Introduction to Mechanics
Free-Fall

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Last time

- kinematics examples

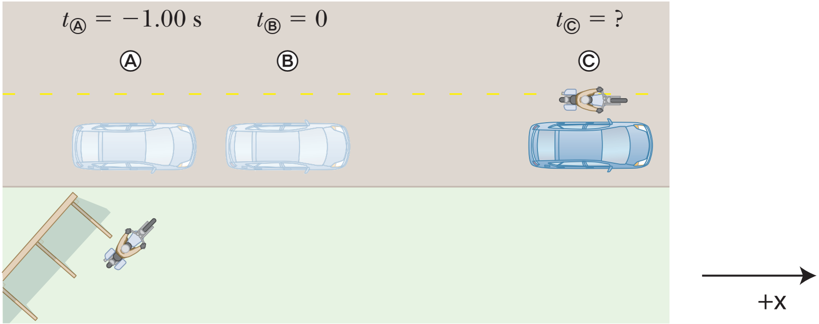
Overview

- harder kinematics example
- inertia
- free fall
- problems with objects in free fall

Example 6

A car traveling at a constant speed of 45.0 m/s passes a trooper on a motorcycle hidden behind a billboard. One second after the speeding car passes the billboard, the trooper sets out from the billboard to catch the car, accelerating at a constant rate of 3.00 m/s^2. How long does it take the trooper to overtake the car?

Sketch:



[1] Serway & Jewett, "Physics for Scientists and Engineers", pg 39.

Example 6

Hypothesis: The trooper accelerates at 3.00 m/s^2. It will take 15 s for him to reach a speed of 45.0 m/s, but then he still needs to catch up to the car. I would guess it will take him about another 15 s to catch up. Guess: about 30 s or so.

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Given: $v_{\text{car}} = 45.0$ m/s, ($a_{\text{car}} = 0$), $a_{\text{tr}} = 3.00$ m/s^2, $v_{\text{tr},0} = 0$ m/s, and the car has a 1 s head start

Asked for: t when they are at the same posititION.

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Strategy: Use the idea that when the trooper has caught up

$$\overrightarrow{\boldsymbol{\Delta x}}_{\text{car}} = \overrightarrow{\boldsymbol{\Delta x}}_{\text{tr}}$$

We can visualize this on a graph, or just do a bit of algebra.

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$$
\begin{aligned}
\overrightarrow{\boldsymbol{\Delta x}}_{\mathrm{car}} &= \overrightarrow{\boldsymbol{\Delta x}}_{\mathrm{tr}} \\
v_{\mathrm{car}}(t+1)\cancel{\hat{\mathbf{i}}} &= \left(\cancelto{0}{(v_{\mathrm{tr},0})}\, t + \frac{1}{2}a_{\mathrm{tr}}t^2\right)\cancel{\hat{\mathbf{i}}} \\
0 &= \frac{1}{2}a_{\mathrm{tr}}t^2 - v_{\mathrm{car}}t - v_{\mathrm{car}}
\end{aligned}
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This is a quadratic expression in t.

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$$\begin{aligned}
0 &= \frac{1}{2}a_{\text{tr}}t^2 - v_{\text{car}}t - v_{\text{car}} \\
t &= \frac{v_{\text{car}} \pm \sqrt{v_{\text{car}}^2 + 2a_{\text{tr}}v_{\text{car}}}}{a_{\text{tr}}} \\
t &= \underline{31.0 \text{ s}}
\end{aligned}$$

Example 6

Answer: $t = 31.0$ s

Analysis: This answer is quite close to the hypothesis! The car is going 45.0 m/s, or 100 mi/h, which is very fast, but the trooper also has a high acceleration of 3.00 m/s^2. Motorcycles can have accelerations even higher than that, so the numbers in this question are reasonable.

Free-Falling Objects

One common scenario of interest where acceleration is constant is objects freely falling.

When we refer to free fall, we mean objects moving under the influence of gravity, and where we are ignoring resistive forces, *eg.* air resistance.

Galileo and the Leaning Tower of Pisa

Aristotle, an early Greek natural philosopher, said that heavier objects fall faster than lighter ones.

Galileo tested this idea and found it was wrong. Any two massive objects accelerate at the same rate.

Galileo studied the motion of objects by **experiment**, as well as by abstract reasoning.

Galileo and Inertia

He considered balls rolling on inclined surfaces and developed the notion of *inertia*.

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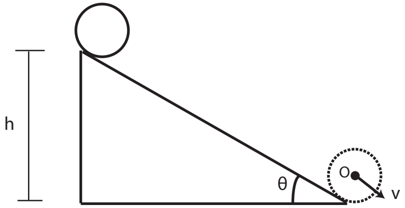
Inertia is the tendency of objects to stay doing whatever they are already doing, unless they are interfered with.

Galileo's idea of inertia:

A body moving on a level surface will continue in the same direction at a constant speed unless disturbed.

Acceleration and Free-Fall

Galileo reasoned about the acceleration due to gravity by thinking more about inclined surfaces.



