Introduction to Mechanics
Vectors in 2 Dimensions
Trigonometery
Vector Addition

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Last time

- freely falling objects
- class activity: measure your reaction time

Overview

- vectors in 2 dimensions
- some trigonometry review
- vector addition

Reminder: Vectors and Scalars

scalar

A scalar quantity indicates an amount. It is represented by a real number. (Assuming it is a physical quantity.)

vector

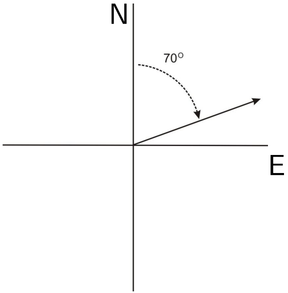
A vector quantity indicates both an amount (magnitude) and a direction. It is represented by a real number for each possible direction, or a real number and (an) angle(s).

In the lecture notes vectors are represented using bold variables with over-arrows.

How can we write vectors? - with angles

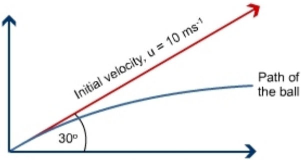
Bearing angles

Example, a plane flies at a bearing of 70°



Generic reference angles

A baseball is thrown at 10 m s^{-1} at 30° above the horizontal.



How can we write vectors? - as a list

A vector in the x, y-plane could be written

$$(2, 1) \quad \text{or} \quad \begin{pmatrix} 2 & 1 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

(In some textbooks it is written $\langle 2, 1 \rangle$, but there are reasons not to write it this way.)

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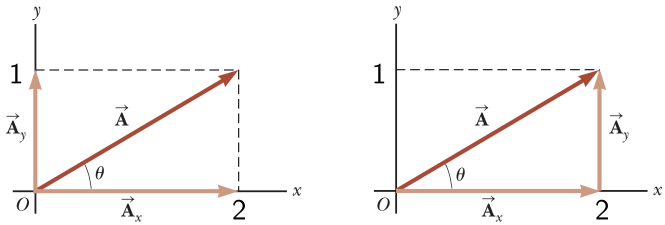
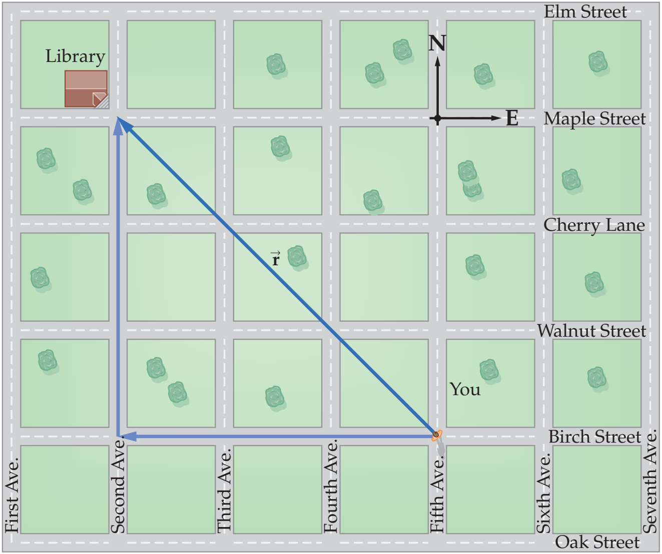
When drawn in the x, y-plane it looks like:



Vector Components



[1]Walker, 4th ed, page 59.

Vector Components

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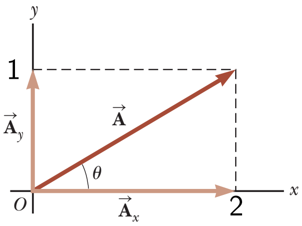
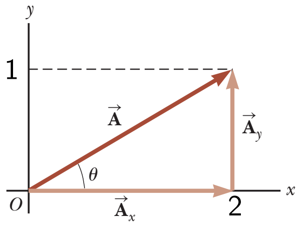
We say that 2 is the ***x*-component** of the vector $(2, 1)$ and 1 is the ***y*-component**.

Each component direction is perpendicular to the others: $x \perp y$.



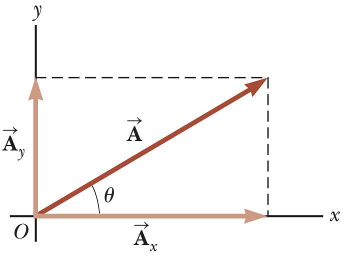


Components

Consider the 2 dimensional vector $\vec{\mathbf{A}}$.

Since the two vectors add together by attaching the head of one to the tail of the other, which is the same as adding the components, we can always write a vector in the x, y-plane as the sum of two component vectors.

$$\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y$$
$$(2,1) = (2,0) + (0,1)$$



Representing Vectors: Unit Vectors

We can write a vector in the x, y-plane as the sum of two component vectors.

