Conceptual Physics
Rotational Motion

Lana Sheridan

De Anza College

July 17, 2017

Last time

- energy sources discussion
- collisions (elastic)

Overview

- inelastic collisions
- circular motion and rotation
- centripetal force
- fictitious forces
- torque
- moment of inertia
- center of mass
- angular momentum

Types of Collision

There are two different types of collisions:

Elastic collisions

are collisions in which none of the kinetic energy of the colliding objects is lost. $(K_i = K_f)$

Types of Collision

There are two different types of collisions:

Elastic collisions

are collisions in which none of the kinetic energy of the colliding objects is lost. $(K_i = K_f)$

Inelastic collisions

are collisions in which energy is lost as sound, heat, or in deformations of the colliding objects.

Types of Collision

There are two different types of collisions:

Elastic collisions

are collisions in which none of the kinetic energy of the colliding objects is lost. $(K_i = K_f)$

Inelastic collisions

are collisions in which energy is lost as sound, heat, or in deformations of the colliding objects.

When the colliding objects stick together afterwards the collision is *perfectly inelastic*.

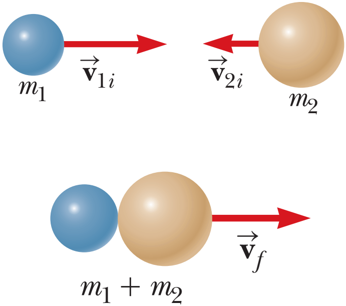
Inelastic Collisions

For general inelastic collisions, some kinetic energy is lost.

But we can still use the conservation of momentum:

$$p_i = p_f$$

Perfectly Inelastic Collisions



Now the two particles stick together after colliding $\Rightarrow$ same final velocity!

$$p_i = p_f \quad \Rightarrow \quad m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = (m_1 + m_2)\mathbf{v}_f$$

Inelastic Collision Example

From page 91-92 of Hewitt:

Two freight rail cars collide and lock together. Initially, one is moving at 10 m/s and the other is at rest. Both have the same mass. What is their final velocity?

Inelastic Collision Example

From page 91-92 of Hewitt:

Two freight rail cars collide and lock together. Initially, one is moving at 10 m/s and the other is at rest. Both have the same mass. What is their final velocity?

$$\begin{aligned} \mathbf{p}_{\text{net},i} &= \mathbf{p}_{\text{net},f} \\ m\mathbf{v}_i &= (2m)\mathbf{v}_f \\ 10m &= 2mv_f \end{aligned}$$

The final mass is twice as much, so the final speed must be only half as much: $v_f = 5$ m/s.

Collision Question

Two objects collide and move apart after the collision. Could the collision be inelastic?

(A) Yes.

(B) No.

Collision Question

Two objects collide and move apart after the collision. Could the collision be inelastic?

(A) Yes. $\longleftarrow$

(B) No.

Question

In a perfectly inelastic one-dimensional collision between two moving objects, what condition alone is necessary so that the final kinetic energy of the system is zero after the collision?

(A) The objects must have initial momenta with the same magnitude but opposite directions.

(B) The objects must have the same mass.

(C) The objects must have the same initial velocity.

(D) The objects must have the same initial speed, with velocity vectors in opposite directions.

[1] Serway & Jewett, page 259, Quick Quiz 9.5.

Question

In a perfectly inelastic one-dimensional collision between two moving objects, what condition alone is necessary so that the final kinetic energy of the system is zero after the collision?

(A) The objects must have initial momenta with the same magnitude but opposite directions. $\leftarrow$

(B) The objects must have the same mass.

(C) The objects must have the same initial velocity.

(D) The objects must have the same initial speed, with velocity vectors in opposite directions.

[1] Serway & Jewett, page 259, Quick Quiz 9.5.

Rotational Motion

Objects can move through space, but they can have another kind of motion too:

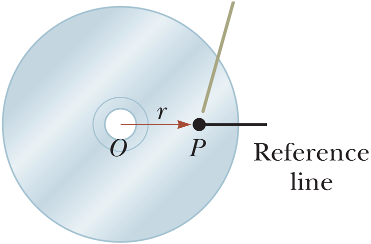
They can rotate about some axis.