The steeper the incline the larger the acceleration.

Free-Fall

When the ball drops straight downward, it gains approximately 9.8 m/s of speed in each second.

Time of fall (s)	Velocity acquired (m/s)
0	0
1	10
2	20
3	30
$\vdots$	$\vdots$

This is a constant acceleration! We call this acceleration g.

$$\boxed{g = 9.8 \text{ m s}^{-2} \approx 10 \text{ m s}^{-2}}$$

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After falling for 6.5 s, (roughly) what is the ball's speed? 65 m/s

Free-Falling Objects

The important point is that at the surface of the Earth, **all objects** experience this same acceleration due to gravity: $g = 9.8$ m s^{-2}.

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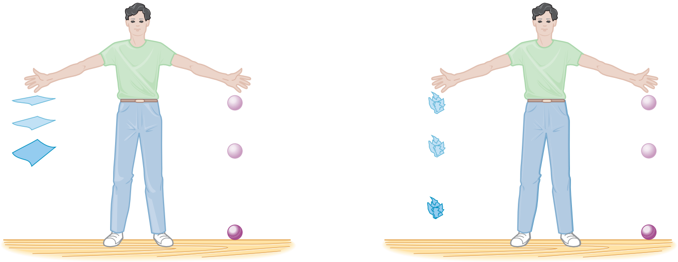
The important point is that at the surface of the Earth, **all objects** experience this same acceleration due to gravity: $g = 9.8 \text{ m s}^{-2}$.

In the absence of air resistance, the acceleration does not depend on an object's mass!



The fact that acceleration due to gravity is independent of mass can be seen in airless environments...

[1]Figure from Walker, "Physics", page 39.

Free-Falling Objects

Objects near the Earth's surface have a **constant** acceleration of $\boldsymbol{g = 9.8\ \mathrm{ms}^{-2}}$. (Or, about 10 ms^{-2})

The kinematics equations for constant acceleration all apply.

Notation and a new Unit Vector

When drawing a vertical axis (*eg.* for a falling object), it is traditional to label the axis y.

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Reminder: Unit vectors are one-unit-long vectors that just give a direction.

$\hat{\mathbf{j}}$ is the unit vector pointing in the $+y$ direction. In the textbook, $\hat{\mathbf{y}}$ is used for this.

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$\vec{\mathbf{v}} = -31 \text{ m/s}\,\hat{\mathbf{j}}$ (2 sig. figs.)

Ex 2-5: Free-Fall Separation - skipping, but can look

You drop a rock from a bridge to the river below. When the rock has fallen 4 m, you drop a second rock. As the rocks continue their free fall, does their separation increase, decrease, or stay the same?

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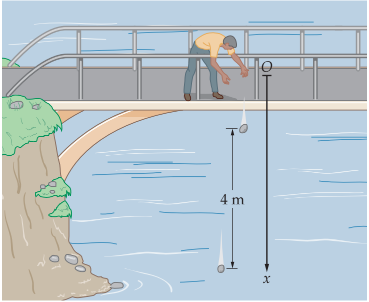

Hypothesis?

[1]Walker, "Physics", page 40.

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Let $\downarrow +y$.

Kinematic equation:

$$\Delta y = v_{0y} t + \frac{1}{2} a_y t^2$$

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Suppose the time taken to fall 4 m is t_4.

$$t_4 = \sqrt{\frac{2(4\text{ m})}{g}} = 0.9035\text{ s}$$

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Ex 2-5: Free-Fall Separation - skipping, but can look

At $t = 0$ s, the second rock is dropped, and the two rocks are 4 m apart. The first rock has already been falling for time t_4.

Position of the first rock:

$$y_1 = \frac{1}{2}g(t + t_4)^2$$

Position of the second rock:

$$y_2 = \frac{1}{2}gt^2$$

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$$\begin{aligned} s &= y_1 - y_2 \\ &= \frac{1}{2}g(t + t_4)^2 - \frac{1}{2}gt^2 \\ s &= \underbrace{(g\, t_4)}_{\text{constant}}\, \underbrace{t}_{\text{time var.}} + \frac{g}{2}t_4^2 \leftarrow \text{constant} \end{aligned}$$

Ex 2-5: Free-Fall Separation - skipping, but can look

The separation of the two rocks, s, is

$$s = (g\, t_4)\, t + \frac{g}{2} t_4^2$$

It increases linearly with time!

Any two objects dropped one before the other will increase their separation.

Implication of Free-Fall Separation

Why does a stream of water get narrower as if falls from a faucet?

Summary

- free-fall

First Test Thursday.

Homework

- **Please bring a 30 cm ruler to class tomorrow!**

Walker Physics:

- prev: Ch 2, onward from page 52. Problems: 65*, 67, 121
- new: Ch 2, Problems: 61, 63, 69, 73, 101

*Part (a) of this problem is unclear. Should read: "(a) How much time *from the moment his friend passes him* does it take until he catches his friend?"