To indicate the components we can use *unit vectors.*

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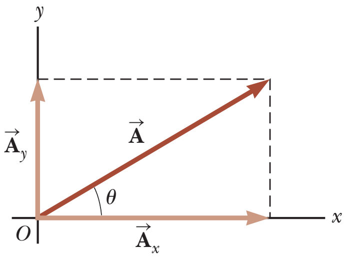
In two dimensions, a pair of perpendicular unit vectors are usually denoted $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ (or sometimes $\hat{\mathbf{x}}, \hat{\mathbf{y}}$).

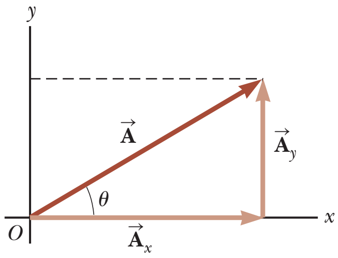
A 2 dimensional vector can be written as $\vec{\mathbf{A}} = (2, 1) = 2\hat{\mathbf{i}} + \hat{\mathbf{j}}$.

Components

Vector $\vec{\mathbf{A}}$ is the sum of a piece along x and a piece along y:
$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$.

A_x is the i-component (or x-component) of $\vec{\mathbf{A}}$ and
A_y is the j-component (or y-component) of $\vec{\mathbf{A}}$.





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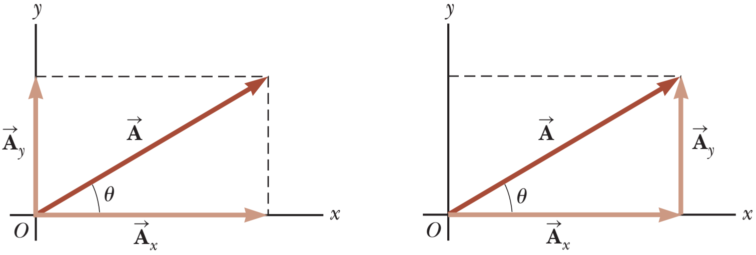
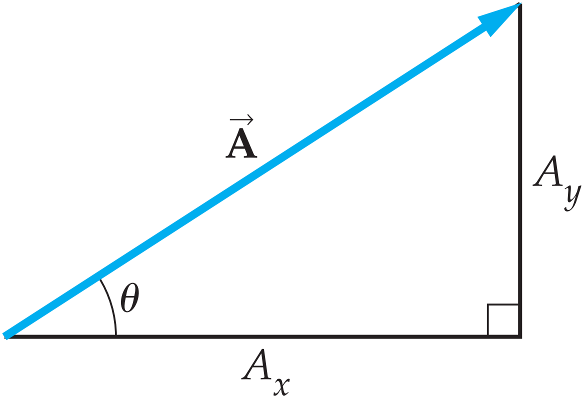


To find a vector's components requires some simple trigonometry.

Trigonometry



$$\sin\theta = \frac{A_y}{A} \;;\quad \cos\theta = \frac{A_x}{A} \;;\quad \tan\theta = \frac{A_y}{A_x}$$

Trigonometry, Ex 3.1

Captain Cyrus Harding wants to find the height of a cliff. He stands with his back to the base of the cliff, then marches straight away from it for 5.00×10^2 ft. At this point he lies on the ground and measures the angle from the horizontal to the top of the cliff. If the angle is 34.0°, (a) how high is the cliff? (b) What is the straight-line distance from Captain Harding to the top of the cliff?

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[1]Walker, 4th ed, page 60.

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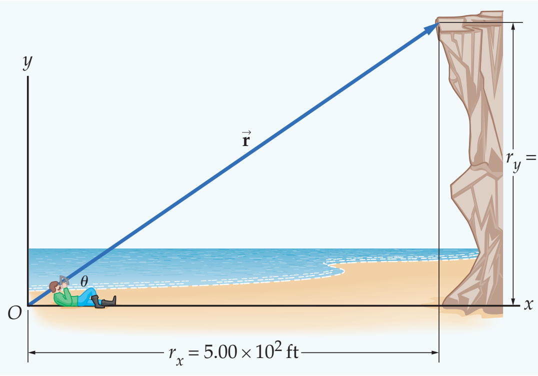
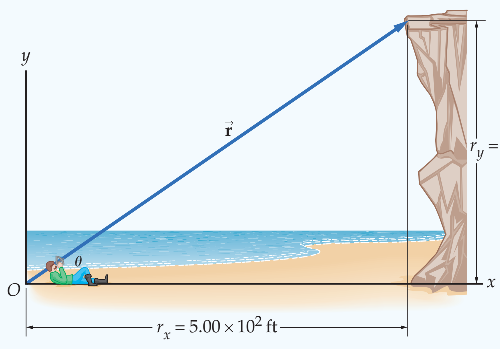


[1]Walker, 4th ed, page 60.