Examples of rotating objects:

- the Earth, makes a complete rotation once per day
- merry-go-rounds
- records / cds on a player

Rotating disk

Consider a marked point P on the disk. As time passes it moves:



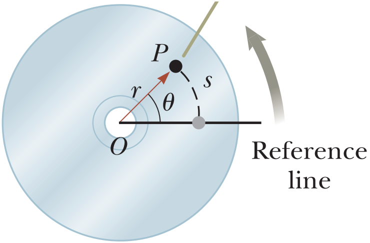
The distance it moves is s, if θ is measured in radians.

$$s = r\theta$$

[1]Figures from Serway & Jewett, 9th ed.

Rotating disk

Consider a marked point P on the disk. As time passes it moves:



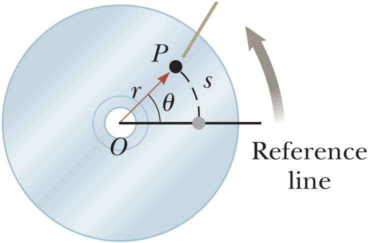
The distance it moves is s, if θ is measured in radians.

$$s = r\theta$$

[1]Figures from Serway & Jewett, 9th ed.

Angular speed

The angle that the disk rotates by is θ, in some amount of time t

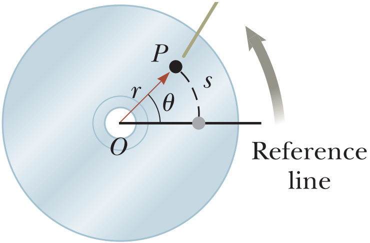


the **angular speed** ("rotational speed") of the disk is

$$\text{angular speed} = \frac{\text{change in angle}}{\text{change in time}}$$

Angular speed

The angle that the disk rotates by is θ, in some amount of time t



the **angular speed** ("rotational speed") of the disk is

$$\text{angular speed} = \frac{\text{change in angle}}{\text{change in time}}$$

In notation:

$$\omega = \frac{\theta}{t}$$

where we let ω represent angular speed.

Angular speed

The angular speed of the Earth's rotation is 2π per day or

$$\omega_{\mathsf{E}} = \frac{2\pi}{86,400\ \mathsf{s}}$$

The units are radians per second. (Or just s^{-1}.)

We can also measure rotational speed in terms of the number of complete rotations in some amount of time.

Records speeds are a good example of this. Typical angular speeds:

- $33\frac{1}{3}$ RPM (called "a 33")
- 45 RPM
- 78 RPM

where RPM means *rotations per minute*.

Angular speed and Tangential speed

The **tangential speed** of point P is its instantaneous speed. We write it as v because it is fundamentally the same thing we called speed before:

$$\text{speed} = \frac{\text{distance traveled}}{\text{change in time}}$$

For the point P it travels a distance s in time t

$$v = \frac{s}{t}$$

Angular speed and Tangential speed

But remember: $s = r\theta$.

We can write

$$v = \frac{s}{t} = \frac{r\theta}{t}$$

Angular speed and Tangential speed

But remember: $s = r\theta$.

We can write

$$v = \frac{s}{t} = \frac{r\theta}{t}$$

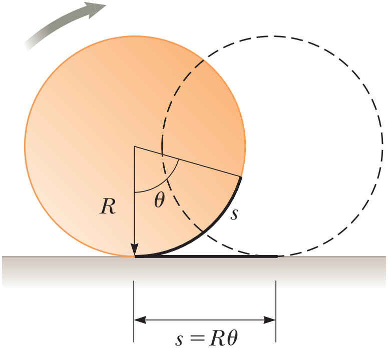
However, $\omega = \frac{\theta}{t}$ so we can make a relation between tangential speed v and angular speed ω:

$$v = r\omega$$

(tangential speed = distance to axis $\times$ angular speed)

Rolling Motion

A rolling object moves along a surface as it rotates. Consider a wheel:



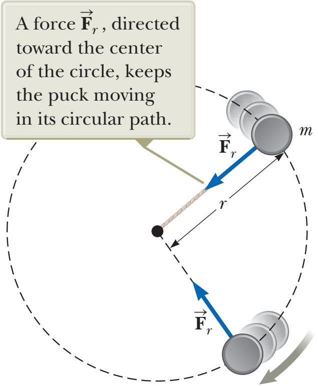
If the outside edge of the wheel does not slip on the surface, then there is a relation between the angular speed of the wheel's rotation and the speed that the wheel itself moves along.

Interestingly, it is also:

$$v_{\text{wheel}} = r\omega$$

Centripetal Force

Now consider an object that is rotating about an axis. For example a puck on a string:



Centripetal Force

If an object moves on a circular path, its velocity must always be changing. $\Rightarrow$ It is accelerating.

