Conceptual Physics
Gravity

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De Anza College

July 18, 2017

Last time

- rotational motion
- centripetal force vs. centrifugal “force”
- torque
- moment of inertia
- center of mass and center of gravity

Overview

- angular momentum
- Gravity!
- gravitational field
- black holes

Comparison of Linear and Rotational quantities

Linear Quantities	Rotational Quantities
$\mathbf{x}$	θ
$\mathbf{v} = \frac{\Delta \mathbf{x}}{\Delta t}$	$\boldsymbol{\omega} = \frac{\Delta \theta}{\Delta t}$
$\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t}$	$\alpha = \frac{\Delta \omega}{\Delta t}$
mass, m	moment of inertia, I
momentum, $\mathbf{p}$	angular momentum, $\mathbf{L}$

Angular Momentum

Angular momentum is a rotational version of momentum.

Remember

$$\text{momentum} = \text{mass} \times \text{velocity}$$

or $\mathbf{p} = m\mathbf{v}$.

Angular momentum, $\mathbf{L}$, can be defined in a similar way:

$$\text{angular momentum} = \text{moment of inertia} \times \text{angular velocity}$$

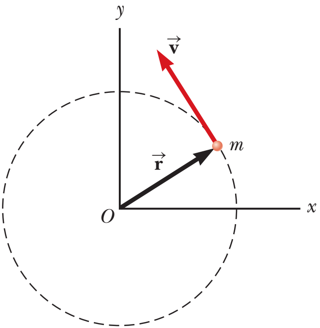
$$\mathbf{L} = I\boldsymbol{\omega}$$

Angular Momentum: special case

The angular momentum of a small object, with mass m, moving in a circle is:

$$L = mvr$$

This is the linear momentum mv times the distance from the center of the circle r.



[1]Figure from Serway & Jewett, 9th ed.

Angular Momentum is Conserved

The momentum of a system is constant (does not change) unless the system is acted upon by an external force.

A similar rule holds for angular momentum:

The angular momentum of a system does not change unless it acted upon by an external torque.

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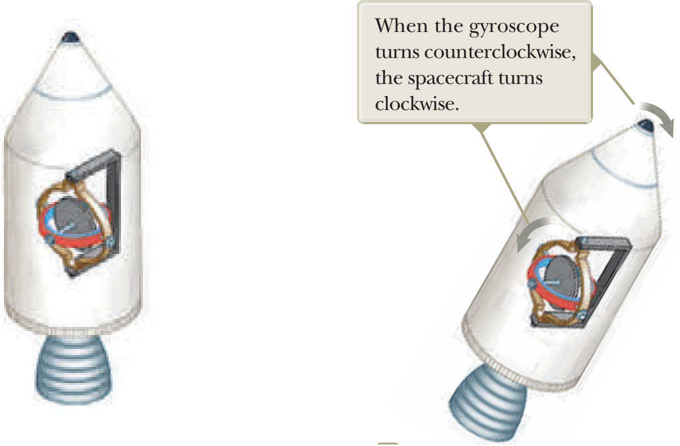
Suppose an object is changing shape, so that its moment of inertia gets smaller: $I_f < I_i$.

$$L_i = L_f \ \rightarrow \ I_i \omega_i = I_f \omega_f$$

That means the angular speed increases! $\omega_f > \omega_i$

Conservation of Angular Momentum

The conservation of angular momentum can be used to orient spacecraft.



[1] Figures from Serway & Jewett, 9th ed.

Conservation of Angular Momentum

The conservation of angular momentum also makes tops and gyroscopes stable when rotating.

[1] From http://www.livescience.com/33614-the-cool-physics-of-7-toys.html

Motion of the stars, planets, and falling apples

Part of Newton's genius was to realize that the same force that dictates the motion of the stars and planets is what holds us on the Earth.

This realization is called the **Newtonian synthesis**.

The planets are also falling, but they are constantly falling around the Sun.

