Introduction to Mechanics
Practice using the Kinematics Equations

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Last time

- using kinematics equations
- problem solving practice

Overview

- more practice using kinematics equations

The Kinematics Equations

For constant acceleration:

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}\,t$$

$$\overrightarrow{\Delta \boldsymbol{x}} = \frac{\vec{\mathbf{v}}_0 + \vec{\mathbf{v}}}{2}\,t$$

$$\overrightarrow{\Delta \boldsymbol{x}} = \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}\,t^2$$

$$\overrightarrow{\Delta \boldsymbol{x}} = \vec{\mathbf{v}}\,t - \frac{1}{2}\vec{\mathbf{a}}\,t^2$$

$$v^2 = v_0^2 + 2\,a\,\Delta x$$

For zero acceleration:

$$\overrightarrow{\Delta \boldsymbol{x}} = \vec{\mathbf{v}}\,t$$

Using the Kinematics Equations to solve problems

Process:

1. Identify which quantity we need to find and which ones we are given.
2. Is there a quantity that we are not given and are not asked for?
    1. If so, use the equation that does *not* include that quantity.
    2. If there is not, more that one kinematics equation may be required or there may be several equivalent approaches.
3. Input known quantities and solve.

Example 4 (Ch 2 #62)

A boat is cruising in a straight line at a constant speed of 2.6 m/s when it is shifted into neutral. After coasting 12 m the engine is engaged again, and the boat resumes cruising at the reduced constant speed of 1.6 m/s. Assuming constant acceleration while coasting,

(a) how long did it take for the boat to coast the 12 m?

(b) What was the boat's acceleration while it was coasting?

(c) When the boat had coasted for 6.0 m, was its speed 2.1 m/s, more than 2.1 m/s, or less than 2.1 m/s? Explain.

[1] Walker, "Physics", pg 52.

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(a) how long did it take for the boat to coast the 12 m?

Draw a sketch.

Hypothesis: less than $(12 \text{ m}) \div (1.6 \text{ m/s}) = 7.5$ s.

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Hypothesis: less than (12 m)$\div$(1.6 m/s) $=$ 7.5 s.

Given: $v_0 =$2.6 m/s, $v =$1.6 m/s, $\Delta x =$12 m.
Asked for: t

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Given: v_0 =2.6 m/s, v =1.6 m/s, Δx =12 m.
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Strategy: use equation

$$\overrightarrow{\Delta x} = \frac{\vec{\mathbf{v}}_0 + \vec{\mathbf{v}}}{2} t$$

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Rearrange:

$$\begin{aligned} t &= \frac{2\,\Delta x}{v + v_0} \\ &= \frac{2\,(12\text{ m})}{1.6\text{ m/s} + 2.6\text{ m/s}} \\ &= 5.71\text{ s} \\ &= \underline{5.7\text{ s}} \quad (2\text{ sig figs}) \end{aligned}$$

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Given: $v_0 =$2.6 m/s, $v =$1.6 m/s, $\Delta x =$12 m; $t = 5.71$ s
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Given: $v_0 =$2.6 m/s, $v =$1.6 m/s, $\Delta x =$12 m; $t = 5.71$ s
Asked for: a

Strategy: there are many ways... Here's one:

$$v^2 = v_0^2 + 2a(\Delta x)$$

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Rearrange:

$$\begin{aligned} a &= \frac{v^2 - v_0^2}{2\,\Delta x} \\ &= \frac{(1.6\text{ m/s})^2 - (2.6\text{ m/s})^2}{2\,(12\text{ m})} \\ &= \underline{-0.175\text{ m/s}^2} \\ &= \underline{-0.18\text{ m/s}^2} \quad \text{(2 sig figs)} \end{aligned}$$

Analysis: This is a very gentle deceleration, but it is reasonable for a boat. We get the same answer if we use $\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}\,t$ instead.

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(c) When the boat had coasted for 6.0 m, was its speed 2.1 m/s, more than 2.1 m/s, or less than 2.1 m/s? Explain.

Hypothesis: after 6.0 m it should be traveling at more than 2.1 m/s, because

$$\Delta v \propto t,\ \Delta v \not\propto \Delta x$$

Distance covered is the area under a velocity-time graph; when moving faster, you cover more distance.

Example 4 (Ch 2 #62)

Check: $v^2 = v_0^2 + 2a(\Delta x)$. Rearrange:

$$\begin{aligned} v^2 &= v_0^2 + 2a(\Delta x) \\ v^2 &= (2.6\text{ m/s})^2 + 2(-0.175\text{ m/s}^2)(6.0\text{ m}) \\ v &= \underline{2.2\text{ m/s}} \end{aligned}$$

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Analysis: For constant acceleration, the velocity changes linearly with time but nonlinearly with distance.