[1] Figures from Serway & Jewett.

Centripetal Force

If an object moves on a circular path, its velocity must always be changing. $\Rightarrow$ It is accelerating.

$$\mathbf{F}_{\text{net}} = m\mathbf{a} \;\Rightarrow\; \mathbf{F}_{\text{net}} \neq 0$$

[1]Figures from Serway & Jewett.

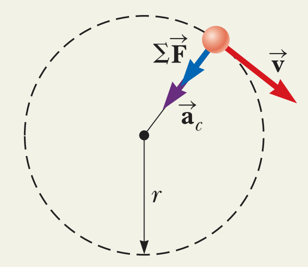
Centripetal Force

If an object moves on a circular path, its velocity must always be changing. $\Rightarrow$ It is accelerating.

$$\mathbf{F}_{\text{net}} = m\mathbf{a} \;\Rightarrow\; \mathbf{F}_{\text{net}} \neq 0$$

Any object moving in a circular (or curved) path must be experiencing a force.

We call this the *centripetal force*.



[1] Figures from Serway & Jewett.

Uniform Circular Motion

For an object moving in a circle at constant speed v,

$$a = a_c = \frac{v^2}{r} = r\omega^2$$

Uniform Circular Motion

For an object moving in a circle at constant speed v,

$$a = a_c = \frac{v^2}{r} = r\omega^2$$

This gives the expression for centripetal force!

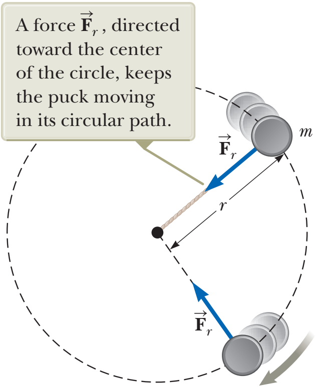
$$\mathbf{F} = m\mathbf{a}$$

so,

$$\mathbf{F}_c = \frac{mv^2}{r}$$

Centripetal Force

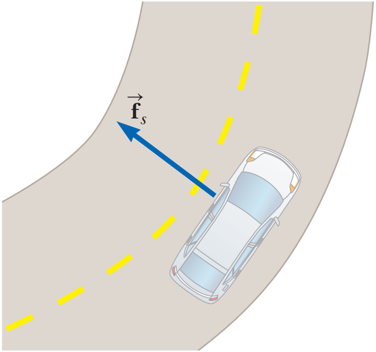
Something must provide this force:



It could be tension in a rope.

Centripetal Force

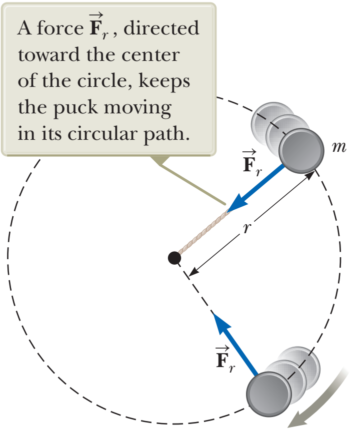
Something must provide this force:



It could be friction.

Centripetal Force

Consider the example of a string constraining the motion of a puck:



Centripetal Force

Question. What will the puck do if the string breaks?

(A) Fly radially outward.

(B) Continue along the circle.

(C) Move tangentially to the circle.

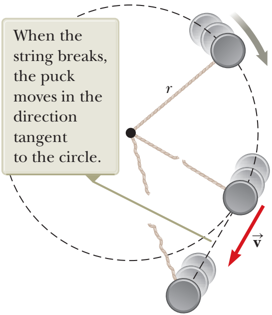
Centripetal Force

Question. What will the puck do if the string breaks?

(A) Fly radially outward.

(B) Continue along the circle.

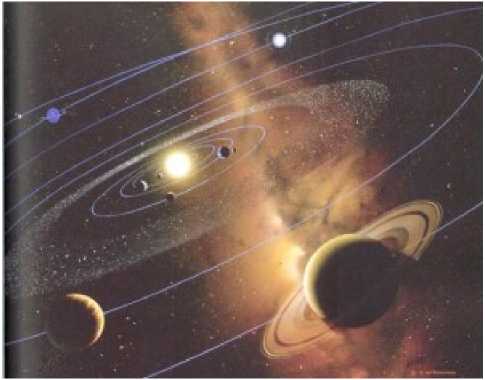
(C) Move tangentially to the circle. ←



Orbits

A centripetal force also holds Earth in orbit around the Sun.