Law of Gravitation

Newton's Universal Law of Gravitation states that any two massive object in the universe interact with each other according to the same rule.

They attract each other with a force that depends on the two masses and the distance between their centers:

$$\text{Force} \sim \frac{\text{mass}_1 \times \text{mass}_2}{\text{distance} \times \text{distance}}$$

As an equation:

$$F = G\,\frac{m_1 m_2}{r^2}$$

(where G is a constant.)

(In the book, distance r is written with the symbol d instead.)

The Universal Gravitational Constant, G

$$F = G \, \frac{m_1 m_2}{r^2}$$

G sets the scale of the force due to gravity (and makes the units come out correctly).

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$$G = 6.67 \times 10^{-11}\text{N m}^2/\text{kg}^2$$

This could also be written:

$$G = 0.0000000000667\text{N m}^2/\text{kg}^2$$

so it is quite a small number.

The Universal Gravitational Constant, G

The fact that G is so small indicates that gravity is a weak force.

Force	$\sim$ Rel. strength	Range (m)	Attract/Repel
Gravitational	10^{-38}	∞	attractive
Electromagnetic	10^{-2}	∞	attr. & rep.
Weak Nuclear	10^{-13}	$< 10^{-18}$	attr. & rep.
Strong Nuclear	1	$< 10^{-15}$	attr. & rep.

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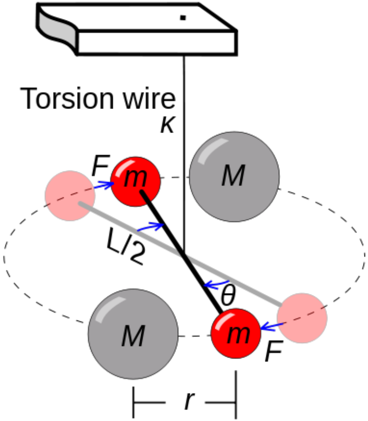
Despite the fact that gravity is a weak force, it is the only one that (typically) matters on large scales.

Measurement of G

G was first measured by Henry Cavendish using a torsion balance.



[1]Diagram from Wikipedia by Chris Burks.

Mass of the Earth

Determining G allowed for a the mass of the Earth M_E to be calculated.

The weight of an object is the force on the object due to gravity, when the distance is the radius of the Earth.

$$
\begin{aligned}
F = G\,\frac{mM_E}{r^2} &= mg \\
G\,\frac{M_E}{R_E^2} &= g \\
M_E &= \frac{gR_E^2}{G} \\
M_E &= 6 \times 10^{24}\ \text{kg}
\end{aligned}
$$

Mass of the Earth

$$M_E = 6 \times 10^{24} \text{ kg}$$

This is a very large number!

And yet, despite the fact that the force on you from the Earth depends on:

$$\text{F} \sim \text{your mass} \times \text{the mass of the Earth}$$

you still have the strength to jump off the floor!

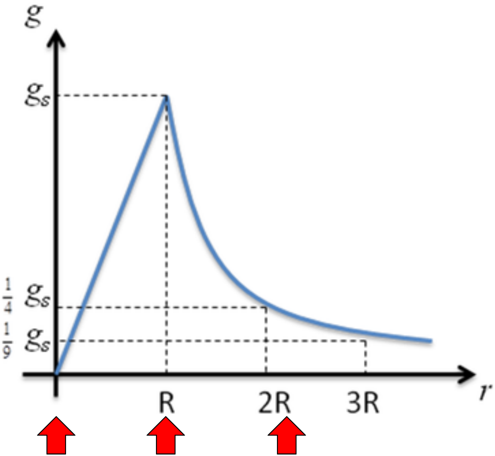
So G must be very small, and the force of gravity very weak.

Value of g

$$
\begin{aligned}
mg &= G\,\frac{mM_E}{r^2} \\
g &= G\,\frac{M_E}{r^2}
\end{aligned}
$$

g takes the value 9.8 m s^{-2} at the Earth's surface, but it can vary with height: as r increases, g decreases.

