Introduction to Mechanics
Kinematics Equations

Lana Sheridan

De Anza College

Jan 22, 2020

Last time

- graphs of kinematic quantities

Overview

- how to solve problems
- the kinematics equations
- derivations and examples

How to solve problems

1. Draw a diagram, sketch, or graph showing the situation in the question.
2. Make a hypothesis or estimate of what the answer will be.
3. Solve the question or problem:
   a. Here, it's a 'problem' -
      i. Write out quantities given in question and quantity asked for.
      ii. Write out the equation(s) you will use. (Start from equations we have discussed in class.)
      iii. Do any required algebra.
      iv. Plug in givens and solve.
      v. Check units.
4. Analyze answer as appropriate.
   a. Compare answer to hypothesis - if it is not the same try to explain why.
   b. Is your answer reasonable? / Compare to other things your are familiar with.
   c. Consider limits or special cases.

Now You Try It

A car is traveling along a straight road at 11 m/s and accelerates at a constant rate of 1.8 m/s^2. How long does it take to reach a speed of 20 m/s?

Kinematics Equations

For an object moving with constant acceleration, we can derive equations that we will be able to use to solve problems.

First we will consider objects moving in a straight line (1-D kinematics), but the equations are useful in 2 or 3 dimensions also.

Vector Equations vs Scalar Equations

I will write the kinematics equations in vector form, for example:

$$\overrightarrow{\Delta \boldsymbol{x}} = \vec{\mathbf{v}}\, t$$

since displacement and velocity are vector quantities.

Vector Equations vs Scalar Equations

I will write the kinematics equations in vector form, for example:

$$\overrightarrow{\boldsymbol{\Delta x}} = \vec{\mathbf{v}}\, t$$

since displacement and velocity are vector quantities.

However, we are looking only at 1-dimension for the moment. We know the displacement will be either in the positive or negative x direction ($\pm\hat{\mathbf{i}}$ direction).

What we can do, is write this equation instead as a scalar equation by factoring out the unit vectors from each side:

$$\Delta x\, \hat{\mathbf{i}} = (v_x\, \hat{\mathbf{i}})t$$

Vector Equations vs Scalar Equations

I will write the kinematics equations in vector form, for example:

$$\overrightarrow{\Delta x} = \vec{\mathbf{v}}\, t$$

since displacement and velocity are vector quantities.

However, we are looking only at 1-dimension for the moment. We know the displacement will be either in the positive or negative x direction ($\pm\hat{\mathbf{i}}$ direction).

What we can do, is write this equation instead as a scalar equation by factoring out the unit vectors from each side:

$$\Delta x\, \cancel{\hat{\mathbf{i}}} = (v_x\, \cancel{\hat{\mathbf{i}}})\, t$$

Vector Equations vs Scalar Equations

I will write the kinematics equations in vector form, for example:

$$\overrightarrow{\boldsymbol{\Delta x}} = \vec{\mathbf{v}}\, t$$

since displacement and velocity are vector quantities.

However, we are looking only at 1-dimension for the moment. We know the displacement will be either in the positive or negative x direction ($\pm\hat{\mathbf{i}}$ direction).

What we can do, is write this equation instead as a scalar equation by factoring out the unit vectors from each side:

$$\Delta x = v_x\, t$$

In that last expression, Δx and v_x are the *signed* magnitudes of the $\overrightarrow{\boldsymbol{\Delta x}}$ and $\vec{\mathbf{v}}$ vectors.

That is, Δx and v_x can be positive or negative.

The Kinematics Equations

For constant acceleration:

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}\,t$$

$$\overrightarrow{\Delta \boldsymbol{x}} = \frac{\vec{\mathbf{v}}_0 + \vec{\mathbf{v}}}{2}\,t$$

$$\overrightarrow{\Delta \boldsymbol{x}} = \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}\,t^2$$

$$\overrightarrow{\Delta \boldsymbol{x}} = \vec{\mathbf{v}}\,t - \frac{1}{2}\vec{\mathbf{a}}\,t^2$$

$$v_x^2 = v_{0x}^2 + 2\,a_x\,\Delta x$$

For zero acceleration:

$$\overrightarrow{\Delta \boldsymbol{x}} = \vec{\mathbf{v}}\,t$$

The Kinematics Equations: the “no-displacement” equation

From the definition of average acceleration:

$$\vec{\mathbf{a}}_{\text{avg}} = \frac{\overrightarrow{\Delta \boldsymbol{v}}}{\Delta t}$$

$$\overrightarrow{\Delta \boldsymbol{v}} = \vec{\mathbf{v}} - \vec{\mathbf{v}}_0$$

and starting at time $t = 0$ means $\Delta t = t - 0 = t$.