What is the force due to?

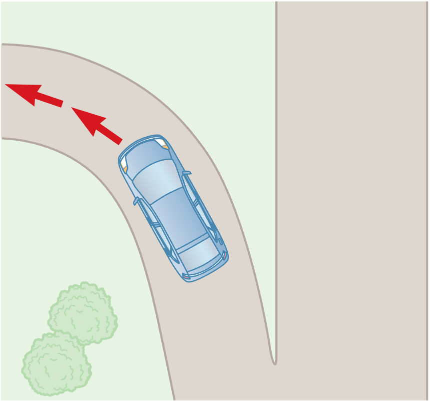


[1]Figure from EarthSky.org.

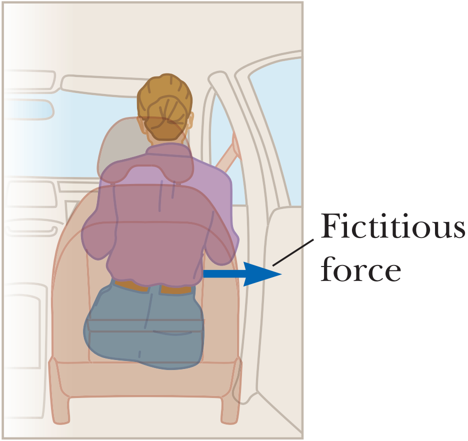
A Fictitious Force: Centrifugal force

"fictitious" $\rightarrow$ fictional.

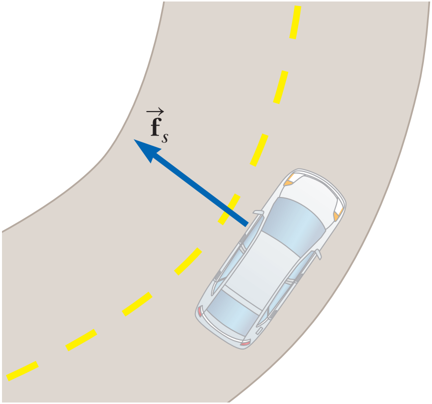
The centrifugal force is the "force" that makes you feel sucked to the outside in a turn:



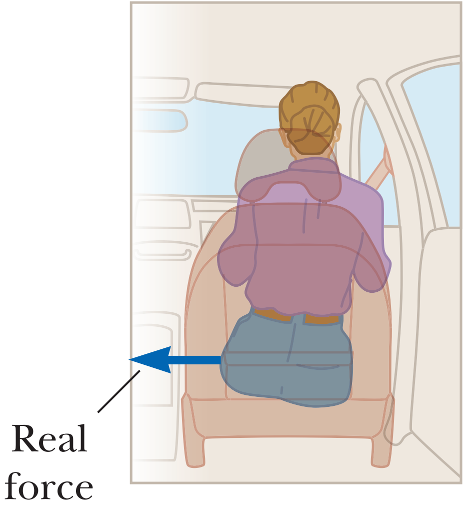
A Fictitious Force: Centrifugal force



The real force is Centripetal



The real force is Centripetal



Rotation and Force Question

Two pennies are place on a circular rotating platform, one near to the center, the other, towards the outside rim. The platform starts at rest and is slowly spun faster and faster (increasing angular speed). Which penny slides off the platform first?

(A) The one near the center.
(B) The one near the rim.

Rotation and Force Question

Two pennies are place on a circular rotating platform, one near to the center, the other, towards the outside rim. The platform starts at rest and is slowly spun faster and faster (increasing angular speed). Which penny slides off the platform first?

(A) The one near the center.

(B) The one near the rim. $\longleftarrow$

Rotating reference frame

If you are in a rotating frame, you can describe your world as if it is at rest by adding a fictitious outward centrifugal force to your physics.

You can use this to simulate gravity: for example, in rotating space stations, *eg.* in the films *2001*, *Elysium*, *Interstellar*.

Rotating reference frame

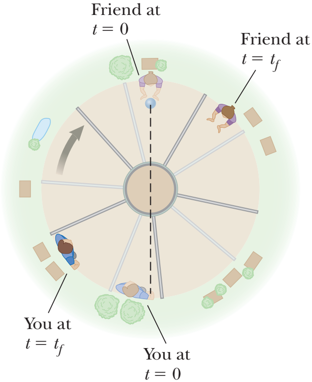
If you are in a rotating frame, you can describe your world as if it is at rest by adding a fictitious outward centrifugal force to your physics.

