Conceptual Physics
Motion and Graphs
Free Fall
Using Vectors

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Last time

- Units
- More about size and scale
- Motion of objects
- Inertia
- Quantities of motion

Overview

- graphs of motion with time
- free fall
- Vectors
- relative motion

Speed, velocity, acceleration

Speed is the rate of distance covered with time.

Velocity, v, is speed with direction specified.

Acceleration, a, is the rate of change of velocity with time.

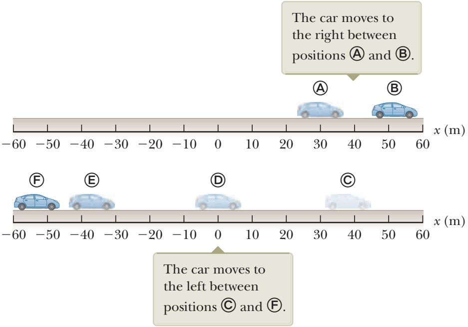
Graphs and Physics

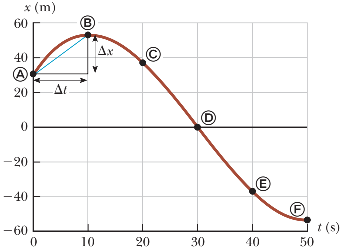
Graphs represent the values of a function (*eg.* $f(x)$) as a variable changes (*eg.* x).

In physics, the function and the variable are physical quantities: things we can measure, *eg.* position and time.

The slope tells us about the rate that one quantity changes when we change the other.

Position vs. Time Graphs

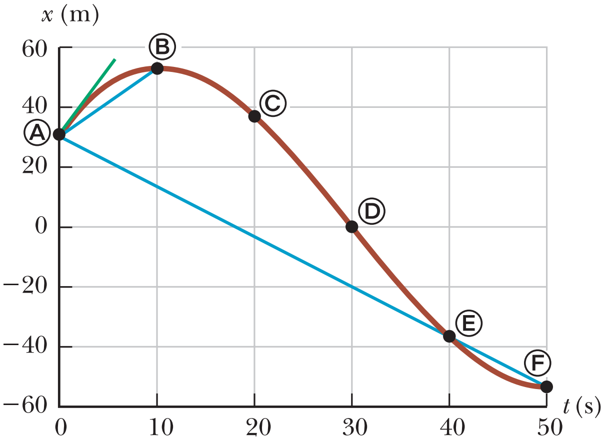




[1]Figures from Serway & Jewett

Average Velocity in Position vs. Time Graphs

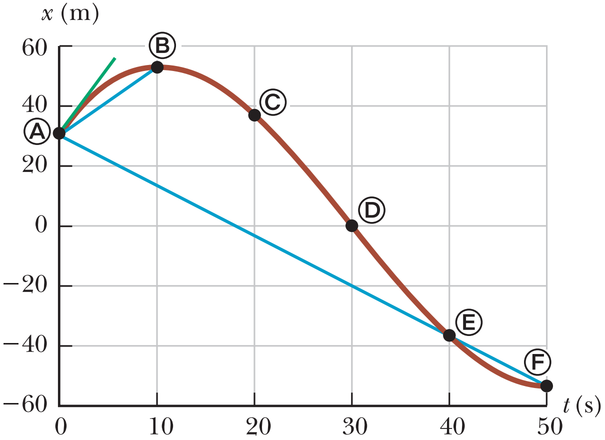
Green line: slope is the instantaneous velocity at point A.



Blue lines: slopes are the average velocities from A→B and A→E.

