Name: Key

Physics 4C Spring 2020 Test 4 (Waves)

Note for part (d). It doesn't work to just put k' into the final answer for (c). Why? Because that would be changing the length of the string. L is const. When $k \to k'$, $L \not\to \frac{5\pi}{k'}$, it stays $\frac{5\pi}{k}$ ← original value.

1. A standing wave on a string is created by sending a sine wave

$$y(x,t) = A\sin(kx - \omega t)$$

down the string and letting that wave reflect back from the end of the string where it is fixed in place with a clamp. The string has a mass m. Suppose a standing wave pattern with 5 loops is produced on the string.

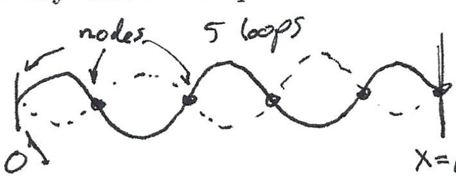
Give your answers in terms of A, k, ω, and m, as needed.

(a) Let one end of the string be at $x = 0$. What are the x coordinates of the nodes? [5 pts]

(b) What is the fundamental frequency of the string? [3 pts]

(c) Find an expression for the tension in the string. [6 pts]

(d) Now imagine the tension in the string is changed so that, keeping the frequency constant, the string now vibrates in its fundamental mode. By what multiplicative factor does k change? By what multiplicative factor does the tension change? [6 pts]



incoming, reflected

$$y_s = A\sin(kx - \omega t) + A\sin(kx + \omega t)$$

$$= 2A\sin(kx)\cos(\omega t) \quad (*)$$

$2A\sin(kx)$: Amplitude @ point x

a) Nodes when $\sin kx = 0$

$\Rightarrow kx = 0, \pi, \ldots 5\pi$

(6 nodes) $x = 0, \frac{\pi}{k}, \frac{2\pi}{k}, \frac{3\pi}{k}, \frac{4\pi}{k}, \frac{5\pi}{k}$

or

5 loops $\Rightarrow$ 5th harmonic $\frac{2L}{5} = \lambda$

$\frac{2L}{5} = \frac{2\pi}{k}$

$L = \frac{5\pi}{k}$

so nodes every $\frac{\pi}{k}$, $0 \to \frac{5\pi}{k}$

b) 5th harmonic so

$5f_1 = f$ ← freq of string now

$5f_1 = \frac{\omega}{2\pi}$ ← ang. freq. of string (*)

$f_1 = \frac{\omega}{10\pi}$

c) $v = \sqrt{\frac{F_T}{\mu}}$ ← Tension $\Rightarrow F_T = v^2\mu$

$= \left(\frac{\omega}{k}\right)^2\left(\frac{m}{L}\right)$

$= \frac{\omega^2 m}{k^2\left(\frac{5\pi}{k}\right)}$

$F_T = \frac{\omega^2 m}{5\pi k}$

d)

ω const

$\lambda' = 5\lambda$

$\frac{2\pi}{k'} = 5\frac{2\pi}{k}$

$k' = \frac{1}{5}k$

factor of $\frac{1}{5}$

$v' = \frac{\omega}{k'} = \frac{\omega}{k/5} = 5v$

$F_T' = (v')^2\mu$ ← const.

$= (5v)^2\mu$

$= 25F_T$

factor of 25