De Anza College

Conceptual Physics
Projectiles
Motion of Planets

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De Anza College

July 19, 2017

Last time

- angular momentum
- gravity
- gravitational field
- black holes

Overview

- projectile motion
- orbital motion
- escape speed
- Kepler's Laws

Projectiles

projectile

Any object that is thrown. We will use this word specifically to refer to thrown objects that experience a vertical acceleration g.

For projectile motion, we assume air resistance is negligible. This gives symmetrical parabolic trajectories.

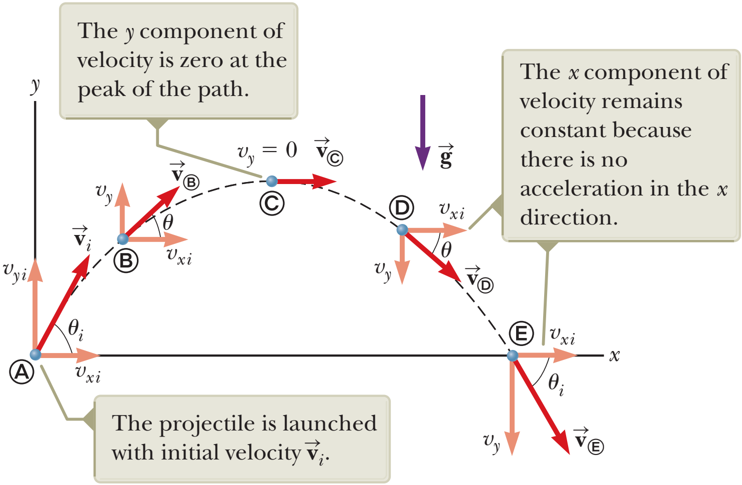
Main Idea

Motion in perpendicular directions can be analyzed separately.

A vertical force (gravity) does not affect horizontal motion.

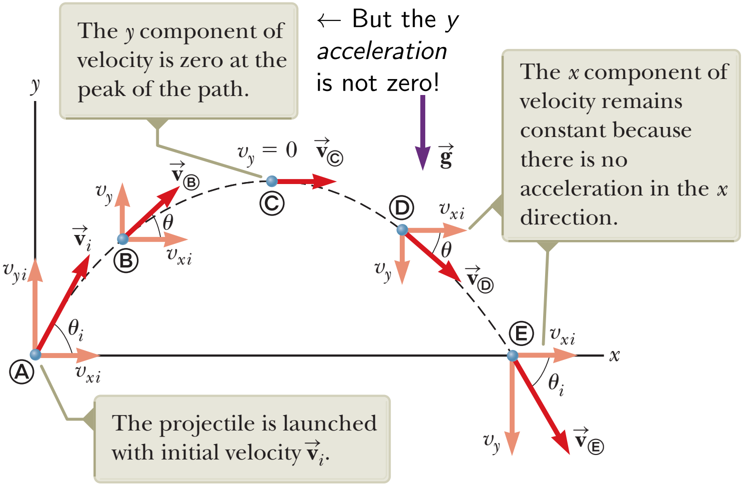
The horizontal component of velocity is constant.

Projectile Velocity



[1]Figure from Serway & Jewett, 9th ed.

Projectile Velocity



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Motion of projectiles

We already considered the motion of objects dropped from rest at time $t = 0$, allowed to fall freely. (Calling up positive.)

Velocity of the object at time t:

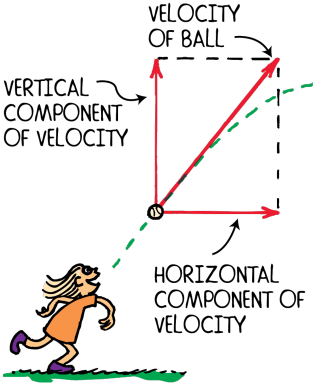
$$v = v_i - gt$$

(Magnitude of the) distance the object falls in time t:

$$d = \frac{1}{2}gt^2$$

Acceleration due to gravity and kinematics

Let's think about the components of the motion separately.



Vertical (y-direction):

$$v_y = v_{i,y} - gt$$

$$d_y = v_{i,y}t - \frac{1}{2}gt^2$$

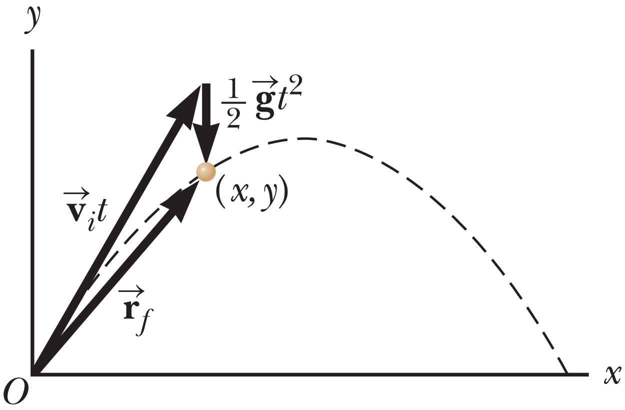
Horizontal (x-direction):

$$a_x = 0\ ,\ \ v_x = v_{i,x}$$

$$d_x = v_{i,x}t$$

[1] Drawing by Hewitt, via Pearson.

