DeAnza College

# Introduction to Mechanics
# Vector Properties and Operations
# Vector Addition

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# Last time

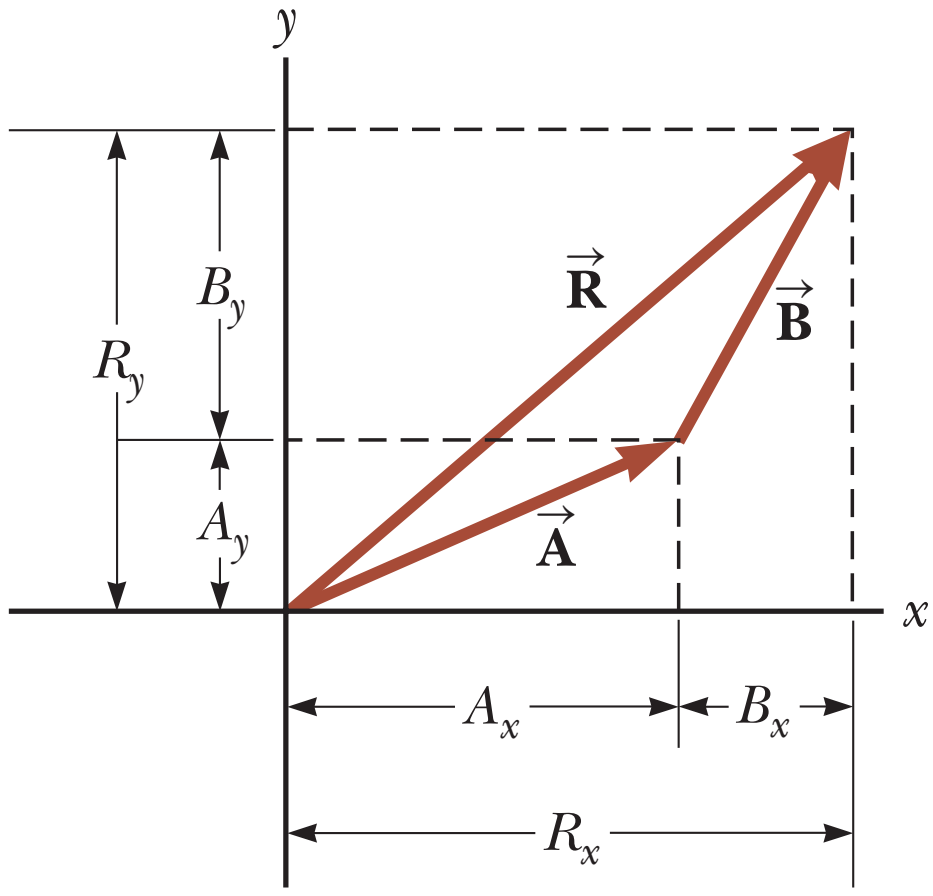
- expressing vectors
- trigonometry

# Overview

- some vector operations
- vector addition

# Vectors Properties and Operations: Addition

To add vectors, break each vector into components and sum each component independently.



# Vector Addition Example

A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower. What is the magnitude and direction of the hiker's resultant displacement $\vec{\mathbf{R}}$ for the trip?

[0]Based on S&J Example 3.5, pg 69.

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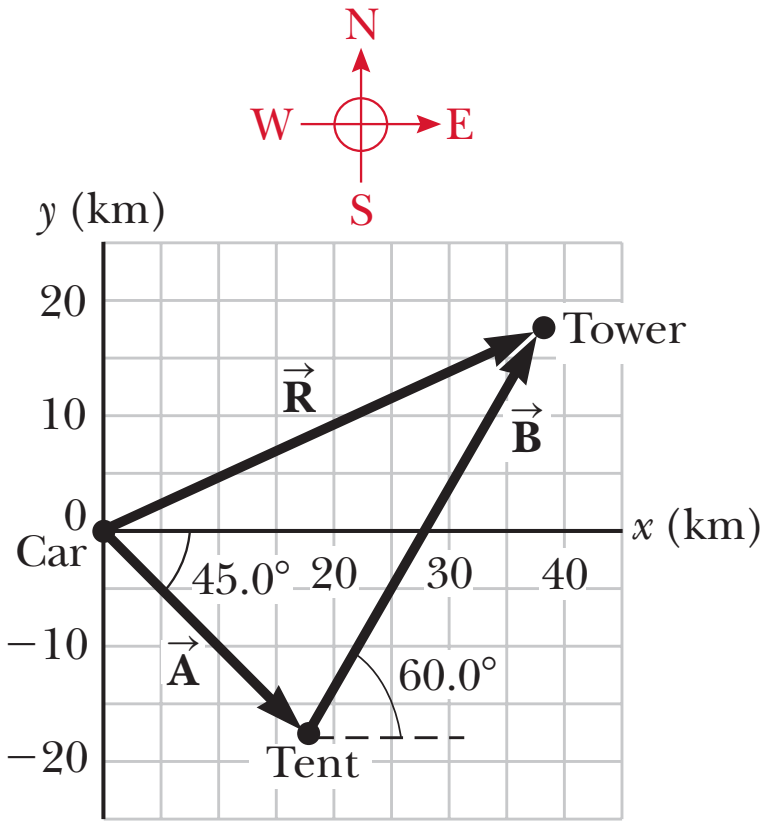