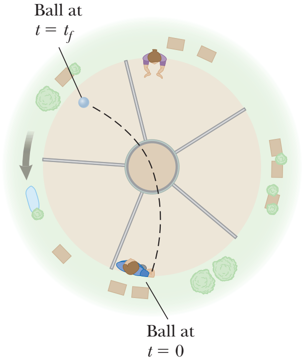
You can use this to simulate gravity: for example, in rotating space stations, *eg.* in the films *2001*, *Elysium*, *Interstellar*.

Water in a bucket...

Rotating Frames: Coriolis "Force"

There is another fictitious force that non-inertial observers see in a rotating frame.



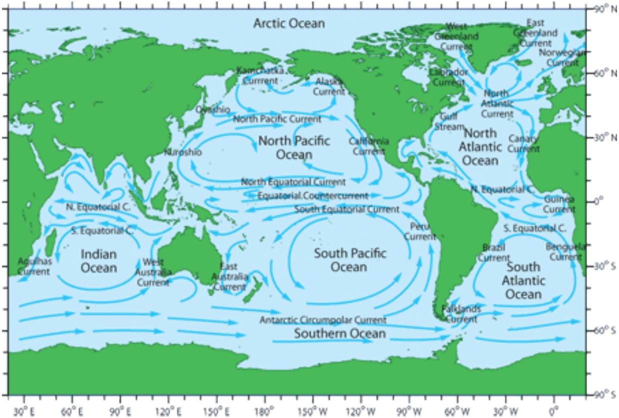


The **Coriolis "force"** appears as a fictitious sideways force to a non-inertial observer.

Rotating Frames: Coriolis “Force”

We can detect the effect of Earth’s rotation: they manifest as Coriolis effects.

eg. ocean surface currents:

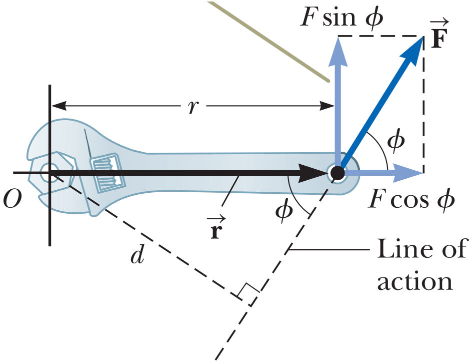


[1]Figure from http://www.seos-project.eu/

Torque

Torque is a force causing a rotation.

$$\text{Torque} = \text{lever arm} \times \text{force}$$



Torque is written τ ("tau")

$$\tau = d\,F = (r \sin \phi) F$$

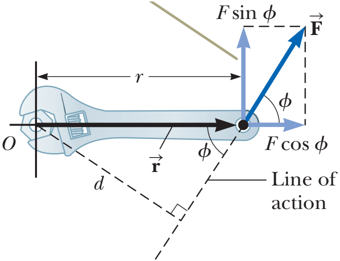
Torque

$$\text{Torque} = \text{lever arm} \times \text{force}$$

$$\tau = (r \sin \phi) F = d\, F$$

The units of torque are N m (Newton meters).

Torque is not Work



If we take the lever arm to be d, as in $d = r \sin \phi$, then we can write:

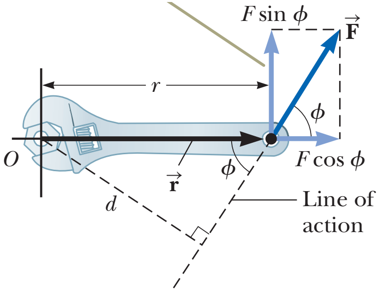
$$\tau = d\,F$$

This looks a lot like the formula for work: $W = Fd\cos\theta$.

Work is measured in Joules, and 1 J = 1 N × 1 m.

But **torque is not the same thing at all!**

Torque is not Work



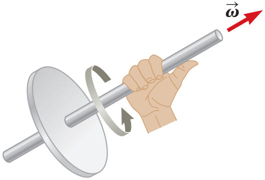
In the diagram, the force does not move the end of wrench in the direction of **r**.

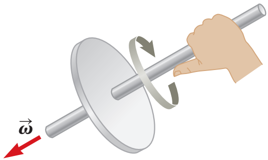
It moves the end of the wrench perpendicular to **r**. This causes a rotation.

[1]Figures from Serway & Jewett.

Torque is a Force causing a Rotation

Technically, torque is a vector: the direction of the vector tells us whether the rotation will be clockwise or counterclockwise.