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Example 5

A car driver sees an obstacle in the road and applies the brakes. It takes him 4.33 s to stop the car over a distance of 55.0 m. Assuming the car brakes with constant acceleration, what was the car's deceleration?

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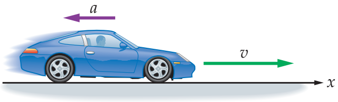

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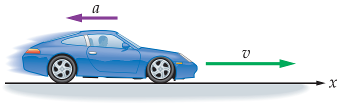

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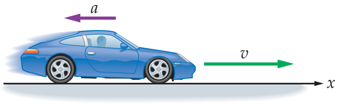

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Given: $t = 4.33$ s, $\Delta x = 55.0$ m, $v = 0$ m/s
Asked for: $\vec{\mathbf{a}}$

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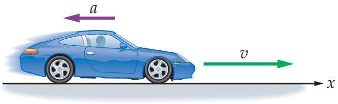

Hypothesis: 55 m is not very far and 4.33 s is not a long time: the deceleration must be high-ish. I would guess maybe 5 m/s^2 which is about half of g.

Given: $t = 4.33$ s, $\Delta x = 55.0$ m, $v = 0$ m/s
Asked for: $\vec{\mathbf{a}}$
Strategy: use

$$\overrightarrow{\boldsymbol{\Delta x}} = \vec{\mathbf{v}}\,t - \frac{1}{2}\vec{\mathbf{a}}\,t^2$$

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$$\begin{aligned} \overrightarrow{\Delta \boldsymbol{x}} &= \vec{\mathbf{v}}\, t - \frac{1}{2} \vec{\mathbf{a}}\, t^2 \\ \frac{1}{2} \vec{\mathbf{a}}\, t^2 &= \vec{\mathbf{v}}\, t - \overrightarrow{\Delta \boldsymbol{x}} \\ \vec{\mathbf{a}} &= \frac{2(\vec{\mathbf{v}}\, t - \overrightarrow{\Delta \boldsymbol{x}})}{t^2} \end{aligned}$$

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$$\begin{aligned}
\overrightarrow{\Delta x} &= \vec{\mathbf{v}}\,t - \frac{1}{2}\vec{\mathbf{a}}\,t^2 \\
\frac{1}{2}\vec{\mathbf{a}}\,t^2 &= \vec{\mathbf{v}}\,t - \overrightarrow{\Delta x} \\
\vec{\mathbf{a}} &= \frac{2(\vec{\mathbf{v}}\,t - \overrightarrow{\Delta x})}{t^2} \\
&= \frac{2(0 - 55.0\ \mathrm{m}\hat{\mathbf{i}})}{(4.33\ \mathrm{s})^2} \\
\vec{\mathbf{a}} &= -5.87\ \mathrm{m/s^2}\,\hat{\mathbf{i}}
\end{aligned}$$

Or, the car's acceleration is <u>5.87 $\mathrm{m/s^2}$, opposite the direction of the car's travel</u>.

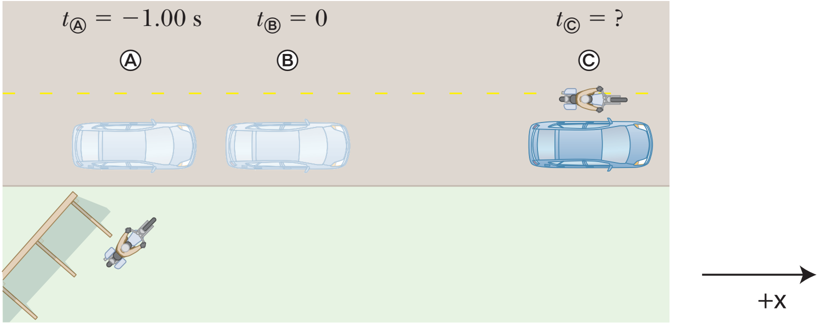
Example 5

Reasonable / Agrees with hypothesis: This is a large deceleration, but a car can manage it. It is reasonable considering he needs to stop before the obstacle and is breaking hard.

Example 6

A car traveling at a constant speed of 45.0 m/s passes a trooper on a motorcycle hidden behind a billboard. One second after the speeding car passes the billboard, the trooper sets out from the billboard to catch the car, accelerating at a constant rate of 3.00 m/s^2. How long does it take the trooper to overtake the car?

Sketch:



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[1]Serway & Jewett, "Physics for Scientists and Engineers", pg 39.

Summary

- practice using the kinematics equations

First Test Thursday, Jan 30.

Homework

- **Please bring a 30 cm ruler to class on Wednesday!**

Walker Physics:

- Ch 2, onward from page 52. Problems: 65*, 67, 121
- Read all of Ch 2.

*Part (a) of this problem is unclear. Should read: "(a) How much time *from the moment his friend passes him* does it take until he catches his friend?"