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$$\begin{aligned}
\vec{\mathbf{R}} &= (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} \\
&= (17.7 + 20)\hat{\mathbf{i}} + (-17.7 + 34.6)\hat{\mathbf{j}} \text{ km} \\
&= 37.7\hat{\mathbf{i}} + 17.0\hat{\mathbf{j}} \text{ km} \\
&= 41.3 \text{ km at } 24.2° \text{ north of east}
\end{aligned}$$

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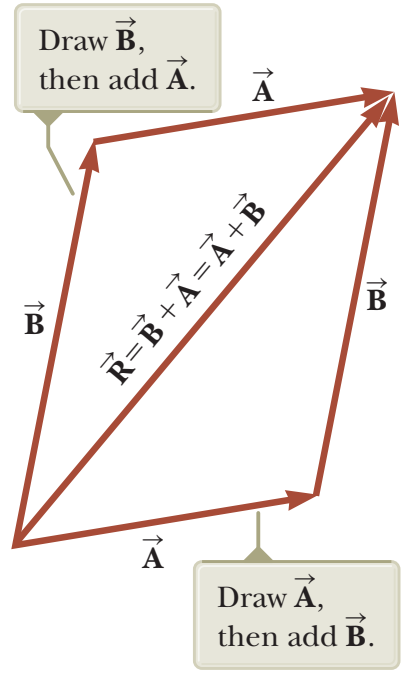
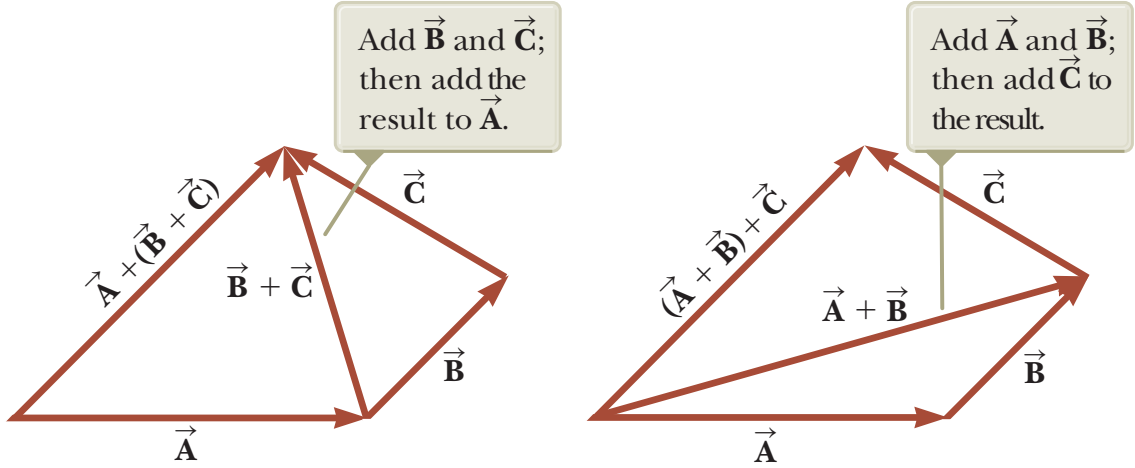
# Vectors Properties and Operations

## Properties of Addition

- $\vec{A} + \vec{B} = \vec{B} + \vec{A}$ (commutative)




- $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$ (associative)



# Thinking about Vectors

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What can you say about two vectors that add together to equal zero?

When can a nonzero vector have a zero horizontal component?

