Introduction to Mechanics
Practice using the Kinematics Equations

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Jan 23, 2020

Last time

- how to solve problems
- the kinematics equations

Overview

- derivation of the last kinematics equation
- using kinematics equations
- problem solving practice

The Kinematics Equations

For constant acceleration:

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}\,t$$

$$\overrightarrow{\Delta \boldsymbol{x}} = \frac{\vec{\mathbf{v}}_0 + \vec{\mathbf{v}}}{2}\,t$$

$$\overrightarrow{\Delta \boldsymbol{x}} = \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}\,t^2$$

$$\overrightarrow{\Delta \boldsymbol{x}} = \vec{\mathbf{v}}\,t - \frac{1}{2}\vec{\mathbf{a}}\,t^2$$

$$v^2 = v_0^2 + 2\,a\,\Delta x$$

For zero acceleration:

$$\overrightarrow{\Delta \boldsymbol{x}} = \vec{\mathbf{v}}\,t$$

The Kinematics Equations: the “no-time” equation

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We could also write this as:

$$(\Delta x)\,\hat{\mathbf{i}} = \left(\frac{v_{0x} + v_x}{2} t\right)\hat{\mathbf{i}}$$

where Δx, v_{0x}, and v_x could each be positive or negative.

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We do the same for equation (1):

$$v\,\hat{\mathbf{i}} = (v_{0x} + a_x t)\,\hat{\mathbf{i}}$$

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$$v = (v_{0x} + a_x t)$$

Rearranging for t:

$$t = \frac{v_x - v_{0x}}{a_x}$$

The Kinematics Equations: the "no-time" equation

$$t = \frac{v_x - v_{0x}}{a_x} \; ; \qquad \Delta x = \left(\frac{v_{0x} + v_x}{2} \right) t$$

Substituting for t in our Δx equation:

$$\begin{aligned} \Delta x &= \left(\frac{v_{0x} + v_x}{2} \right) \left(\frac{v_x - v_{0x}}{a_x} \right) \\ 2a_x \Delta x &= (v_{0x} + v_x)(v_x - v_{0x}) \end{aligned}$$

so,

$$v_x^2 = v_{0x}^2 + 2\, a_x\, \Delta x \tag{5}$$

Example 2-7, pg 34

Jets at JFK International Airport accelerate from rest at one end of a runway, and must attain takeoff speed before reaching the other end of the runway.

(a) Plane A has acceleration a and takeoff speed v_{to}. What is the minimum length of runway, Δx_A, required for this plane? Give a symbolic answer.

(b) Plane B has the same acceleration as plane A, but requires twice the takeoff speed. Find Δx_B and compare with Δx_A.

(c) Find the minimum runway length for plane A if $a = 2.20$ m/s^2 and $v_{to} = 95.0$ m/s. (These values are typical for a 747 jetliner.)

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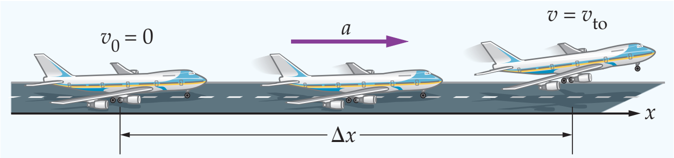
Sketch:



Example 2-7, pg 34

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(c) Find the minimum runway length for plane A if $a = 2.20$ m/s^2 and $v_{to} = 95.0$ m/s.

Hypothesis: I guess a minimum runway length would be about 2 football fields, about 200 m.

Example 2-7, pg 34

Given: a, $v = v_{to}$, $v_0 = 0$.

Asked for: Δx.

Strategy: Use equation

$$v^2 = v_0^2 + 2a(\Delta x)$$

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(a) Rearrange for Δx:

$$\begin{aligned} v_{to}^2 &= \cancelto{0}{v_0^2} + 2a(\Delta x_A) \\ \Delta x_A &= \frac{v_{to}^2}{2a} \end{aligned}$$

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(b) Now for plane B, $v = 2v_{to}$:

$$\begin{aligned} (2v_{to})^2 &= 2a(\Delta x_B) \\ \Delta x_B &= \frac{4v_{to}^2}{2a} \\ \underline{\Delta x_B} &\underline{= 4\Delta x_A} \end{aligned}$$

Example 2-7, pg 34

(c) Find the minimum runway length for plane A if $a = 2.20$ m/s^2 and $v_{to} = 95.0$ m/s.