The Kinematics Equations: the "no-displacement" equation

From the definition of average acceleration:

$$\vec{\mathbf{a}}_{\text{avg}} = \frac{\overrightarrow{\Delta \boldsymbol{v}}}{\Delta t}$$

$$\overrightarrow{\Delta \boldsymbol{v}} = \vec{\mathbf{v}} - \vec{\mathbf{v}}_0$$

and starting at time $t = 0$ means $\Delta t = t - 0 = t$.

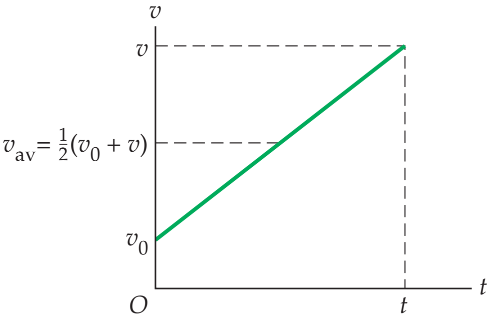
For constant acceleration $\vec{\mathbf{a}}_{\text{avg}} = \vec{\mathbf{a}}$, so $\vec{\mathbf{a}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_0}{t}$

$$\vec{\mathbf{v}}(t) = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}\,t \tag{1}$$

where v_0 is the velocity at $t = 0$ and $\vec{\mathbf{v}}(t)$ is the velocity at time t.

The Kinematics Equations: the "no-acceleration" equation

IF the acceleration of an object is constant, then the velocity-time graph is a straight line,



Area under graph is the displacement!

Area of a trapezoid:

$$A = \frac{1}{2}(b_1 + b_2)h$$

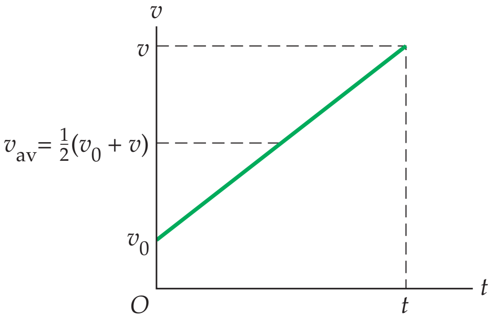
$$\overrightarrow{\Delta x} = \left(\frac{\vec{\mathbf{v}}_0 + \vec{\mathbf{v}}}{2}\right)t \quad (2)$$

Average Velocity

Average of a function (math)

The average value of a function over an interval from t_1 to t_2 is

$$f_{\text{avg}} = \frac{1}{t_2 - t_1}\left[\text{Area under } f(t) \text{ from } t_1 \text{ to } t_2\right]$$

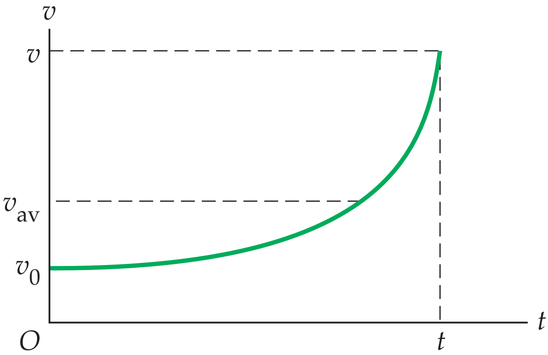


$$\vec{\mathbf{v}}_{\text{avg}} = \frac{1}{t}\left[\overrightarrow{\Delta x}\right]$$

$$\vec{\mathbf{v}}_{\text{avg}} = \frac{1}{2}(\vec{\mathbf{v}}_0 + \vec{\mathbf{v}})$$

Average Velocity

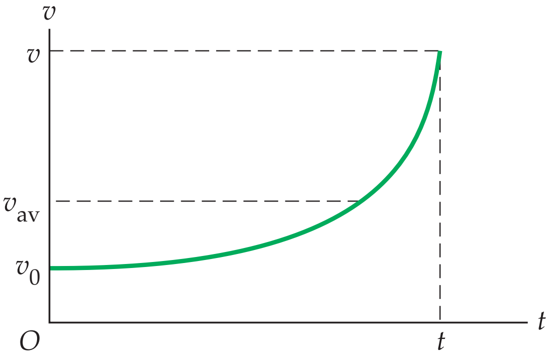
IF the acceleration of an object is not constant, and the velocity-time graph is NOT a straight line,



is the average velocity $\frac{1}{2}(\vec{\mathbf{v}}_0 + \vec{\mathbf{v}})$?