Projectile's Trajectory



The object would move in a straight line, but the force of gravity causes it to fall as it moves to the right.

[1]Figure from Serway & Jewett, 9th ed.

Example Problem

Suppose the pellet-gun on the previous slide can fire the pellet with an extremely high velocity.

(a) How many meters below the line of sight would the pellet be after 5 seconds?

(b) If the horizontal component of the pellet's velocity is 20 m/s, how far downrange is the pellet after those 5 seconds?

[0] See Hewitt, page 175.

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(Hint: we can consider each component of the velocity separately.)
answers: (a) 125 m, (b) 100 m

[0]See Hewitt, page 175.

Another example, problem 1, page 192

A ball is thrown horizontally from a cliff at a speed of 10 m/s. What will its speed be (roughly) after 1s?

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answer: 14 m/s

[0]See Hewitt, page 192.

Last one, # 4, page 192

A steel ball is fired horizontally at 8.0 m/s from from a 1.0 m-high table top.

Show that a 20 cm tall coffee can placed on the floor 3.2 m from the base of the table will catch the ball.

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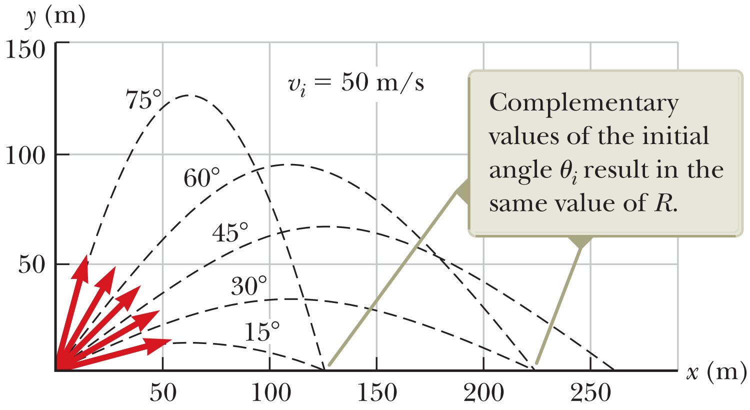
for a falling object

$$d = \frac{1}{2}gt^2$$

Effect of changing launch angle

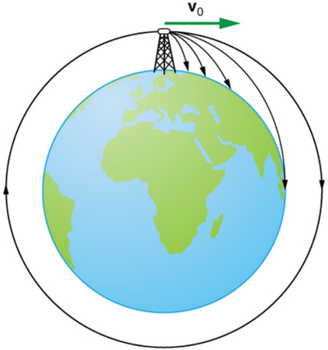


[1]Figure from Serway & Jewett, 9th ed.

Satellites

Imagine now that you could throw an object straight out horizontally so fast that as it falls it matches the Earth's curvature.

The object falls 1 m in 0.45 s. If it goes forward 3,570 m in that time, then the Earth's surface has also curved one meter away over that distance.



This corresponds to a speed of $7,900$ m/s.

[1]Figure from "College Physics", OpenStax College, Ch 3, pg 110.

Satellites

This is the same principle by which the Moon stays in orbit around the Earth. The Moon is the Earth's natural satellite.

The same idea is applied to artificial satellites that are put into orbit around the Earth.

The are moving at very high speeds and continuously falling around the Earth.

Satellites can have many purposes, *eg.*

- telecommunications
- weather and Earth-monitoring
- GPS (global positioning system)
- human spaceflight and experiments

Weightlessness

Astronauts aboard the ISS experience weightlessness (or "microgravity").

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The astronauts still have a force due to gravity on them of

$$W' = mg'$$

(their weight).

Why do they feel weightless?

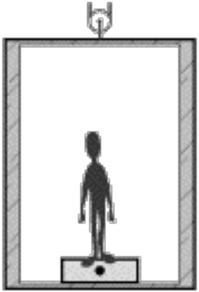
The Sensation of Weight

The feeling of your weight comes from the normal force that surfaces exert on you.

This is also the force that a scale measures by compressing an internal spring.

