Introduction to Mechanics
Motion in 2 Dimensions
Relative Motion

Lana Sheridan

De Anza College

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Last time

- some vector operations
- vector addition

Overview

- introduction to motion in 2 dimensions
- constant velocity in 2 dimensions
- relative motion

Quick review of Vector Expressions

Let $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ be vectors. Let n be a scalar.

Could this possibly be a valid equation?

$$\vec{\mathbf{a}} = \vec{\mathbf{b}}$$

(A) yes

(B) no

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Motion in 2 Dimensions

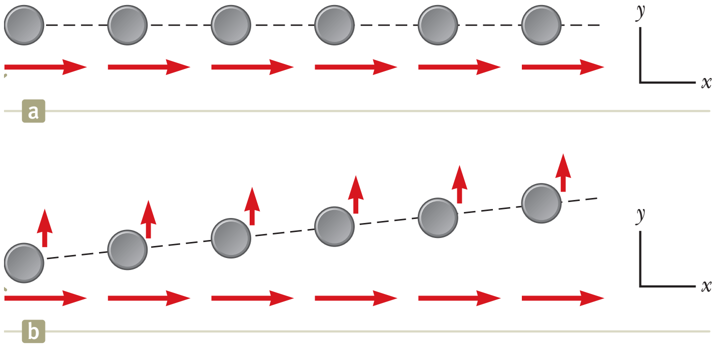
So far we have looked at motion in 1 dimension only, motion along a straight line.

However, motion on a plane (2 dimensions), or through space (3 dimensions) obeys the same equations.

We will now focus on 2 dimensional motion.

Motion in 2 directions

Imagine an air hockey puck moving with horizontally constant velocity:



If it experiences a momentary upward (in the diagram) acceleration, it will have a component of velocity upwards. The horizontal motion remains unchanged!

[1] Figure from Serway & Jewett, 9th ed.

Direction and Motion

When we say something is moving, we mean that it is moving **relative** to something else.

In order to describe measurements of

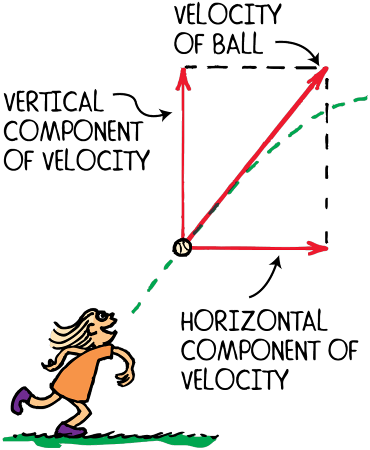
- where something is
- how fast it is moving

we must have **reference frames**.

In 2 dimensions we need to choose a pair of perpendicular directions to be our x and y axes.

Motion in 2 directions: Components of velocity

Motion in perpendicular directions can be analyzed separately.



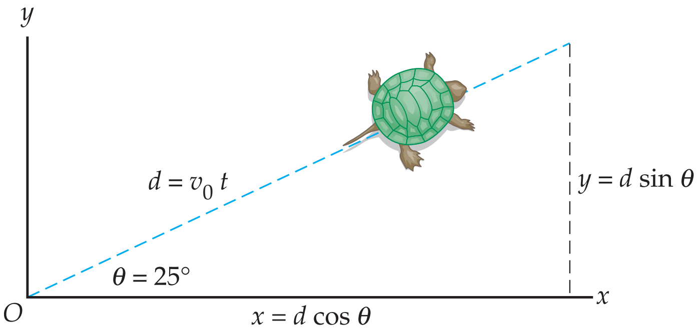
A vertical force (gravity) does not affect horizontal motion.

The horizontal component of velocity is constant.

[1] Drawing by Paul Hewitt, via Pearson.

Constant Velocity in 2 Dimensions

Consider a turtle that moves with a constant velocity.