Average Velocity

IF the acceleration of an object is not constant, and the velocity-time graph is NOT a straight line,



is the average velocity $\frac{1}{2}(\vec{\mathbf{v}}_0 + \vec{\mathbf{v}})$?   No.

The Kinematics Equations: the "no-final-velocity" equation

Using the equation

$$\overrightarrow{\Delta x} = \left(\frac{\vec{\mathbf{v}}_0 + \vec{\mathbf{v}}}{2}\right) t$$

and the equation

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}\, t$$

replace $\vec{\mathbf{v}}$ in the first equation.

$$\begin{aligned} \overrightarrow{\Delta x} &= \left(\frac{\vec{\mathbf{v}}_0 + (\vec{\mathbf{v}}_0 + \vec{\mathbf{a}}\, t)}{2}\right) t \\ &= \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}\, t^2 \end{aligned}$$

For constant acceleration:

$$\vec{\mathbf{x}}(t) = \vec{\mathbf{x}}_0 + \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}\, t^2 \tag{3}$$

Example 2-6, page 34

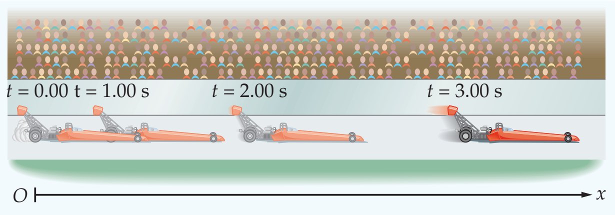
A drag racer starts from rest and accelerates at 7.40 m/s^2. How far has it traveled in (a) 1.00 s, (b) 2.00 s, (c) 3.00 s?

[1] Walker "Physics", pg 33.

Example 2-6, page 34

A drag racer starts from rest and accelerates at 7.40 m/s^2. How far has it traveled in (a) 1.00 s, (b) 2.00 s, (c) 3.00 s?

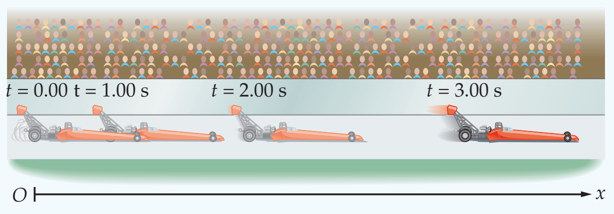
Sketch:



[1]Walker "Physics", pg 33.

Example 2-6, page 34

A drag racer starts from rest and accelerates at 7.40 m/s^2. How far has it traveled in (a) 1.00 s, (b) 2.00 s, (c) 3.00 s?

Sketch: 

Hypothesis:

- For part (a) the car will have travelled 3.7 m in part (a) because after one second the car will be moving at 7.40 m/s, but its average velocity will be less.
- The car will have travelled more than twice as far for part (b) as for part (a).
- The answer for part (c) will be greater than part (b).

[1]Walker "Physics", pg 33.

Example 2-6, page 34

A drag racer starts from rest and accelerates at 7.40 m/s^2. How far has it traveled in (a) 1.00 s, (b) 2.00 s, (c) 3.00 s?

Given: $a = 7.40$ m/s^2, $v_0 = 0$ m/s, t.
Asked for: Δx

[1] Walker "Physics", pg 33.

Example 2-6, page 34

A drag racer starts from rest and accelerates at 7.40 $\mathrm{m/s^2}$. How far has it traveled in (a) 1.00 s, (b) 2.00 s, (c) 3.00 s?

Given: $a = 7.40\ \mathrm{m/s^2}$, $v_0 = 0\ \mathrm{m/s}$, t.
Asked for: Δx

Strategy: Use equation

$$\overrightarrow{\boldsymbol{\Delta x}} = \vec{\mathbf{x}}(t) - \vec{\mathbf{x}}_0 = \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}t^2$$

(a) Letting the x-direction in my sketch be positive:

$$\begin{aligned}
\Delta x &= \cancelto{0}{v_0}\, t + \frac{1}{2}at^2 \\
&= \frac{1}{2}(7.40\ \mathrm{m/s^2})(1.00\ \mathrm{s})^2 \\
&= \underline{3.70\ \mathrm{m}}
\end{aligned}$$

---

[1]Walker "Physics", pg 33.

Example 2-6, page 34

A drag racer starts from rest and accelerates at 7.40 m/s^2. How far has it traveled in (a) 1.00 s, (b) 2.00 s, (c) 3.00 s?

