightarrow \underline{f_1 = 100 \text{ Hz}}$

(Pipe 2)

c) Consider pipe 1:

$L = \frac{n\lambda_n}{2}$ (Pipe contains a whole # of half wavelengths)

and $\lambda_n = \frac{v}{f_n}$

$L = \frac{nv}{2f_n}$

For the fundamental $n=1$

$L = \frac{v}{2f}$

$= \frac{343 \text{ m/s}}{2(200 \text{ Hz})}$

$= 0.8575 \text{ m}$

$\underline{L = 0.858 \text{ m}}$

(Check: pipe 2 fundamental

$L = \frac{\lambda_1}{4}$ $\lambda_1 = \frac{v}{f_1}$

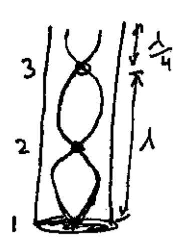
$L = \frac{v}{4f_1}$

$= \frac{(343 \text{ m/s})}{4(100 \text{ Hz})}$

$= 0.858 \text{ m}$ ✓)

d) $500 \text{ Hz} = 5f_1$

$\Rightarrow$ ⓐ 500 Hz is 5^{th} harmonic and the tube contains 5 quarter wavelenths or $\frac{5\lambda}{4}$.



$\underline{\text{3 displacement nodes.}}$

3. At a distance d from a loudspeaker the sound level is β.

   (a) Give an expression for the average power output by the loudspeaker. (You may give your answer in terms of I_0.) [5 pts]

   (b) How far must one go away from the speaker so that the sound level is reduced to $\beta/2$? [6 pts]

a) $I = \frac{P_{sp}}{4\pi d^2}$ avg power output by speaker

$P_{sp} = 4\pi d^2 I$

$\beta = 10 \log_{10}\left(\frac{I}{I_0}\right)$

$\frac{\beta}{10} = \log_{10}\left(\frac{I}{I_0}\right)$

$I = 10^{\beta/10} I_0$

$\underline{P_{sp} = 4\pi d^2 \left(10^{\beta/10} I_0\right)}$

b) Let r be the distance from the speaker where the sound level is $\frac{\beta}{2}$ and I' be the intensity at that point.

Two ways to solve:

way 1:

$\beta - \frac{\beta}{2} = 10 \log_{10}\left(\frac{I}{I_0}\right) - 10 \log_{10}\left(\frac{I'}{I_0}\right)$

$\frac{\beta}{2} = 10 \log_{10}\left(\frac{I}{\cancel{I_0}} \frac{\cancel{I_0}}{I'}\right)$

$10^{\beta/20} = \frac{I}{I'}$

using ★:

$P_{sp} = 4\pi d^2 I = 4\pi r^2 I'$

$\Rightarrow \frac{I}{I'} = \frac{r^2}{d^2}$

$10^{\beta/20} = \frac{r^2}{d^2}$

$\underline{r = 10^{\beta/40} d}$

way 2:

$\frac{\beta}{2} = 10 \log_{10}\left(\frac{I'}{I_0}\right), \quad I' = \frac{P_{sp}}{4\pi r^2}$

$\frac{\beta}{2} = 10 \log_{10}\left(\frac{\frac{\cancel{4\pi} d^2 (10^{\beta/10} \cancel{I_0})}{\cancel{4\pi} r^2}}{\cancel{I_0}}\right)$ using Part a)

$\frac{\beta}{20} = \log_{10}\left(\frac{d^2}{r^2} 10^{\beta/10}\right)$

$10^{\beta/20} = 10^{\beta/10} \frac{d^2}{r^2}$

$r^2 = 10^{\beta/20} d^2$

$\underline{r = 10^{\beta/40} d}$

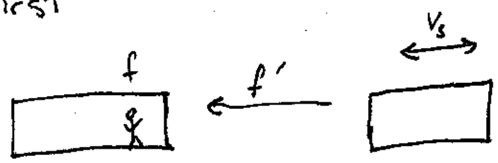
4. Two trains are nearby each other and both are blowing their whistles which have the same frequency f. A passenger on one of the trains hears beats with constant frequency when her train is stationary (at rest). Then her train starts to move *toward* the other train and accelerates until it reaches a speed u. During the acceleration she hears the beat frequency decrease steadily by an amount Δf_b (without ever increasing). Assume the other train has been moving with a constant velocity the entire time. Let the speed of sound in air be v.

   (a) Is the other train moving toward or away from the passenger's train? [3 pts]

   (b) Find an expression for the other train's speed. [8 pts]

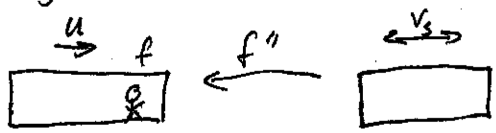
passenger hears frequency f from the whistle on her train interfering with the Doppler shifted whistle of the other train.

at first



$f_{b,i} = |f' - f|$ $\qquad f' = \left(\frac{V}{V \mp V_s}\right) f$

passengers train moves @ speed u



$f_{b,f} = |f'' - f|$ $\qquad f'' = \left(\frac{V+u}{V \mp V_s}\right) f$

a) away from. The beat frequency decreases $\Rightarrow f''$ is closer to f than f' is to f.

Since the effect of the observer's train moving towards the other train is to raise the freq. heard from the other train $f'' > f'$ $\Rightarrow f' < f$, so the other train moves away and $f' = \left(\frac{V}{V+V_s}\right) f$. [Note that if $\vec{u} = \vec{V}_s$, $f_b = 0$.]

b) Δf_b is the amount of change in beat freq. (i.e. assume a +ve #)

$$\therefore \Delta f_b = |f_{b,f} - f_{b,i}|$$
$$= f_{b,i} - f_{b,f}$$
$$= (\cancel{f} - f') - (\cancel{f} - f'')$$
$$= f'' - f'$$
$$= \left[\frac{\cancel{V} + u}{V + V_s} - \frac{\cancel{V}}{V + V_s}\right] f$$
$$\Delta f_b = \frac{u}{V + V_s} f$$

$$\frac{\Delta f_b}{f} = \frac{u}{V + V_s}$$
$$V + V_s = \frac{uf}{\Delta f_b}$$
$$V_s = \frac{uf}{\Delta f_b} - V$$