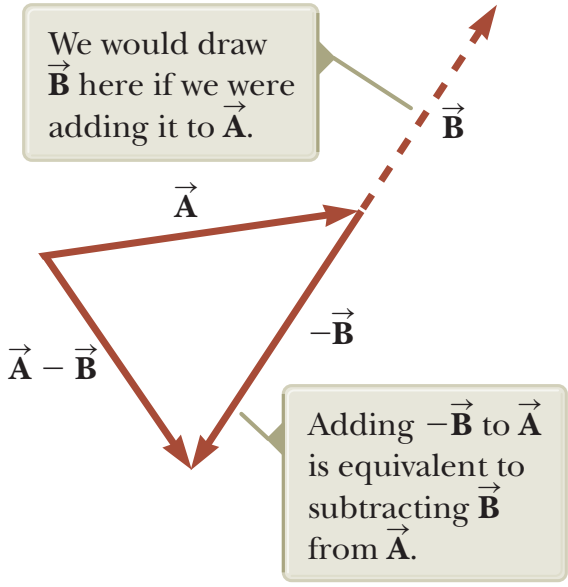
# Vectors Properties and Operations

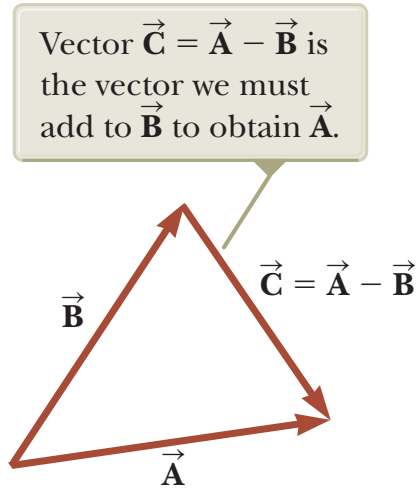
## Negation

If $\vec{\mathbf{u}} = -\vec{\mathbf{v}}$ then $\vec{\mathbf{u}}$ has the same magnitude as $\vec{\mathbf{v}}$ but points in the **opposite** direction.

## Subtraction

$$\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + (-\vec{\mathbf{B}})$$



# Vectors Properties and Operations

There are several different multiplicative operations on vectors.

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## Multiplication by a scalar

Suppose we want to multiply a scalar, like the number 5, by the vector:

$$\vec{\mathbf{v}} = 2\hat{\mathbf{i}} + 1\hat{\mathbf{j}}$$

The result is:

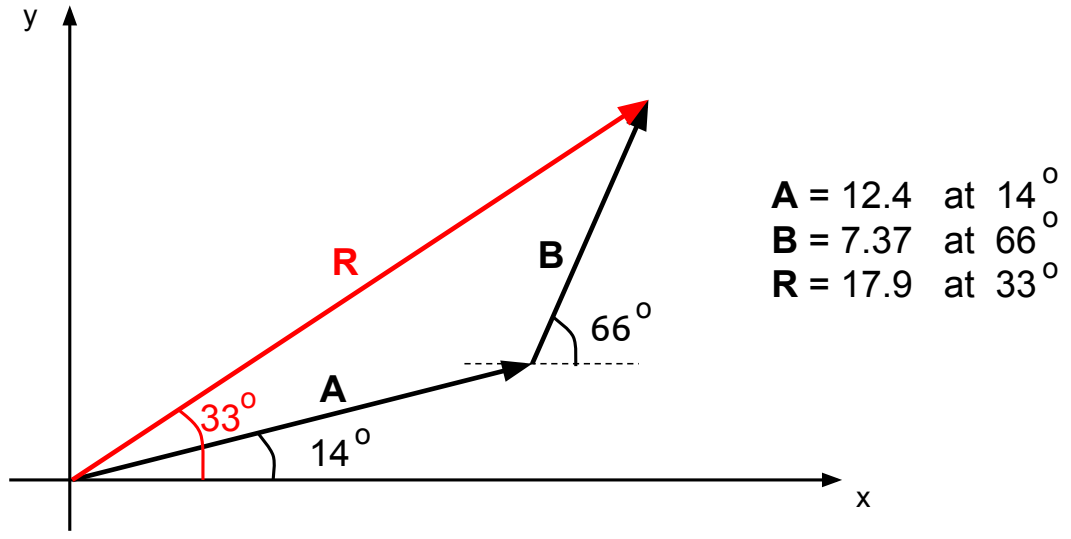
$$5\vec{\mathbf{v}} = (5 \times 2)\hat{\mathbf{i}} + (5 \times 1)\hat{\mathbf{j}} = 10\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$$

Each component is multiplied by the scalar. The direction of the vector doesn't change, but its magnitude increases by a factor of 5.