Variation of g



Center of the Earth | Surface of the Earth | Outside of the Earth

[1]Figure from www.physbot.co.uk

Fields

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A field is any kind of physical quantity that has values specified at every point in space and time.

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Fields were first introduced as a calculation tool. A force-field can be used to identify the force a particular particle will feel at a certain point in space and time based on the other objects in its environment that it will interact with.

Fields are drawn with lines showing the direction of force that a particle will feel at that point. The density of the lines at that point in the diagram indicates the approximate magnitude of the force at that point.

Gravitational Field

The gravitational field is a force-field.

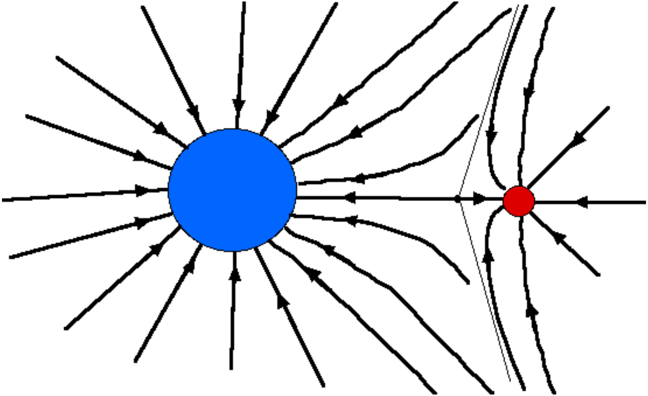
Gravity attracts two masses even though they are far apart.

A large mass can be thought of as altering the space around it.

Another mass feels a force because of the distortion to the space where it is located.

Gravitational Field

The gravitational field caused by the Sun-Earth system looks something like:



[1]Figure from http://www.launc.tased.edu.au

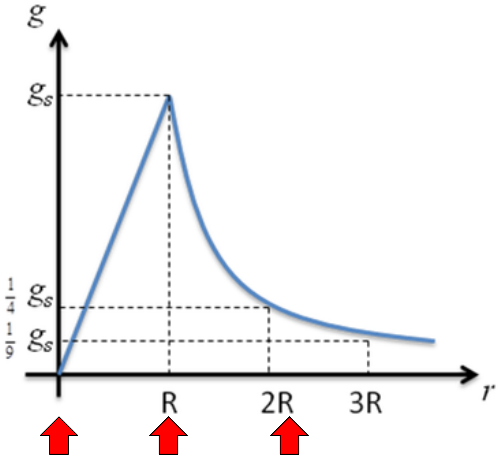
What about the Gravitational Field inside the Earth?

At the very center of the Earth, the Earth's mass is distributed all around equally in each direction.

At that point, you would be equally attracted in all directions.

The net force on you is zero, and the acceleration due to gravity would also be zero.

What about the Gravitational Field inside the Earth?



Center of the Earth

Surface of the Earth

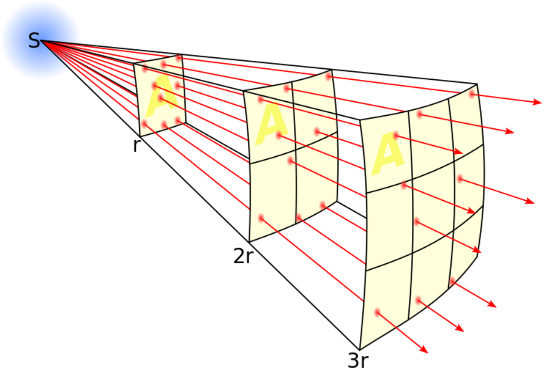
Outside of the Earth

[1] Figure from www.physbot.co.uk

Form of Newton's Law of Gravitation

$$F = G\,\frac{m_1 m_2}{r^2}$$

This kind of law is called an **inverse square law**.



[1]Figure from Wikipedia.

