Introduction to Mechanics
Projectiles
Launched Horizontally
Launched at an Angle

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Last time

- relative motion problem
- motion in 2D with constant acceleration
- projectile motion
- projectiles launched horizontally

Overview

- projectiles launched horizontally
- projectiles launched at an angle

Another Horizontal Launch Example

A steel ball is fired horizontally at 8.0 m/s from from a 1.0 m-high table top.

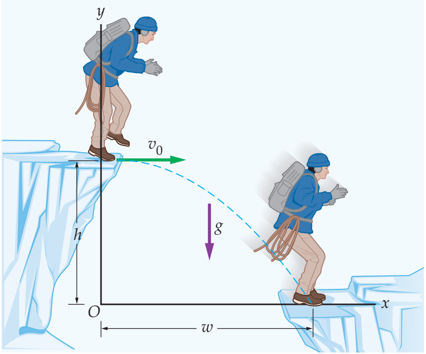
Show that a 20 cm tall coffee can placed on the floor 3.2 m from the base of the table will catch the ball.

[1]See Hewitt "Conceptual Physics", page 192.

Question

A mountain climber leaps horizontally across a crevasse of width w. The opposite side of the crevasse is lower by a distance h. Imagine that the climber jumps with the minimum speed necessary to reach the far side.

If the height h is **increased**, but the width w remains the same, does the minimum speed needed to cross the crevasse



(A) increase,

(B) decrease, or

(C) stay the same?

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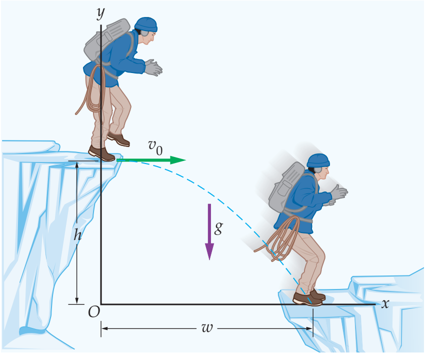

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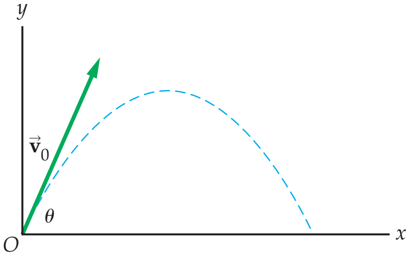
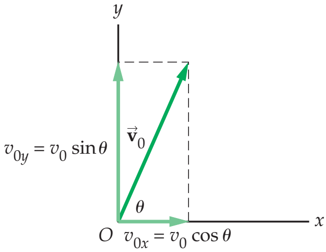
Principle Equations of Projectile Motion

(Notice, these are just special cases of the kinematics equations!)

$$\boxed{\begin{aligned} \Delta x &= v_{0x} t \\ \Delta y &= v_{0y} t - \frac{1}{2} g t^2 \end{aligned}} \quad \boxed{\begin{aligned} v_x &= v_{0x} \\ v_y &= v_{0y} - g t \end{aligned}} \quad \boxed{\begin{aligned} v_x^2 &= v_{0x}^2 \\ v_y^2 &= v_{0y}^2 - 2g(\Delta y) \end{aligned}}$$

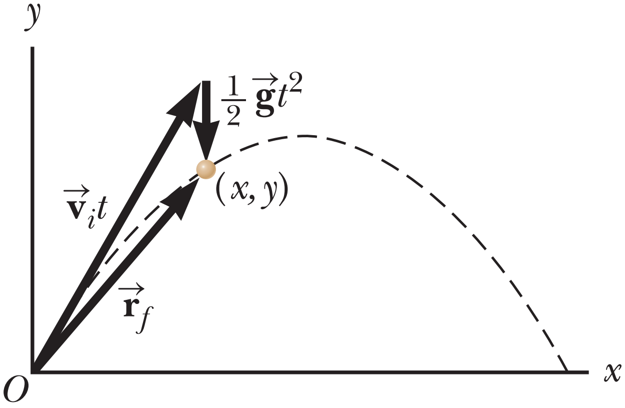
Projectiles Launched at an Angle





$$v_{0x} = v_0 \cos\theta \qquad v_{0y} = v_0 \sin\theta$$

Projectile's Trajectory



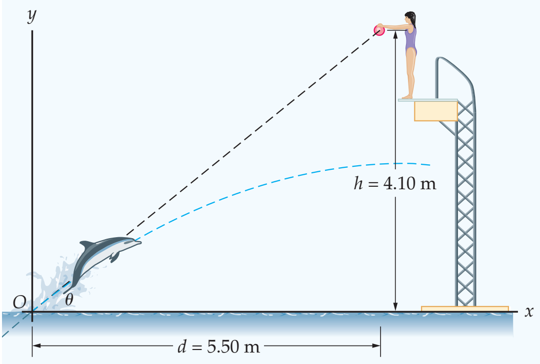
The object would move in a straight line, but the force of gravity causes it to fall as it moves to the right.

$$\Delta \boldsymbol{r} = \mathbf{r}_f - 0 = \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2$$

[1]Figure from Serway & Jewett, 9th ed.