Average Velocity in Position vs. Time Graphs

A→B: $v_{avg} = \frac{\Delta x}{\Delta t} = \frac{50\text{m}-30\text{m}}{10\text{s}-0\text{s}} = 2$ m/s



A→E: $v_{avg} = \frac{\Delta x}{\Delta t} = \frac{-35\text{m}-30\text{m}}{40\text{s}-0\text{s}} = -1.6$ m/s

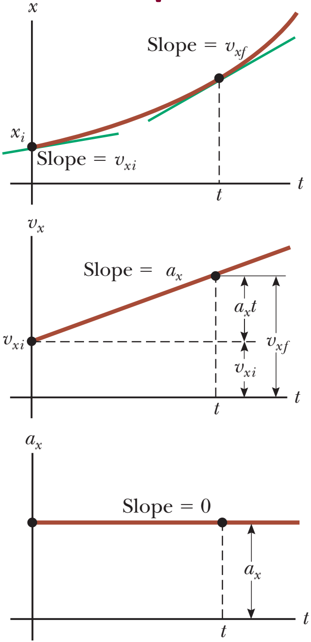
Relating Position, Velocity, Acceleration graphs

For a single moving object, the graphs of its position, velocity, and acceleration are not independent!

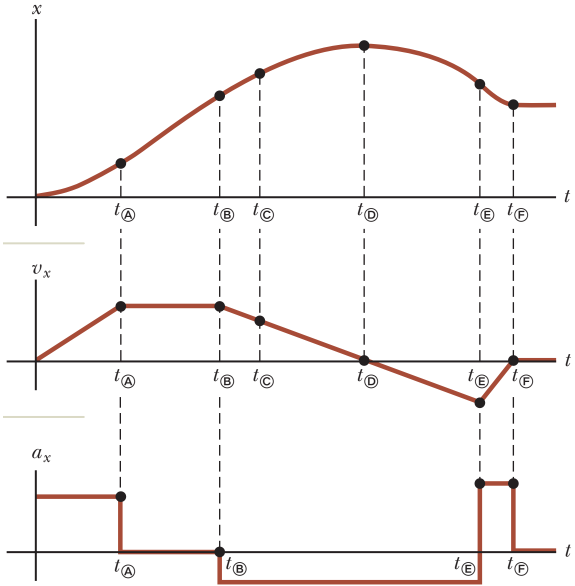
The slope of the position-time graph is the velocity.

The slope of the velocity-time graph is the acceleration.

Constant Acceleration Graphs

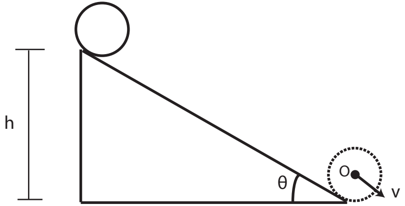


Acceleration vs. Time Graphs



Acceleration and Free Fall

Galileo also reasoned about the **acceleration due to gravity** by thinking more about inclined surfaces.



The steeper the incline the larger the acceleration.

Starting from rest:

final velocity $=$ acceleration $\times$ time.

Free Fall

When the ball drops straight downward, it gains approximately 10 m/s of speed in each second.

Time of fall (s)	Velocity acquired (m/s)
0	0
1	10
2	20
3	30
⋮	⋮

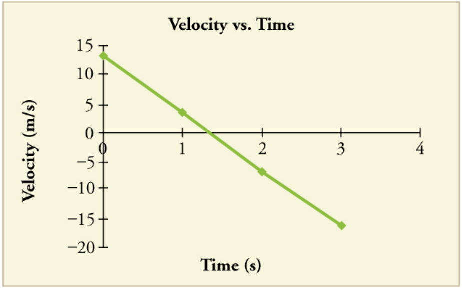
This is a constant acceleration! We call this acceleration g.

$$g = 9.8 \text{ m s}^{-2} \approx 10 \text{ m s}^{-2}$$

g is about 10 meters-per-second-per-second

Free Fall

Constant acceleration corresponds to a straight line on a graph of velocity and time.



Calling up positive and down negative:

$$v = v_i - gt$$

Object's velocity changes by 10 m/s each second.

When dropped from rest, after 6.5 s, (roughly) what is the ball's speed?