Other effects of gravity: Waves

Waves in the ocean are the result of displacement of water.

The force of gravity restores the denser fluid to a lower position but the kinetic energy involved (and also wind) causes these disturbances to propagate.

Other effects of gravity: Tides

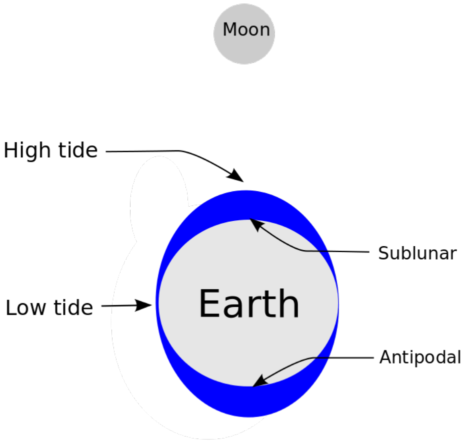
The height of the ocean at a particular point changes with the time of the day.

In a single day there are two high tides and two low tides.

High tides occur when the Moon is overhead, or on the opposite side of the Earth.

The Moon's mass attracts the water of the oceans which pool beneath the Moon. The Earth is also attracted to the Moon, and being nearer, is more strongly accelerated than the water on the far side.

Tides



[1]Diagram from Wikipedia.

Stronger and weaker tides

Strong tides called **spring tides** occur when the Sun is also aligned on the Earth-Moon axis.

Weaker than usual tides called **neap tides** occur when the Sun is perpendicular to the Earth-Moon axis and partially cancels out the Moon's effect.

Tides

While we tend to notice ocean tides, there are also tidal forces on the Earth's crust causing land to rise and fall with the Moon's orbit also, albeit to a lesser degree.

The atmosphere also experiences tides, but this is most noticeable in the upper atmosphere.

Moon tides

The Earth's force on the Moon also deforms the Moon causing it to be not-quite spherical.

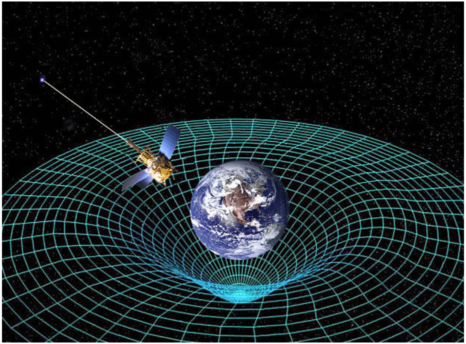
Since the Earth's gravitational field is not uniform across the Moon, the Moon's center of gravity is not in the same place as its center of mass: the Earth exerts a torque on the Moon if the Moon's long axis is not aligned with the Earth.

This is called **tidal locking** and is the reason that the same side of the Moon always faces the Earth.

Einstein's General Relativity and Gravity

According to relativity, space and time are not really separate but should be thought of together as spacetime.

General relativity predicts that gravitational fields warp spacetime.

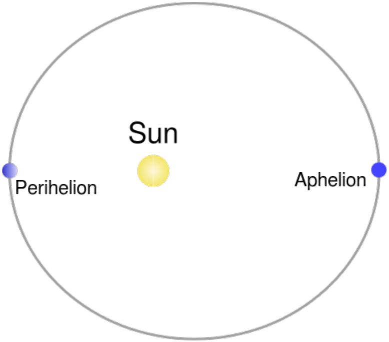


[1] Image by NASA, http://www.nasa.gov/mission_pages/gpb/gpb_012.html

General Relativity and the Orbit of Mercury

General relativity correctly predicts the orbit of Mercury.

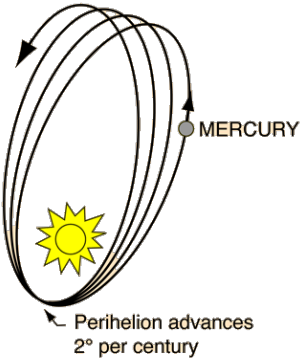
The *perihelion* of a planet's orbit is the closest point in the orbit to the sun.