Trigonometry, Ex 3.1

(a) how high is the cliff?



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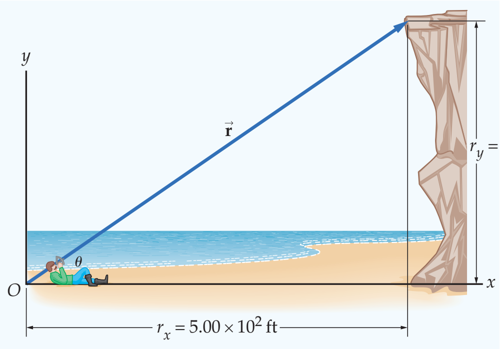

$$\tan\theta = \frac{r_y}{r_x}$$

Multiply both sides by r_x:

$$r_y = r_x \tan\theta$$

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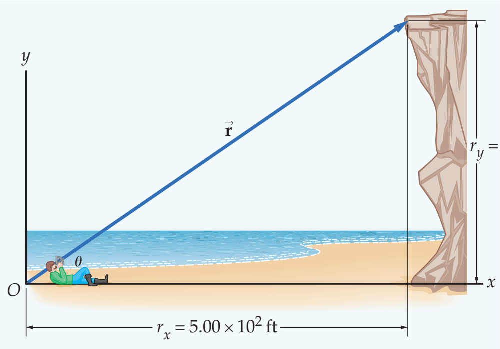


$$\tan\theta = \frac{r_y}{r_x}$$

Multiply both sides by r_x:

$$r_y = r_x \tan\theta$$

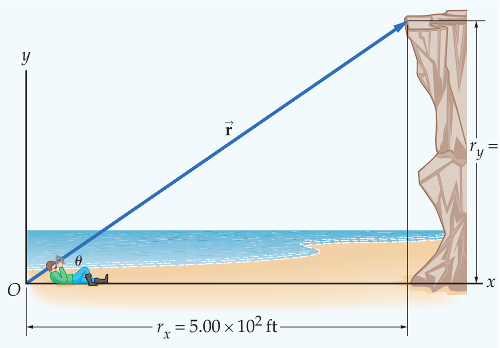
Solve:

$$\begin{aligned} r_y &= (500\text{ ft})\tan(34.0^\circ) \\ &= \underline{337\text{ ft}} \quad \text{(to 3 s.f.)} \end{aligned}$$

[1]Walker, 4th ed, page 60.

Trigonometry, Ex 3.1

(b) What is the straight-line distance from Captain Harding to the top of the cliff?



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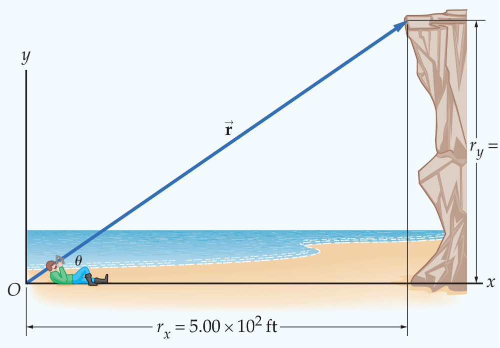

Pythagorean theorem

$$r = \sqrt{r_x^2 + r_y^2}$$

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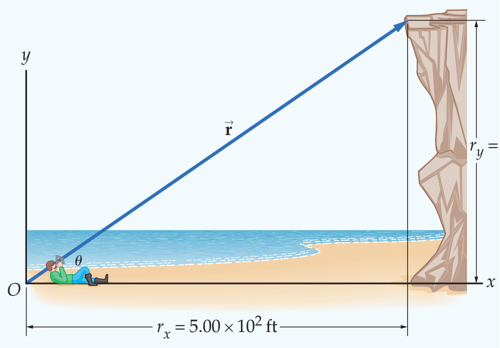

Pythagorean theorem

$$r = \sqrt{r_x^2 + r_y^2}$$

Solve:

$$\begin{aligned} r &= \sqrt{(500\text{ ft})^2 + (337\text{ ft})^2} \\ &= \underline{603\text{ ft}} \quad \text{(to 3 s.f.)} \end{aligned}$$

(Or use $r = \frac{r_x}{\cos\theta}$)

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Magnitude-and-Angle Notation to Components

Vector $\vec{\mathbf{A}}$ is the sum of a piece along x and a piece along y:
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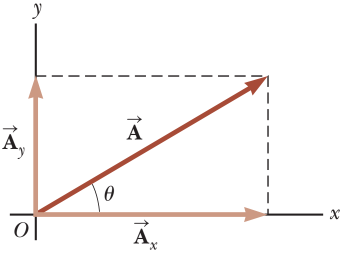

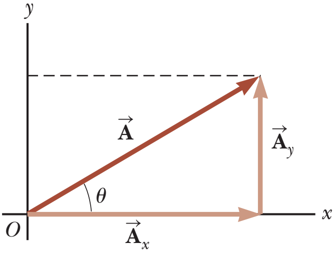

Notice that $A_x = A\cos\theta$ and $A_y = A\sin\theta$.