Use the same equation for (b), (c)

$$\overrightarrow{\boldsymbol{\Delta x}} = \vec{\mathbf{x}}(t) - \vec{\mathbf{x}}_0 = \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}t^2$$

$$\text{(b)} \qquad \begin{aligned} \Delta x &= \frac{1}{2}at^2 \\ &= \frac{1}{2}(7.40 \text{ m/s}^2)(2.00 \text{ s})^2 \\ &= \underline{14.8 \text{ m}} \end{aligned}$$

$$\text{(c)} \qquad \begin{aligned} \Delta x &= \frac{1}{2}at^2 \\ &= \frac{1}{2}(7.40 \text{ m/s}^2)(3.00 \text{ s})^2 \\ &= \underline{33.3 \text{ m}} \end{aligned}$$

Example 2-6, page 34

(a) 3.70 m, (b) 14.8 m, (c) 33.3 m

Analysis:
My hypotheses for (a), (b), and (c) were correct.

It makes sense that the distances covered by the car increases with time, and it makes sense that the distance covered in each one second interval is greater than the distance covered in the previous interval since the car is still accelerating.

The distance covered over 3 seconds is 9 times the distance covered in 1 second.

The car covers ~ 30 m in 3 s, giving an average speed of ~ 10 m/s. We know cars can go much faster than this, so the answer is not unreasonable.

[1] Walker "Physics", pg 33.

The Kinematics Equations: the "no-initial-velocity" equation

We can build a very similar equation to that last one.
This time we rearrange $\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}\,t$ to give:

$$\vec{\mathbf{v}}_0 = \vec{\mathbf{v}} - \vec{\mathbf{a}}\,t$$

And put that into the equation

$$\overrightarrow{\Delta \boldsymbol{x}} = \left(\frac{\vec{\mathbf{v}}_0 + \vec{\mathbf{v}}}{2}\right) t$$

$$\begin{aligned} \overrightarrow{\Delta \boldsymbol{x}} &= \left(\frac{(\vec{\mathbf{v}} - \vec{\mathbf{a}}\,t) + \vec{\mathbf{v}}}{2}\right) t \\ &= \vec{\mathbf{v}}\,t - \frac{1}{2}\vec{\mathbf{a}}\,t^2 \end{aligned}$$

For constant acceleration:

$$\vec{\mathbf{x}}(t) = \vec{\mathbf{x}}_0 + \vec{\mathbf{v}}\,t - \frac{1}{2}\vec{\mathbf{a}}\,t^2 \qquad (4)$$

The Kinematics Equations: the “no-time” equation

The last equation we will derive is a scalar equation.

$$v_x^2 = v_{0x}^2 + 2\, a_x\, \Delta x \tag{5}$$

See next lecture for this.

The Kinematics Equations

For constant acceleration:

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}\,t$$

$$\overrightarrow{\Delta \boldsymbol{x}} = \frac{\vec{\mathbf{v}}_0 + \vec{\mathbf{v}}}{2}\,t$$

$$\overrightarrow{\Delta \boldsymbol{x}} = \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}\,t^2$$

$$\overrightarrow{\Delta \boldsymbol{x}} = \vec{\mathbf{v}}\,t - \frac{1}{2}\vec{\mathbf{a}}\,t^2$$

$$v_x^2 = v_{0x}^2 + 2\,a_x\,\Delta x$$

For zero acceleration:

$$\overrightarrow{\Delta \boldsymbol{x}} = \vec{\mathbf{v}}\,t$$

The Kinematics Equations Summary

For constant acceleration:

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}\,t$$

$$\overrightarrow{\Delta \boldsymbol{x}} = \frac{\vec{\mathbf{v}}_0 + \vec{\mathbf{v}}}{2}\,t$$

$$\overrightarrow{\Delta \boldsymbol{x}} = \vec{\mathbf{v}}_0\,t + \frac{1}{2}\vec{\mathbf{a}}\,t^2$$

$$\overrightarrow{\Delta \boldsymbol{x}} = \vec{\mathbf{v}}\,t - \frac{1}{2}\vec{\mathbf{a}}\,t^2$$

$$v_x^2 = v_{0x}^2 + 2\,a_x\,\Delta x$$

For zero acceleration:

$$\vec{\mathbf{x}} = \vec{\mathbf{v}}\,t$$

Summary

- kinematics equations for constant acceleration
- some derivations and an example

First Test next week **Thursday, Jan 30**.

Homework Walker Physics:

- Ch 2, onward from page 47. Questions: 12, 13; Problems: 49