[1]Figure by Wikipedia user Chris55.

General Relativity and the Orbit of Mercury

The perihelion of Mercury advances over time.



Newtonian mechanics is unable to explain the amount of advance observed, but general relativity can.

[1] Figure from HyperPhyiscs.

Einstein's General Relativity and Gravity

Light follows paths of least time from one point to another.

However, if spacetime is curved, these paths do not appear to us to be straight lines.

Why?

Non-Euclidean Geometry

Fight paths in the northern hemisphere seem to curve very far northward when viewed with a flat projection of Earth:

[1] Globe map from http://web.mit.edu/dsheehan/www/MapsAPIexamples

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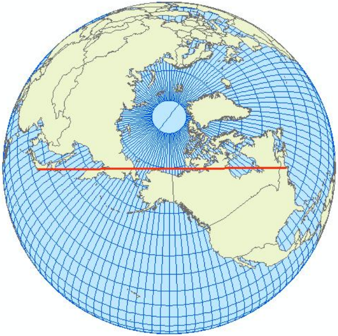

Yet on a sphere, these paths are close to the shortest path between the two points.

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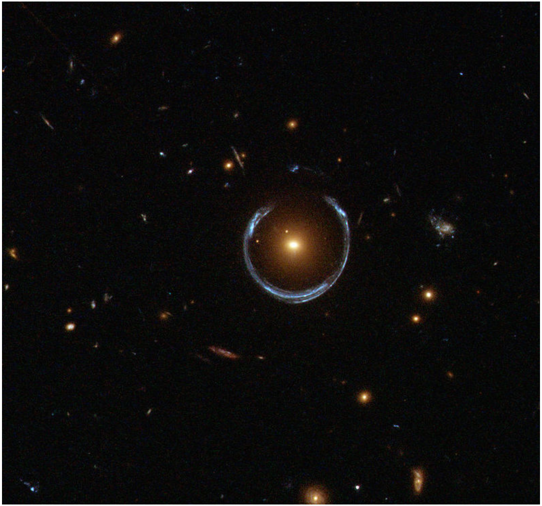
[1]Globe map from http://web.mit.edu/dsheehan/www/MapsAPIexamples

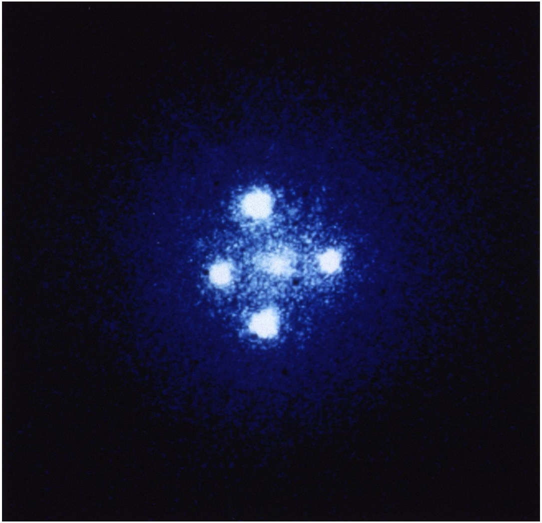
Visible effects of curved spacetime



[1] Image from ESA / Hubble & NASA,
http://apod.nasa.gov/apod/image/1112/lensshoe_hubble_3235.jpg