In situations where the normal force on you changes you feel "lighter" or "heavier"

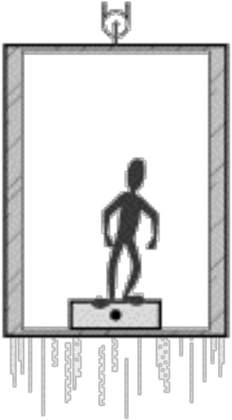
Elevators



$$\mathbf{a} = 0$$

Elevator is at rest or moving with constant velocity. You feel the same as you normally do. Your weight and normal force are both of magnitude mg.

Elevators



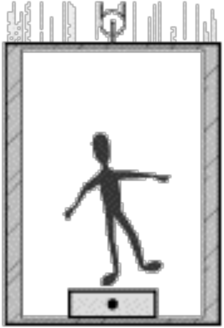
$\mathbf{a} = +a\,\mathbf{j}$ (a is a positive number)

Elevator could be moving upward increasing speed or downward decreasing speed. You feel as if your weight has increased.

Your weight is $-mg\,\mathbf{j}$, but the normal force is $\mathbf{n} = m(g + a)\,\mathbf{j}$.

The normal force is increased!

Elevators



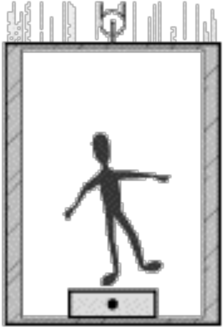
$\mathbf{a} = -a\,\mathbf{j}$ (a is a positive number)

Elevator could be moving upward and slowing down or moving downward increasing speed. You feel as if your weight has decreased.

Your weight is $-mg\,\mathbf{j}$, but the normal force is $\mathbf{n} = m(g - a)\,\mathbf{j}$.

The normal force is decreased!

Elevator in Free Fall



$$\mathbf{a} = -g\,\mathbf{j} \quad (g = 9.8 \text{ m s}^{-2})$$

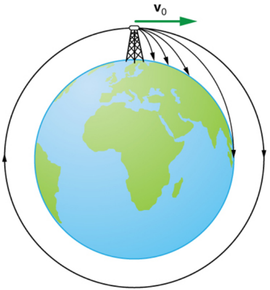
The elevator cable is cut and the elevator falls freely.

You feel weightless! Your weight is $-mg\,\mathbf{j}$, but the normal force is $\mathbf{n} = 0$.

Weightlessness

Astronauts aboard the ISS still have a force due to gravity on them, but they are freely falling together with the ISS around the Earth!

This is the same as being in the falling elevator. There is no normal force on them (unless they push off a wall), so they can drift about the inside of the ISS.



Some Types of Earth-Orbit

- Low Earth Orbit (LEO) - altitude from 160 km to 2,000 km with a period of 90–130 minutes; all human spaceflight except Apollo has been in this orbit (or suborbital), also Earth observation satellites

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- Geosynchronous Orbit (GSO) - altitude of 35,786 km, the satellite orbits the Earth in one day - if it is in an equatorial orbit (GEO), it stays above a fixed point on the Earth's surface and appears fixed in the sky; telecommunications satellites

Escape Speed

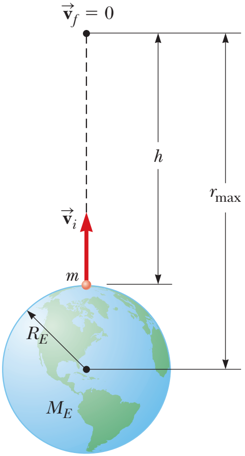
escape speed

The speed an object would need to have at Earth' surface to have enough kinetic energy to leave Earth, overcoming the attraction of Earth's gravity.

The object will convert its kinetic energy to potential energy as it moves higher and higher, away from the Earth.

Escape Speed

How fast does an object need to be projected with to escape Earth's gravity?



Escape Speed

To escape the surface of the Earth, an object must have a starting speed of

$$v_e = 1.12 \times 10^4 \text{ m/s} = 11.2 \text{ km/s}$$

For other planets / celestial bodies the escape speeds can also be defined.

The escape speed depends on the radius of the body and its mass.

For the Sun, the escape speed at its surface is much higher:

$$v_e = 620 \text{ km/s}$$

Motion of the Planets

The planets in our solar system orbit the Sun. (As planets in other systems orbit their stars.)

This is called a *heliocentric* model.

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Nicolaus Copernicus (early 1500s A.D.) is credited with the paradigm since he developed a mathematical model and took seriously the idea that the implication was that the Earth moved around the Sun, but others had similar thoughts:

- Aristarchus of Samos (c. 270 BCE)
- Martianus Capella (400s A.D.)
- Aryabhata (500s A.D.), Nilakantha Somayaji (1500s A.D.)
- Maragha school of astronomy in Persia (1200s A.D.)

Motion of the Planets

After Copernicus's proposal, Tycho Brahe gathered a lot of data about the positions of stars and planets.