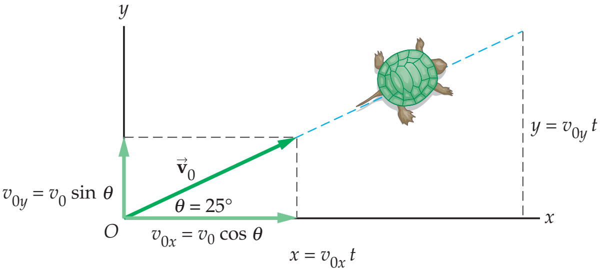
We can find the distance it travels by using the equation $d = v_0 t$.

How far it travels in the x-direction: $x = d\cos\theta$.

And in the y-direction: $y = d\sin\theta$.

[1]Figure from Walker, "Physics".

Constant Velocity in 2 Dimensions



Or, we can find the distance it travels in the x-direction by considering what is its rate of change of x-position with time!

$$v_{0x} = \frac{\Delta x}{\Delta t} = v_0 \cos\theta \quad \Rightarrow \quad x = v_{0x} t = (v_0 \cos\theta) t$$

And in the y-direction:

$$v_{0y} = \frac{\Delta y}{\Delta t} = v_0 \sin\theta \quad \Rightarrow \quad y = v_{0y} t = (v_0 \sin\theta) t$$

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To indicate which way a vector (a force, acceleration, *etc.*) points, we need to have another direction that we can compare to.

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For example, driving, you can say the direction you are driving relative to cardinal directions, North, South, East, West.

North-South and West-East can be **reference axes.**

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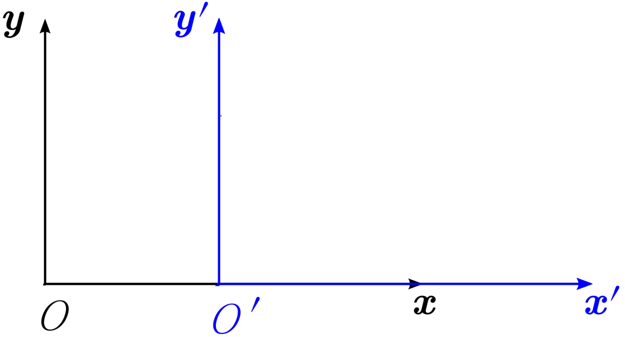
North-South and West-East can be **reference axes.**

We could also choose axes "up" and "down", and parallel to the horizon.

Reference Frames

We could agree to choose directions as North (y) and East (x). However, two different people might pick different origins, O and O', for their axes.

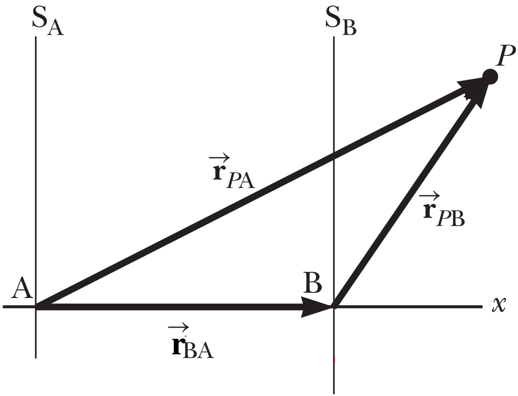


In this case, each person would describe the location of a particle slightly differently. Can we relate those descriptions?

[1] Image modified from work of Wikipedia user Krea.

Reference Frames

We can relate the descriptions using vector addition!



$$\vec{\mathbf{r}}_{PA} = \vec{\mathbf{r}}_{PB} + \vec{\mathbf{r}}_{BA}$$

$\vec{\mathbf{r}}_{PA}$ is the position of particle P *relative to* frame A.

Relative Motion

We can use the notion of motion in 2 dimensions to consider how one object moves **relative** to something else.

All motion is relative.

Our **reference frame** tells us what is a fixed position.

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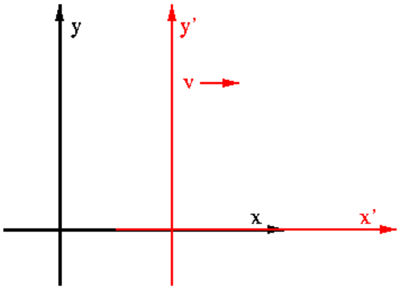
An example of a reference might be picking an object, declaring that it is at rest, and describing the motion of all objects relative to that.