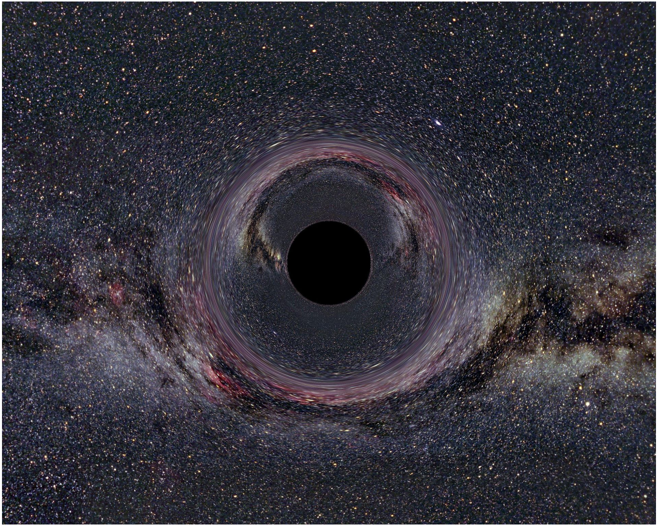
Visible effects of curved spacetime

An Einstein cross: the same object imaged in four places.



[1] Image from NASA, ESA, and STScI, http://hubblesite.org/newscenter/archive/releases/1990/20/image/a/

Black holes



[1]Image by Ute Kraus, Physics education group Kraus, Universität Hildesheim, Space Time Travel

Black holes

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When a star's fuel runs out, the fusion ends and the gas making up the star begin to rapidly cool and collapse under the force of gravity.

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Stars with masses greater than about **1.5 times** the mass of the Sun can collapse so strongly that the repulsion of electrons cannot keep atoms separate: they collapse together to form **neutron stars**.

Stars with masses greater than about **3 times** the mass of the Sun experience even more gravitational force and can collapse even further. These form **black holes**. Nothing that falls in can come back out.

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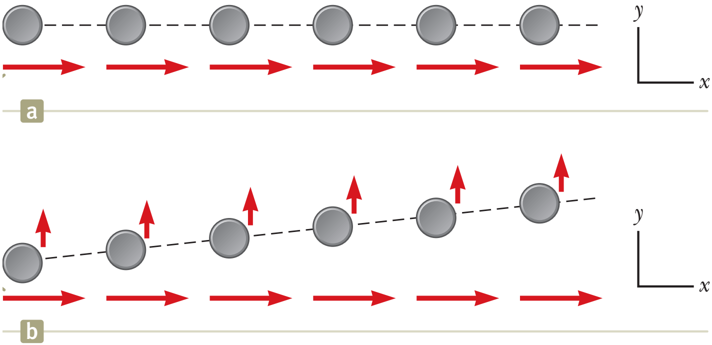
Supermassive black holes are found at the centers of galaxies. These are millions of times the mass of our sun.

Motion in 2 directions

Imagine an air hockey puck moving with horizontally constant velocity:

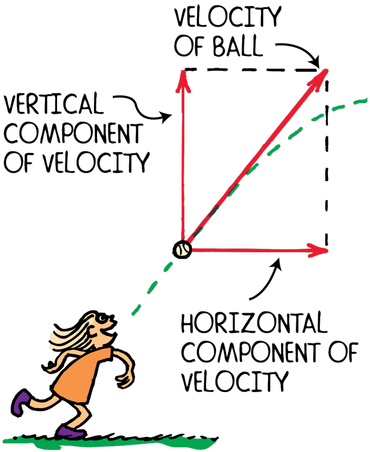


If it experiences a momentary upward (in the diagram) acceleration, it will have a component of velocity upwards. The horizontal motion remains unchanged!

[1] Figure from Serway & Jewett, 9th ed.

Motion in 2 directions

Motion in perpendicular directions can be analyzed separately.



A vertical force (gravity) does not affect horizontal motion.

The horizontal component of velocity is constant.

[1] Drawing by Hewitt, via Pearson.

Projectiles

projectile

Any object that is thrown. We will use this word specifically to refer to thrown objects that experience a vertical acceleration g.

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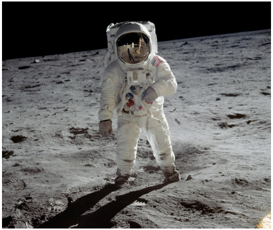
For projectile motion, we assume air resistance is negligible. This gives symmetrical parabolic trajectories.