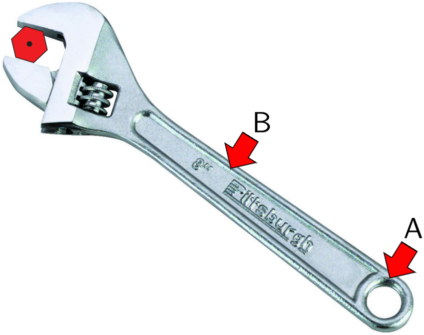


So, to keep it separate:

- units of torque: N m
- units of work or energy: J

[1]Figures from Serway & Jewett.

Question



A torque is supplied by applying a force at point A. To produce the same torque, the force applied at point B must be:

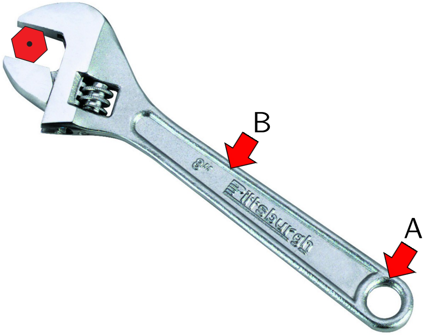
(A) greater

(B) less

(C) the same

[1] Image from Harbor Freight Tools, www.harborfreight.com

Question



A torque is supplied by applying a force at point A. To produce the same torque, the force applied at point B must be:

(A) greater ←

(B) less

(C) the same

[1]Image from Harbor Freight Tools, www.harborfreight.com

Moment of Inertia

A solid, rigid object like a ball, a record (disk), a brick, etc. has a moment of inertia.

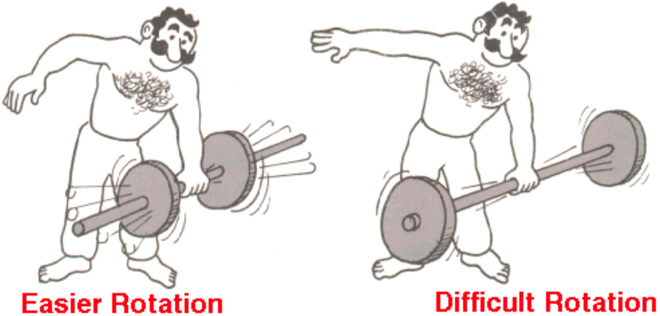
Moment of inertia, I, is similar to mass.

A net torque causes an object to rotate, and moment of inertia measures the object's resistance to rotation.

Mass behaves the same way! A net force causes the motion (acceleration) of an object and the mass measures the object's resistance to changes in its motion.

Moment of Inertia

If the object's mass is far from the point of rotation, more torque is needed to rotate the object (with some angular acceleration).

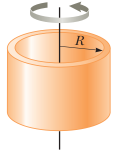


The barbell on the right has a greater moment of inertia.

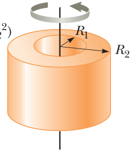
[1]Diagram from Dr. Hunter's page at http://biomech.byu.edu (by Hewitt?)

Different shapes have different Moments of Inertia

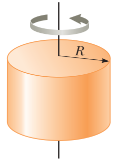
Hoop or thin cylindrical shell
$I_{CM} = MR^2$



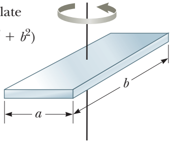
Hollow cylinder
$I_{CM} = \frac{1}{2}M(R_1^{\,2} + R_2^{\,2})$



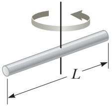
Solid cylinder or disk
$I_{CM} = \frac{1}{2}MR^2$



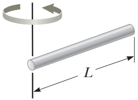
Rectangular plate
$I_{CM} = \frac{1}{12}M(a^2 + b^2)$



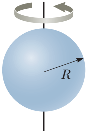
Long, thin rod with rotation axis through center
$I_{CM} = \frac{1}{12}ML^2$



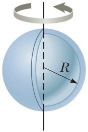
Long, thin rod with rotation axis through end
$I = \frac{1}{3}ML^2$



Solid sphere
$I_{CM} = \frac{2}{5}MR^2$



Thin spherical shell
$I_{CM} = \frac{2}{3}MR^2$



Different shapes have different Moments of Inertia

If an object changes shape, its moment of inertia can change also.