# Adding Vectors Graphically

We will draw the vectors to scale on graph paper.

1. Pick a scale so the vectors fit on the paper (*eg.* 1 cm = 2 km).
2. Draw axes.
3. Starting at the origin, use the protractor to find the angle of the first vector ($\vec{\mathbf{A}}$) from the $x$-direction, then using the ruler, draw its length to scale and in the proper direction.
4. From the end of the first vector, draw the second vector ($\vec{\mathbf{B}}$) to the same scale and in the proper direction. The angle of $\vec{\mathbf{B}}$ is measured from the $x$-direction.




**A** = 12.4 at 14°
**B** = 7.37 at 66°
**R** = 17.9 at 33°

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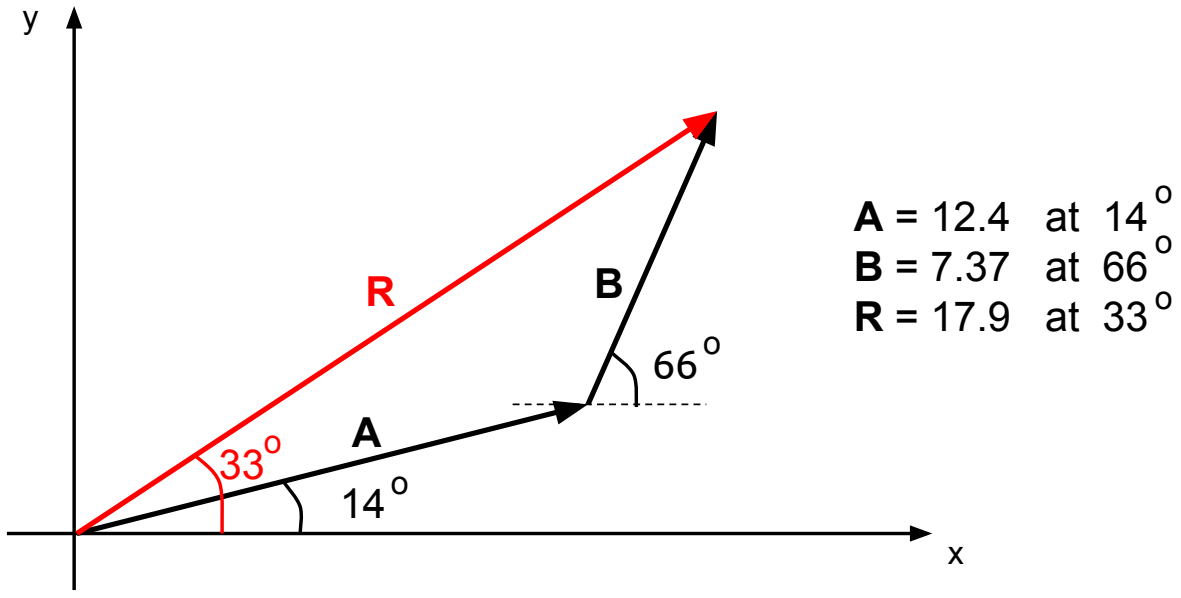


5. The resultant vector $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ is the vector drawn from the tail of vector $\vec{\mathbf{A}}$ to the tip of vector $\vec{\mathbf{B}}$.
6. Measure the **length** of the vector $\vec{\mathbf{R}}$ on your graph paper **with your ruler**. Find the magnitude of the resultant vector $\vec{\mathbf{R}}$ from your chosen scale. Measure its **direction** (relative to the $x$-direction) **with a protractor**.

# Vector Assignment

For this problem, first add the vectors **graphically** (pencil and graph paper, using your ruler and protractor) to find the magnitude and direction of the resultant vector, $\vec{\mathbf{R}}$.

Then **calculate** the magnitude and direction of the resultant vector by the finding and adding the components of each vector. Check that your answers agree.

**A car travels 20.0 km at 60.0° north of west, then 35.0 km at 45.0° north of east. Find the resultant displacement of the car.**

# Summary

- vector operations
- vector addition

**Quiz** Thursday.

## Homework

- finish off the Vector Assignment, to turn in Thursday

Walker Physics:

- Ch 3, onward from page 76. Questions: 7, 8, 9. Problems: 1, 17, 25, 77