Why do we care?



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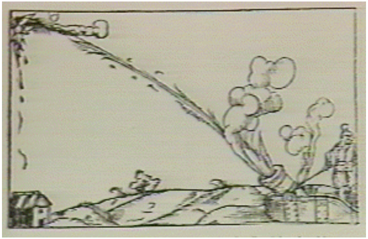
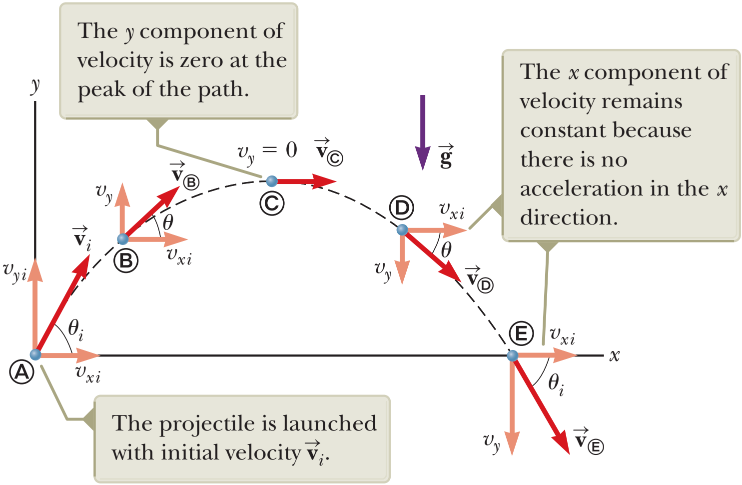


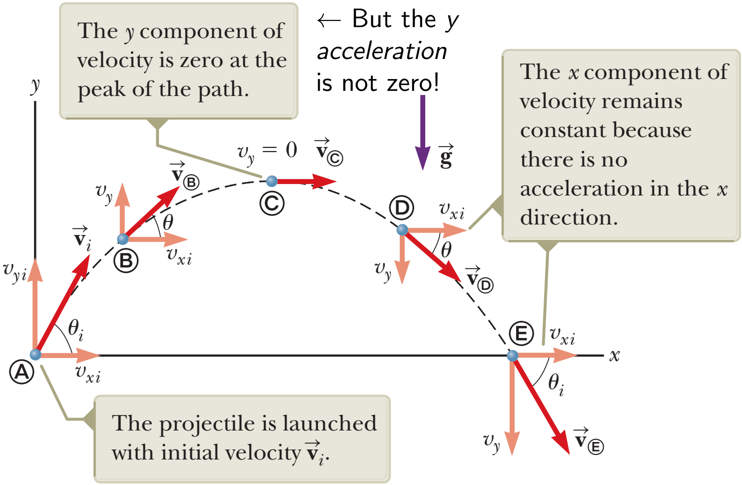
Projectile Velocity



[1]Figure from Serway & Jewett, 9th ed.

Projectile Velocity

← But the y *acceleration* is not zero!



[1]Figure from Serway & Jewett, 9th ed.

Summary

- center of mass
- angular momentum
- Newton's Universal Law of Gravitation
- the gravitational field

Essay Homework! due tomorrow.

Midterm Thursday, July 20th.

Homework Hewitt,

- Ch 8, onward from page 145. Plug and Chug: 5.
- Ch 9, onward from page 167. Plug and chug: 1, 3; Ranking 3, 5; Exercises: 7, 11, 53; Problems: 1, 3
- (Will set Tomorrow: Ch 10, onward from page 190. Exercises: 15, 19, 23, 45, 59; Problems: 3)

Essay

Essay Homework due tomorrow.

- Describe the design features of cars that make them safer for passengers in collisions. Comment on how the design of cars has changed over time to improve these features. In what other circumstances might people be involved in collisions? What is / can be done to make those collisions safer for the people involved? Make sure use physics principles (momentum, impulse) in your answers!