Using our expression from (a):

$$\begin{aligned} \Delta x_A &= \frac{v_{to}^2}{2a} \\ &= \frac{(95.0 \text{ m/s})^2}{2(2.20 \text{ m/s}^2)} \\ &= \underline{2.05 \times 10^3 \text{ m}} \end{aligned}$$

Example 2-7, pg 34

Analysis: **My hypotheses were not correct!**

For part (a), the distance was inversely proportional to the acceleration, but it was proportional to the **square** of the takeoff velocity.

For part (b), Δx_B is **four times** as big as Δx_A. This makes sense because the distance is proportional to the square of the speed.

In part (c), my guess was an order of magnitude too small. The 747 is one of the biggest commercial jets, so it makes sense that it needs a long runway. Looking at Google Maps, the length of the SJC airport is about 3 km, which makes sense if runways need to be at least 2 km.

Using the Kinematics Equations to solve problems

Process:

1. Identify which quantity we need to find and which ones we are given.
2. Is there a quantity that we are not given and are not asked for?
   1. If so, use the equation that does *not* include that quantity.
   2. If there is not, more that one kinematics equation may be required or there may be several equivalent approaches.
3. Input known quantities and solve.

Example 1

How long would it take a car, starting from rest and accelerating uniformly in a straight line at 5.00 m/s^2, to cover a distance of 200.0 m?[1]

[1] Leduc, "Cracking the AP Physics B Exam" Princeton Review.

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Hypothesis: Less than 10 s.

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Asked for: t

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$$\begin{aligned}
(\Delta x)\hat{\mathbf{i}} &= (0 + \frac{1}{2} a t^2)\hat{\mathbf{i}} \\
t^2 &= \frac{2\,\Delta x}{a} \\
t &= \sqrt{\frac{2\,\Delta x}{a}} \\
t &= \sqrt{\frac{2\,(200\text{ m})}{5\text{ m/s}^2}} \\
t &= \underline{8.94\text{ s}}
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A little less than 10s, as expected.

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Example 2 (Ch 2 #44)

When you see a traffic light turn red, you apply the brakes until you come to a stop. If your initial speed was 12 m/s, and you were heading due west, what was your average velocity during braking? Assume constant deceleration.

[1] Walker, “Physics”, pg 50.

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$$= \frac{(12+0)\text{m/s}}{2} \;\; \text{West}$$

$$= \underline{6 \text{ m/s} \;\; \text{West}}$$

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\end{aligned}
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Analysis: Yup, agreed with hypothesis. With constant acceleration the average velocity is just the average of the initial and final velocities.

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Example 3

A car driver sees a speed trap ahead, while driving at 30.0 m/s ($\sim$ 67 mph). Assume the car brakes with constant deceleration of 2.30 m/s^2. It slows to 20.0 m/s, what was the time taken to slow?

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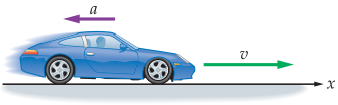

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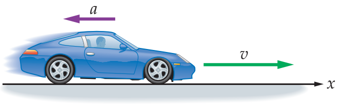

Hypothesis: He's slowing by 10 m/s at about 2 $\mathrm{m/s^2}$, perhaps around $10 \div 2 = 5$ s.

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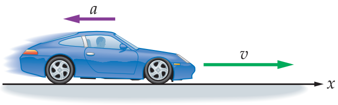

Hypothesis: He's slowing by 10 m/s at about 2 m/s^2, perhaps around $10 \div 2 = 5$ s.

Given: $\vec{\mathbf{v}}_0 = +30.0 \text{ m/s } \hat{\mathbf{i}}$, $\vec{\mathbf{v}} = +20.0 \text{ m/s } \hat{\mathbf{i}}$, $\vec{\mathbf{a}} = -2.30 \text{ m/s}^2 \; \hat{\mathbf{i}}$
Asked for: t

Example 3

Strategy: Use

$$v = v_0 + at$$

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Rearrange:

$$\begin{aligned} t &= \frac{v - v_0}{a} \\ &= \frac{20.0 \text{ m/s} - 30.0 \text{ m/s}}{-2.30 \text{ m/s}^2} \\ &= \underline{4.35 \text{ s}} \end{aligned}$$

Reasonable / Agrees with hypothesis: yes, just a bit smaller. That makes sense because the acceleration had magnitude 2.30 m/s^2 rather than 2 m/s^2. It is a reasonable amount of time to need to slow for a speed trap.

Summary

- practice using the kinematics equations

First Test Thursday, Jan 30.

Homework

Walker Physics:

- Ch 2, onward from page 47. Questions: 11; Problems: 43, 47, 55, 57