Two different people could pick different reference objects and end up with two reference frames *moving* relative to each other.

Relative Motion

When comparing two frames (A and B) moving relative to each other with constant velocity:



$$\vec{\mathbf{v}}_{PA} = \vec{\mathbf{v}}_{PB} + \vec{\mathbf{v}}_{BA}$$

where $\vec{\mathbf{v}}_{BA}$ is the constant velocity of frame B relative to frame A.

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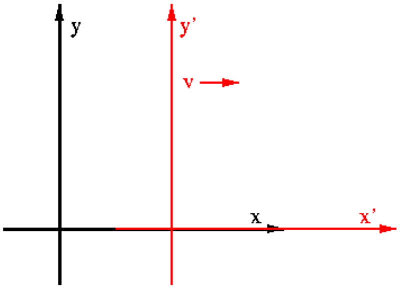

Imagine

$$\vec{\mathbf{v}}_{PA} = \vec{\mathbf{v}}_{P\cancel{B}} + \vec{\mathbf{v}}_{\cancel{B}A}$$

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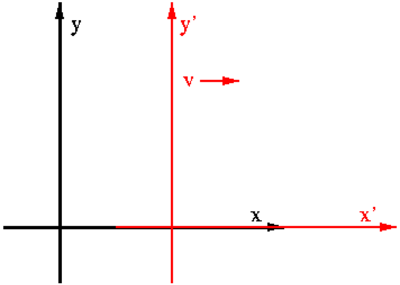

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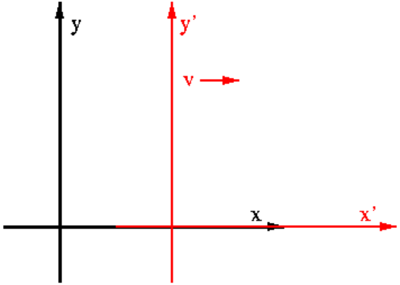

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Comparing two frames A and B, if

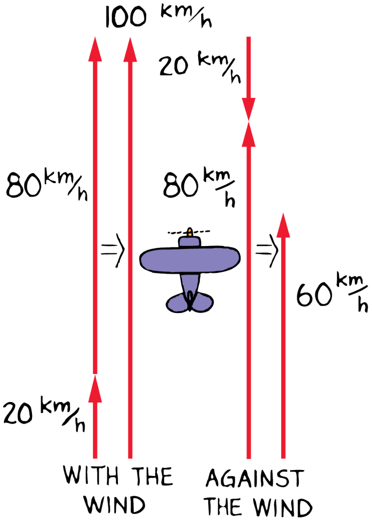
$\vec{\mathbf{v}}_{BA}$ is the velocity of frame B relative to frame A, then

$\vec{\mathbf{v}}_{AB}$ is the velocity of frame A relative to frame B.

$$\vec{\mathbf{v}}_{AB} = -\vec{\mathbf{v}}_{BA}$$

Swapping the subscripts gives a sign flip.

Intuitive Example for Relative Velocities



[1]Figure by Paul Hewitt.

Intuitive Example

Now, imagine an airplane that is flying North at 80 km/h but is blown off course by a cross wind going East at 60 km/h.

How fast is the airplane moving relative to the ground? In which direction?

Sketch:

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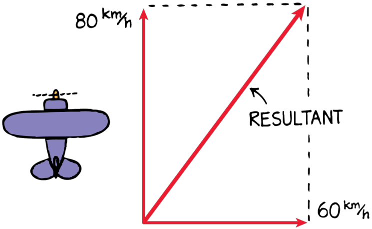

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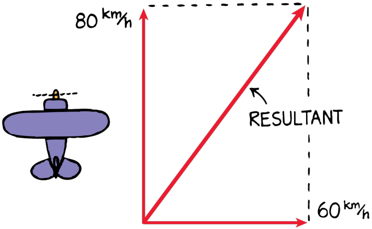

Hypothesis: It will travel to the North-East, at a speed greater than 80 km/h, but less than 80+60 = 140 km/h.