Johannes Kepler inherited Brahe's data and did the calculations to deduce a complete model.

Galileo gathered additional data that supported the heliocentric model and popularized it.

Kepler's Laws

Kepler's Three laws give simple rules for predicting stable planetary orbits.

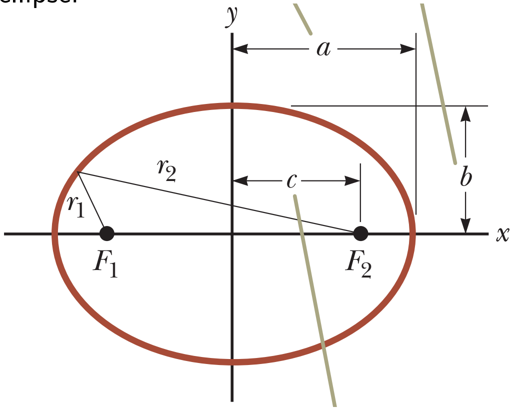
1. All planets move in elliptical orbits with the Sun at one focus.

2. The a line from the Sun to a planet sweeps out equal areas of space in equal time intervals.

3. The square of the orbital period of any planet is proportional to the cube of the distance from the planet to the Sun. ($T^2 \sim r^3$)

Kepler's First Law: Elliptical Orbits

Defining an ellipse:



a is the *semimajor axis*

$$a^2 = b^2 + c^2$$

b is the *semiminor axis*

c is the distance from the center of the ellipse to the focus

e is the *eccentricity* of the ellipse $e = c/a$

Kepler's First Law: Elliptical Orbits

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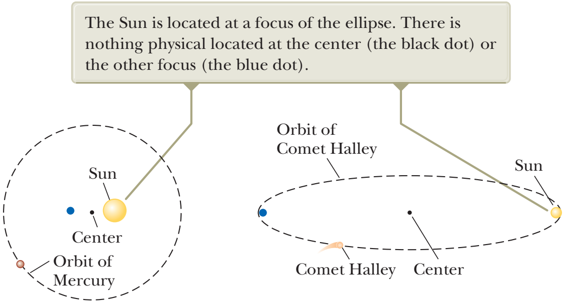
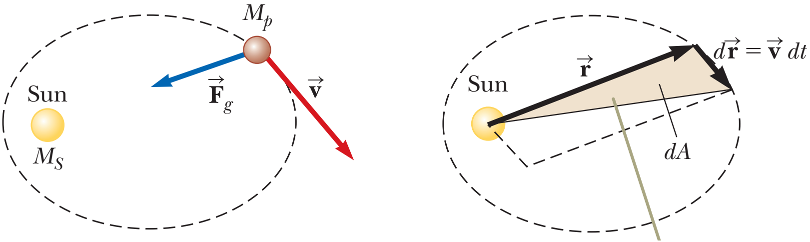


- The planets' orbits are close to circular. (Mercury's is the least circular.)
- Halley's Comet has an orbit with a high eccentricity.

Kepler's Second Law: Equal Areas in Equal Time

"The radius vector drawn from the Sun to a planet sweeps out equal areas in equal time intervals."

What does it mean?



When the planet is closer to the Sun, it must be moving faster.

Kepler's Third Law: $T^2 \sim r^3$

"The square of the orbital period of any planet is proportional to the cube of the distance from the planet to the Sun."

$$T^2 = K_s \, r^3$$

where K_s is a constant,

$K_s = \frac{4\pi^2}{GM_s} = 2.97 \times 10^{-19} \text{ s}^2 \text{ m}^{-3}$.

Kepler's Third Law: $T^2 \sim r^3$

Planets orbiting further from the Sun take longer to complete an orbit.

Body	Period of Revolution (s)	Mean Distance from the Sun (m)
Mercury	7.60×10^6	5.79×10^{10}
Venus	1.94×10^7	1.08×10^{11}
Earth	3.156×10^7	1.496×10^{11}
Mars	5.94×10^7	2.28×10^{11}
Jupiter	3.74×10^8	7.78×10^{11}
Saturn	9.29×10^8	1.43×10^{12}
Uranus	2.65×10^9	2.87×10^{12}
Neptune	5.18×10^9	4.50×10^{12}

Kepler's Laws

By 1621 Kepler had published all three of his laws.

By 1687, Newton was able to show that all three of Kepler's laws were a consequence of his laws of motion and his law of gravitation.

Summary

- projectile motion
- orbits
- motion of planets and satellites
- Kepler's laws
- escape speed

Midterm tomorrow, July 20th.

Homework Hewitt,

- Ch 10, onward from page 190. Exercises: 15, 19, 23, 45, 59; Problems: 3
- Study!
- read through the textbook
- Bring a calculator and a scantron!