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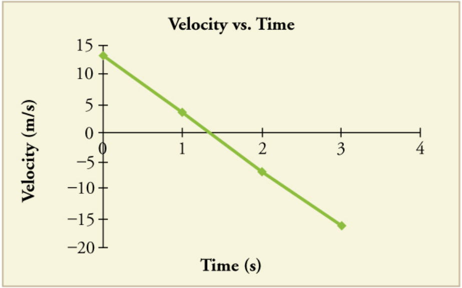

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When dropped from rest, after 6.5 s, (roughly) what is the ball's speed? 65 m/s

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65 m/s is very fast. ($\approx$ 145 mi/hr)

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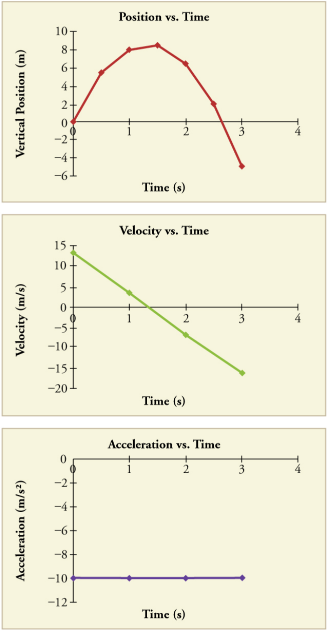
How *far* does the ball travel in those 6.5 s (ignoring air resistance)?

distance an object falls (starting from rest) in time t:

$$d = \frac{1}{2}gt^2$$

This corresponds to the area under the velocity-time graph.

Falling Objects



[1] OpenStax Physics

Free Fall Questions

A free-falling object has a speed of 30 m/s (downward) at one instant. Exactly 1 s later its speed will be

A the same.

B 35 m/s.

C more than 35 m/s.

D 60 m/s.

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D 60 m/s.

It will be 40 m/s. (Assuming $g = 10$ m/s^2)

Free Fall Questions

What is the distance covered by a freely falling object starting from rest after 4 s?

A 4 m

B 16 m

C 40 m

D 80 m

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Review questions for motion

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Why does a stream of water get narrower as if falls from a faucet?

Is the acceleration due to gravity always 9.8 m s^{-2}?

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[1] Hewitt, page 49.

Describing Vectors: Axes

To indicate which way an arrow (or a force, acceleration, *etc.*) points, we need to have another arrow that we can compare to.

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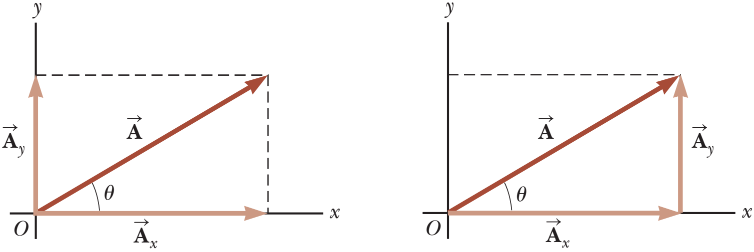
For example, North-South and West-East can be **reference axes.**

We could also choose axes “up” and “down” (vertical), and parallel to the horizon (horizontal).

We typically call the direction axes x and y.

Vectors

To describe where the vector **A** points, we can say, you count some distance along the x direction, the some distance along the y direction.



[1]Figure from Serway & Jewett, Physics for Scientists and Engineers, 9th ed.

Representing Vectors: Unit Vectors

We can count along the directions with special units: *unit vectors.*

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Unit vectors have a length of one unit.

A unit vector in the x direction is usually written $\mathbf{i}$

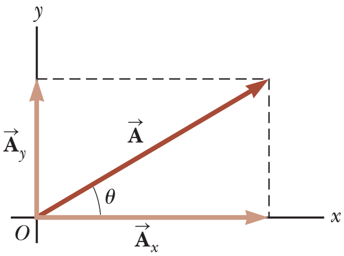
A unit vector in the y direction is usually written $\mathbf{j}$.