Components to Magnitude-and-Angle Notation

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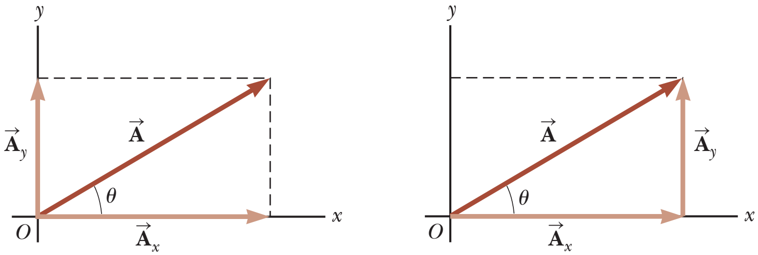

Also notice,

$$A = |\vec{\mathbf{A}}| = \sqrt{A_x^2 + A_y^2}$$

and

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

if the angle is given as shown.

Vectors Properties and Operations

Equality

Vectors $\vec{\mathbf{A}} = \vec{\mathbf{B}}$ if and only if the magnitudes and directions are the same. (Each component is the same.)

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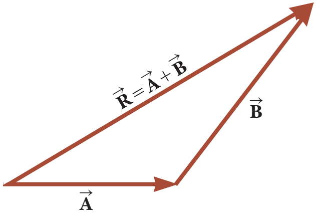
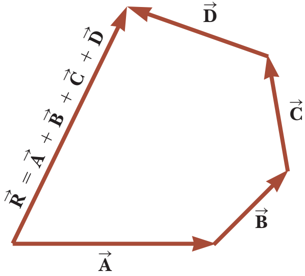
Addition

$\vec{\mathbf{A}} + \vec{\mathbf{B}}$





To calculate the addition of vectors, we usually break them into components.

Using Vectors: Example

(This is a simple example of vector addition where the vectors are already in components.)

Andy runs 100 m south then turns west and runs another 50.0 m. All this takes him 15.0 s.

What is his displacement from his starting point?

Using Vectors: Example

(This is a simple example of vector addition where the vectors are already in components.)

Andy runs 100 m south then turns west and runs another 50.0 m. All this takes him 15.0 s.

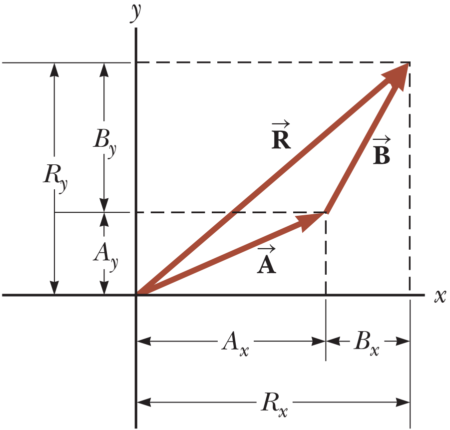
What is his displacement from his starting point?

answer: displacement: 112 m, in a direction 26.6° west of south.

Vectors Properties and Operations

Doing addition:

To add vectors, break each vector into components and sum each component independently.



Vector Addition Example

A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower. What is the magnitude and direction of the hiker's resultant displacement $\vec{\mathbf{R}}$ for the trip?

[0] Based on Serway & Jewett Example 3.5, pg 69.

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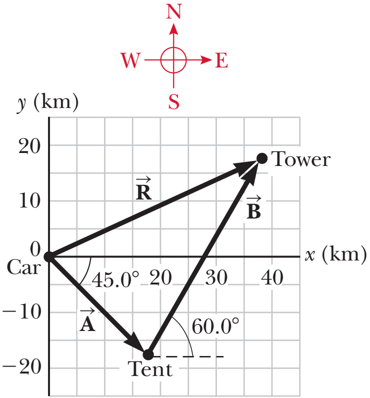

[0] Based on Serway & Jewett Example 3.5, pg 69.

Summary

- vectors in 2 dimensions
- trigonometry review
- vector addition

Next Quiz Thurs.

Homework

- **For next lecture, please bring a ruler, a protractor, and 2 sheets of graph paper.**

Walker Physics:

- start reading Chapter 3
- Ch 3, onward from page 76. Questions: 2, 4, 11. Problems: 5, 7, 11, 15