Center of Mass

For a solid, rigid object:

center of mass

the point on an object where we can model all the mass as being, in order to find the object's trajectory; a freely moving object rotates about this point

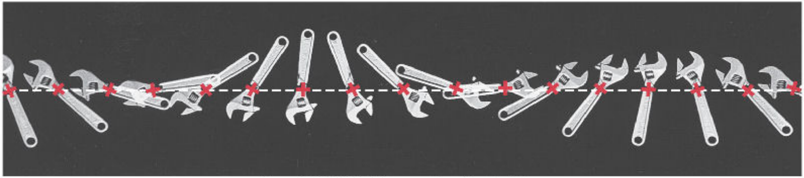
Center of Mass

For a solid, rigid object:

center of mass

the point on an object where we can model all the mass as being, in order to find the object's trajectory; a freely moving object rotates about this point

The center of mass of the wrench follows a straight line as the wrench rotates about that point.

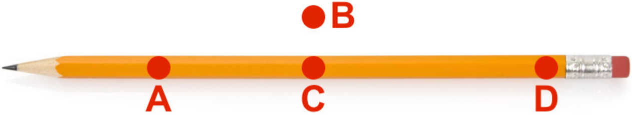


Center of Mass

[1]Figure from
http://www4.uwsp.edu/physastr/kmenning/Phys203/Lect18.html

Center of Mass Questions

Where is the center of mass of this pencil?



(A) Location A.

(B) Location B.

(C) Location C.

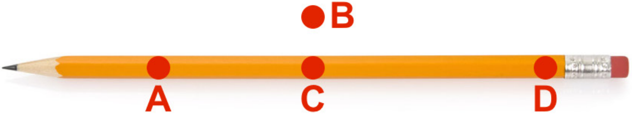
(D) Location D.

---

[1] Pencil picture from kingofwallpapers.com.

Center of Mass Questions

Where is the center of mass of this pencil?



(A) Location A.

(B) Location B.

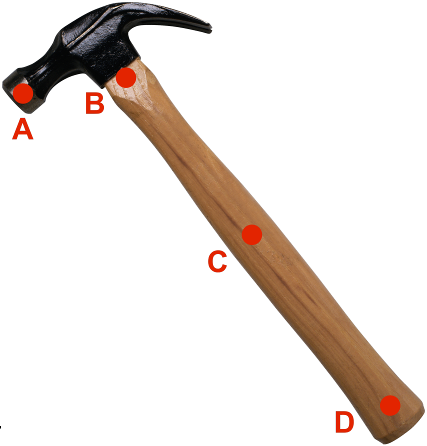
(C) Location C. ←

(D) Location D.

[1] Pencil picture from kingofwallpapers.com.

Center of Mass Questions

Where is the center of mass of this hammer?

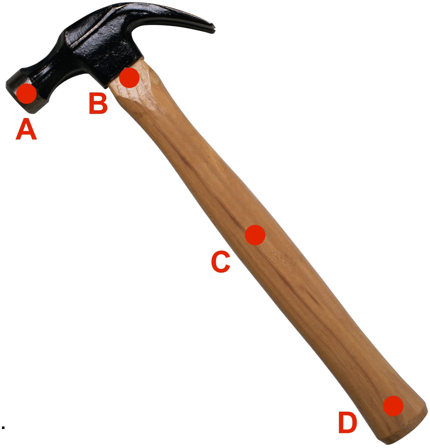


(A) Location A.
(B) Location B.
(C) Location C.
(D) Location D.

[1] Hammer picture from pngimg.com.

Center of Mass Questions

Where is the center of mass of this hammer?



(A) Location A.

(B) Location B. ←

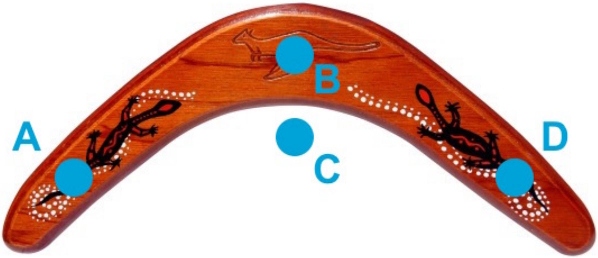
(C) Location C.

(D) Location D.

[1]Hammer picture from pngimg.com.

Center of Mass Questions

Where is the center of mass of this boomerang?



(A) Location A.

(B) Location B.