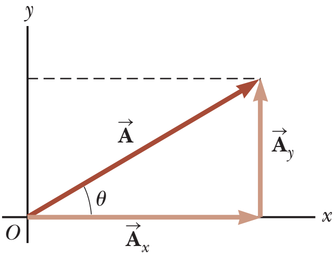
Components

Vector **A** is the sum of a piece along x and a piece along y:

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j}.$$

x component **y component**

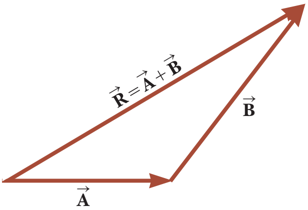


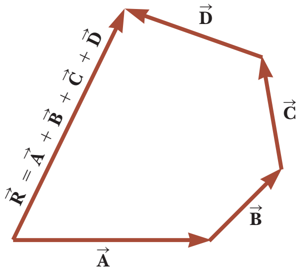


Notice that $A_x = A\cos\theta$ and $A_y = A\sin\theta$.

Adding Vectors

A + **B**

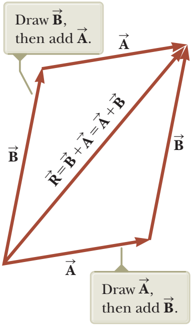




Adding Vectors

Order of addition doesn't matter!

$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$



Using Vectors

Example

Andy runs 100 m south then turns west and runs another 50.0 m. All this takes him 15.0 s.

What is his displacement from his starting point?

What is his average velocity?

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Example

Andy runs 100 m south then turns west and runs another 50.0 m. All this takes him 15.0 s.

What is his displacement from his starting point?

What is his average velocity?

answer: displacement: 112 m

average velocity: 7.45 m/s, in a direction 26.6° west of south.

Thinking about Vectors

What can you say about two vectors that add together to equal zero?

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What can you say about two vectors that add together to equal zero?

When can a nonzero vector have a zero horizontal component?

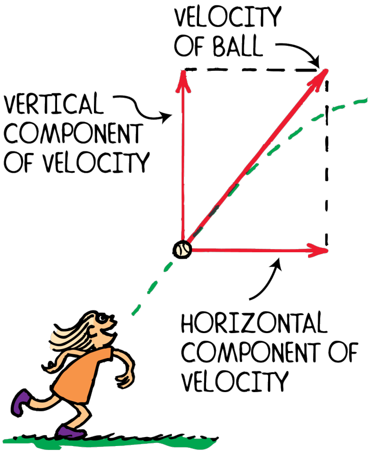
Using Vectors

We've seen how to add vectors, but what do we use this for?

We'll look at examples.

- Finding components of velocity.
- Relative motion.
- Finding net force.

Components of Velocity

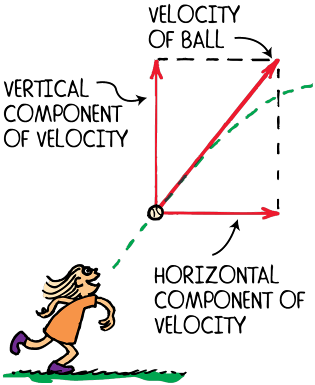


If we know the vertical component of velocity, we can find the time of flight, the maximum height, etc. of the ball.

[1]Drawing by Hewitt, via Pearson.

Acceleration due to gravity and kinematics

Let's think about the components of the motion separately.



Vertical (y-direction):

$$v_y = v_{i,y} - gt$$

$$d_y = v_{i,y}t - \frac{1}{2}gt^2$$

Horizontal (x-direction):

$$a_x = 0\ ,\ \ v_x = v_{i,x}$$

$$d_x = v_{i,x}t$$

We will return to this in a later chapter!

[1] Drawing by Hewitt, via Pearson.

Linear Motion

When we say something is moving, we mean that it is moving **relative** to something else.

Motion is relative.