Jumping Dolphin, Ex 4-6, pg 94

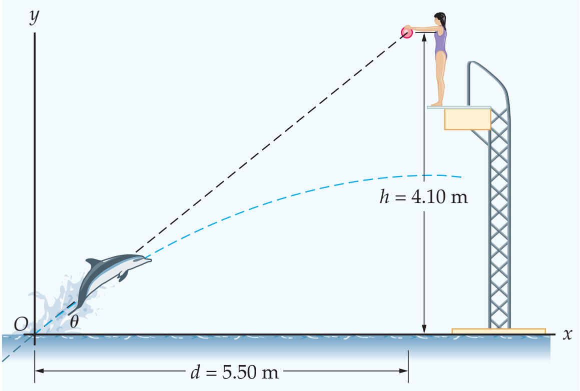
A trained dolphin leaps from the water with an initial speed of 12.0 m/s. It jumps directly toward a ball held by the trainer a horizontal distance of 5.50 m away and a vertical distance of 4.10 m above the water.



If the trainer releases the ball the instant the dolphin leaves the water, show that the dolphin and the falling ball meet.

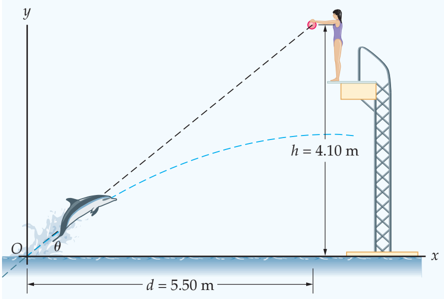
Jumping Dolphin, Ex 4-6, pg 94

Show that the dolphin and the falling ball meet.



Strategy: We need to show that when the dolphin's x-coordinate matches the ball's x-coordinate, their y-coordinates also match.

Jumping Dolphin, Ex 4-6, pg 94



Make an expression for how long it takes the dolphin to cover the horizontal distance to the ball.

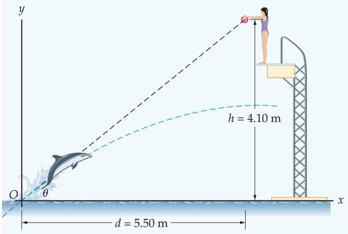
$$d = \Delta x = v_{0,x} t$$

Rearrange for t:

$$t = \frac{d}{v_0 \cos \theta}$$

When t takes this value, the x-coordinates match. Do the y-coordinates also match?

Jumping Dolphin, Ex 4-6, pg 94



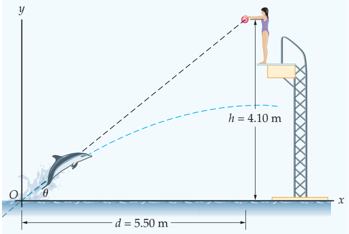
How far does the ball fall in this time?

$$\Delta y = y_b - h = -\frac{1}{2}gt^2$$

So the ball's y-coordinate is:

$$y_b \quad = \quad h - \frac{1}{2}gt^2$$

Jumping Dolphin, Ex 4-6, pg 94



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So the ball's y-coordinate is:

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What is the dolphin's y coordinate?

$$\Delta y = y_d - 0 = v_{0y}t - \frac{1}{2}gt^2$$

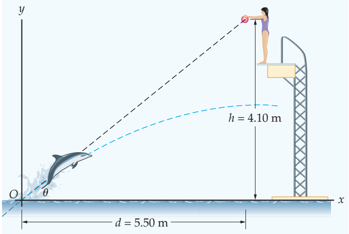
The dolphin's y-coordinate is:

$$y_d = (v_0 \sin\theta)t - \frac{1}{2}gt^2$$

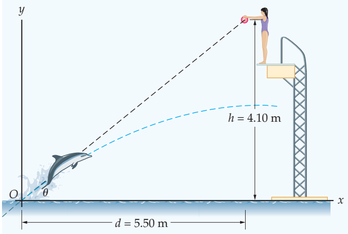
Jumping Dolphin, Ex 4-6, pg 94

$$y_b \stackrel{?}{=} y_d$$

$$h - \frac{1}{2}gt^2 \stackrel{?}{=} (v_0 \sin\theta)t - \frac{1}{2}gt^2$$



Jumping Dolphin, Ex 4-6, pg 94

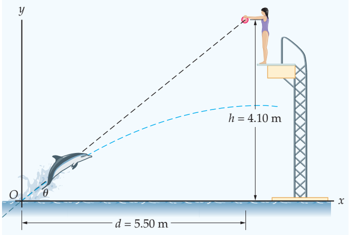


$$y_b \stackrel{?}{=} y_d$$

$$h - \cancel{\frac{1}{2}gt^2} \stackrel{?}{=} (v_0 \sin\theta)t - \cancel{\frac{1}{2}gt^2}$$

$$h \stackrel{?}{=} (v_0 \sin\theta)\left(\frac{d}{v_0 \cos\theta}\right)$$

Jumping Dolphin, Ex 4-6, pg 94



$$
\begin{aligned}
y_b &\stackrel{?}{=} y_d \\
h - \cancel{\frac{1}{2}gt^2} &\stackrel{?}{=} (v_0 \sin\theta)t - \cancel{\frac{1}{2}gt^2} \\
h &\stackrel{?}{=} (v_0 \sin\theta)\left(\frac{d}{v_0 \cos\theta}\right) \\
h &\stackrel{?}{=} d \tan\theta \\
h &= h \quad \checkmark
\end{aligned}
$$

So,

$$y_b = y_d \quad \text{when} \quad x_b = x_d$$

Yes, the dolphin will be able to catch the ball.

Summary

- projectiles launched at an angle

Quiz Thursday

Homework

- relative motion worksheet (answer on scantron and hand in, due Thurs)

Walker Physics:

- Ch 4, onward from page 100. Problems: 31, 33