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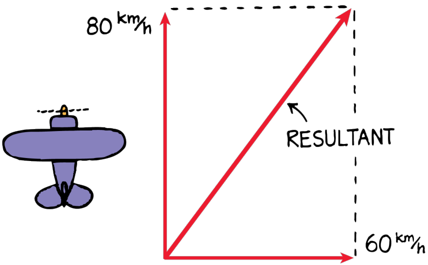

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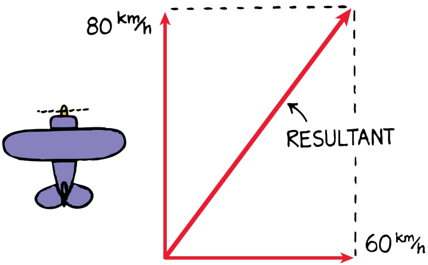

Strategy: vector addition! $\vec{\mathbf{v}}_{pg} = \vec{\mathbf{v}}_{pa} + \vec{\mathbf{v}}_{ag}$

(p - plane, g - ground, a - air, so $\vec{\mathbf{v}}_{ag}$ is the wind velocity)

In this case, the two vectors are at right-angles. We can use the Pythagorean theorem.

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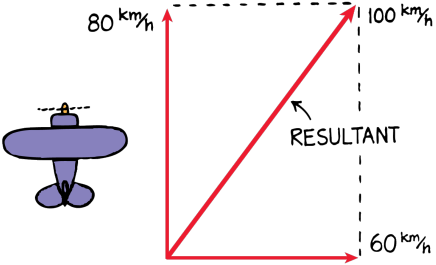

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$\vec{\mathbf{v}} = 100$ km/h at 36.9° East of North (or 53.1° North of East)

Relative Motion Example

A boat crossing a wide river moves with a speed of 10.0 km/h relative to the water. The water in the river has a uniform speed of 5.00 km/h due east relative to the Earth. If the boat heads due north, determine the velocity of the boat relative to an observer standing on either bank.[1]

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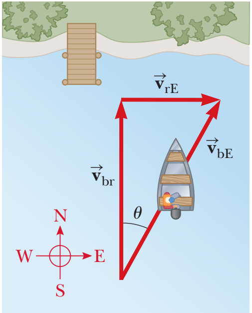

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[2]Page 97, Serway & Jewett

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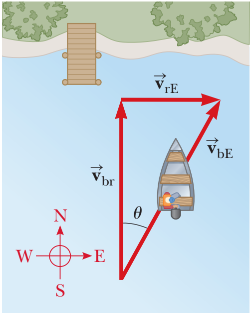

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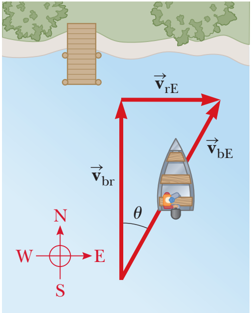

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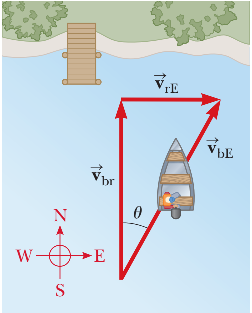

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$$\vec{\mathbf{v}}_{bE} = \vec{\mathbf{v}}_{br} + \vec{\mathbf{v}}_{rE}$$

Simply use vector addition to find $\vec{\mathbf{v}}_{bE}$.

$$\begin{aligned} v_{bE} &= \sqrt{10^2 + 5^2} \\ &= 11.2 \text{ km/h} \end{aligned}$$

$$\theta = \tan^{-1}\left(\frac{5}{10}\right) = 26.6^\circ$$

[2]Page 97, Serway & Jewett

Summary

- motion in 2-dimensions
- motion with constant velocity
- relative motion

Quiz Thursday. (Will NOT be on relative motion.)

Homework

- finish off the Vector Assignment, due tomorrow

Walker Physics:

- Ch 3, onward from page 76. Questions: 7, 8, 9. Problems: 1, 17, 25, 77 (set yesterday)