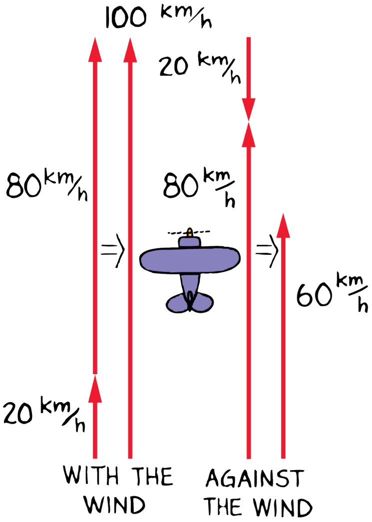
In order to describe measurements of

- where something is
- how fast it is moving

we must have **reference frames**.

An example of references for time an space might be picking an object, declaring that it is at rest, and describing the motion of all objects relative to that.

Intuitive Example for Relative Velocities



Intuitive Example

Now, imagine an airplane that is flying North at 80 km/h but is blown off course by a cross wind going East at 60 km/h.

How fast is the airplane moving relative to the ground? In which direction?

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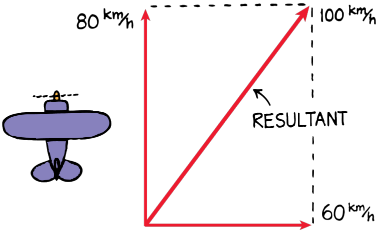

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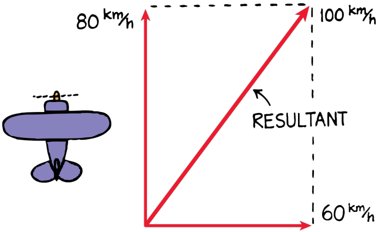


$\mathbf{v} = 100$ km/h at 36.9° East of North (or 53.1° North of East)

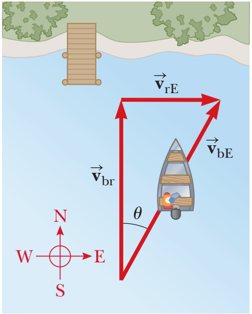
Relative Motion and Components of Velocity

Hewitt, page 79, ranking question 4.

Three motorboats crossing a river.

Relative Motion Example

A boat crossing a wide river moves with a speed of 10.0 km/h relative to the water. The water in the river has a uniform speed of 5.00 km/h due east relative to the Earth. If the boat heads due north, determine the velocity of the boat relative to an observer standing on either bank.[1]



[2]Page 97, Serway & Jewett

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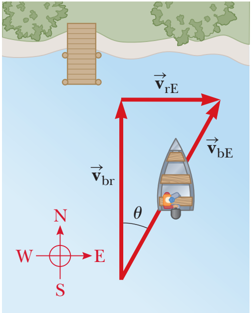

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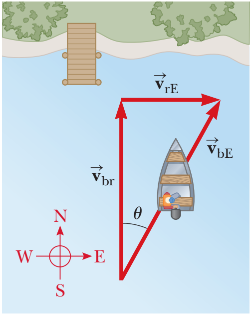

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Simply use vector addition to find v_{bE}.

$$v_{bE} = \sqrt{10^2 + 5^2} = 11.2 \text{ km/h}$$

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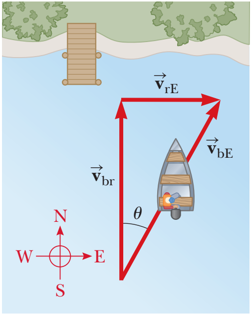

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$$\theta = \tan^{-1}\left(\frac{5}{10}\right) = 26.6^\circ$$

[2]Page 97, Serway & Jewett

Summary

- graphing motional quantities
- free fall
- vectors

Homework

Worksheets,

- graphs worksheet (for Tues)
- vector worksheet (for Tues)

Hewitt, Ch 3, onward from page 47

- Review Questions: 3, 9, 21
- Plug and Chug: 5, 13, 21
- Exercises: 15, 17, 19, 41
- read pages 74-77, in Ch 5
- Ch 5, onward from page 78. Exercises: 31, 33