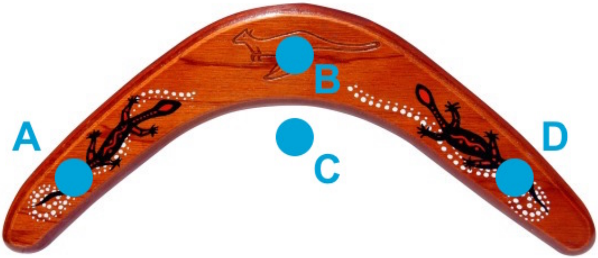
(C) Location C.

(D) Location D.

[1]Boomerang picture from http://motivatedonline.com.

Center of Mass Questions

Where is the center of mass of this boomerang?



(A) Location A.

(B) Location B.

(C) Location C. ←

(D) Location D.

---

[1]Boomerang picture from http://motivatedonline.com.

Center of Gravity

The center of gravity is the single point on an object where we can model the force of gravity as acting on the object.

Near the surface of the Earth, the Earth's gravitational field is uniform, so this is the same as the center of mass.

The center of gravity is the point at which you can balance an object on a single point of support.

[1]Figure from http://dev.physicslab.org/

Center of Mass vs Center of Gravity

For a solid, rigid object:

center of mass

the point on an object where we can model all the mass as being, in order to find the object's trajectory; a freely moving object rotates about this point

center of gravity

the single point on an object where we can model the force of gravity as acting on the object; the point at which you can balance the object.

Angular Momentum

Angular momentum is a rotational version of momentum.

Remember

$$\text{momentum} = \text{mass} \times \text{velocity}$$

or $\mathbf{p} = m\mathbf{v}$.

Angular momentum, $\mathbf{L}$, can be defined in a similar way:

$$\text{angular momentum} = \text{moment of inertia} \times \text{angular velocity}$$

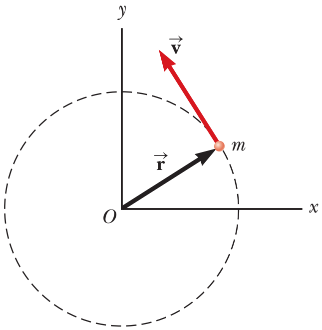
$$\mathbf{L} = I\boldsymbol{\omega}$$

Angular Momentum: special case

The angular momentum of a small object, with mass m, moving in a circle is:

$$L = mvr$$

This is the linear momentum mv times the distance from the center of the circle r.



[1]Figure from Serway & Jewett, 9th ed.

Angular Momentum is Conserved

The momentum of a system is constant (does not change) unless the system is acted upon by an external force.

A similar rule holds for angular momentum:

The angular momentum of a system does not change unless it acted upon by an external torque.

Angular Momentum is Conserved

The angular momentum of a system does not change unless it acted upon by an external torque.

This means $\Delta L = 0$ and so $L_i = L_f$.

Angular Momentum is Conserved

The angular momentum of a system does not change unless it acted upon by an external torque.

This means $\Delta L = 0$ and so $L_i = L_f$.

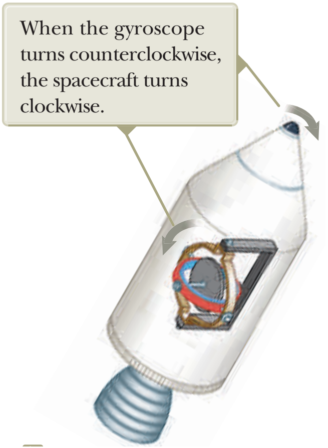
Suppose an object is changing shape, so that its moment of inertia gets smaller: $I_f < I_i$.

$$L_i = L_f \;\rightarrow\; I_i \omega_i = I_f \omega_f$$

That means the angular speed increases! $\omega_f > \omega_i$

Conservation of Angular Momentum

The conservation of angular momentum can be used to orient spacecraft.



[1]Figures from Serway & Jewett, 9th ed.

Conservation of Angular Momentum

The conservation of angular momentum also makes tops and gyroscopes stable when rotating.

[1]From http://www.livescience.com/33614-the-cool-physics-of-7-toys.html

Summary

- inelastic collisions
- circular motion / rotation
- torque
- moment of inertia
- center of mass
- angular momentum

Essay Homework due July 19th.

Midterm Thursday, July 20th.

Homework Hewitt,

- prev: Ch 6, onward from page 96, Plug and chug: 1, 3, 5, 7; Ranking: 1; Exercises: 5, 7, 19, 31, 47
- NEW: Ch 8, onward from page 145. Plug and Chug: 1, 3, 5. Exercises: 1, 3, 19, 39, 41; Problems: 5.