Introduction to Mechanics
Projectiles
Time of Flight
Range of a Projectile

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De Anza College

Feb 18, 2020

Public Service Announcement: Vote!

https://registertovote.ca.gov/

Today is the deadline to register to vote in this election.

However, if you miss the deadline, you can still vote "conditionally".

Election day: Tuesday, March 3. ("Super Tuesday")

Last time

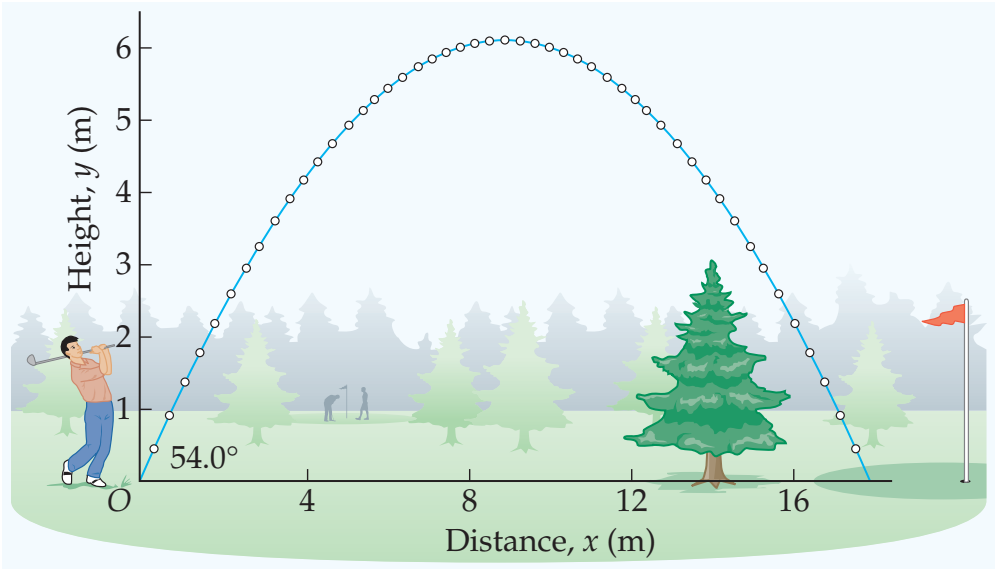
- max height of a projectile

Overview

- time-of-flight of a projectile
- range of a projectile
- trajectory of a projectile

Not Using the Max Height Equation

"How high was the ball when it passed over the tree?" Suppose $v_0 = 13.5$ m/s, $\theta = 54.0°$ and tree is 14.0 m from golfer. How can we find the answer?

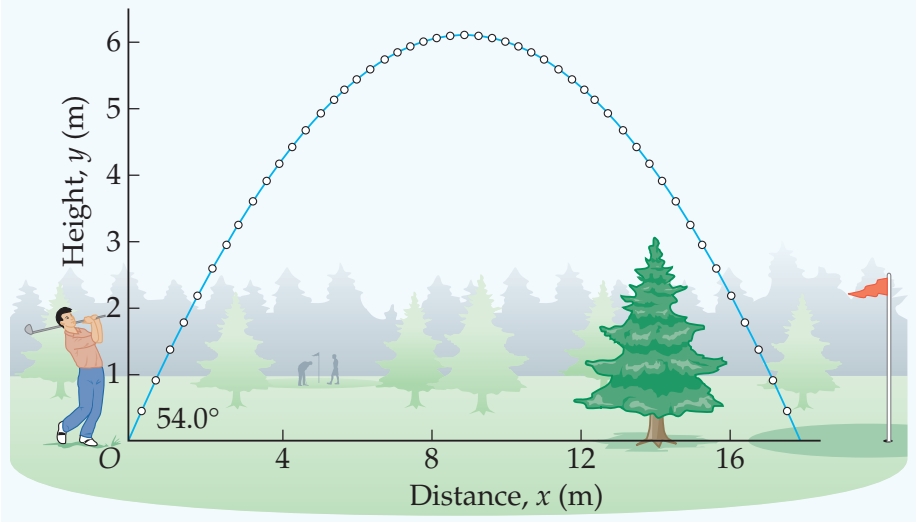


Go back to the kinematics expressions!

$$\Delta y = v_{0y} t - \frac{1}{2} g t^2$$

We can find the height if we know the time the ball was over the tree.

Not Using the Max Height Equation



$$\Delta x = v_{0x} t$$

Rearranging, ball was over the tree when

$$\begin{aligned} t &= \frac{\Delta x}{v_0 \cos\theta} \\ &= \frac{(14\ \text{m})}{(13.5\ \text{m/s})\cos(54.0^\circ)} \\ &= 1.764\ \text{s} \end{aligned}$$

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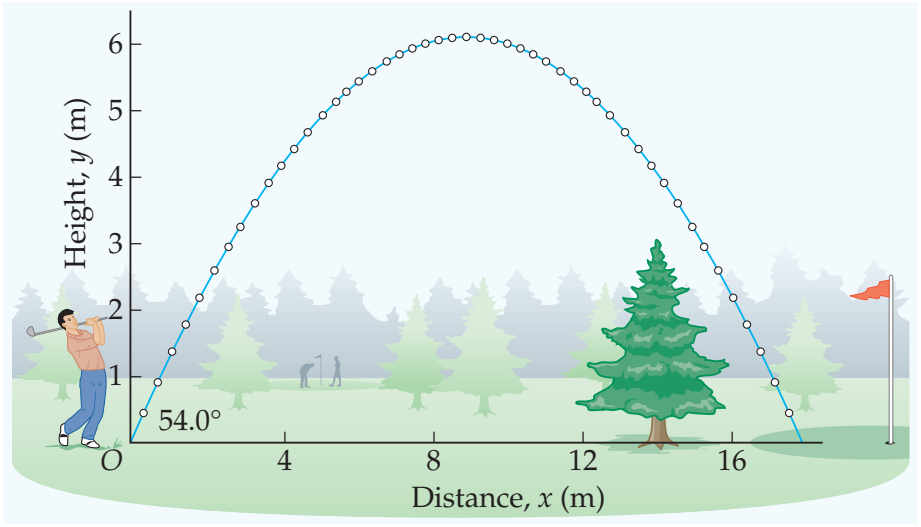

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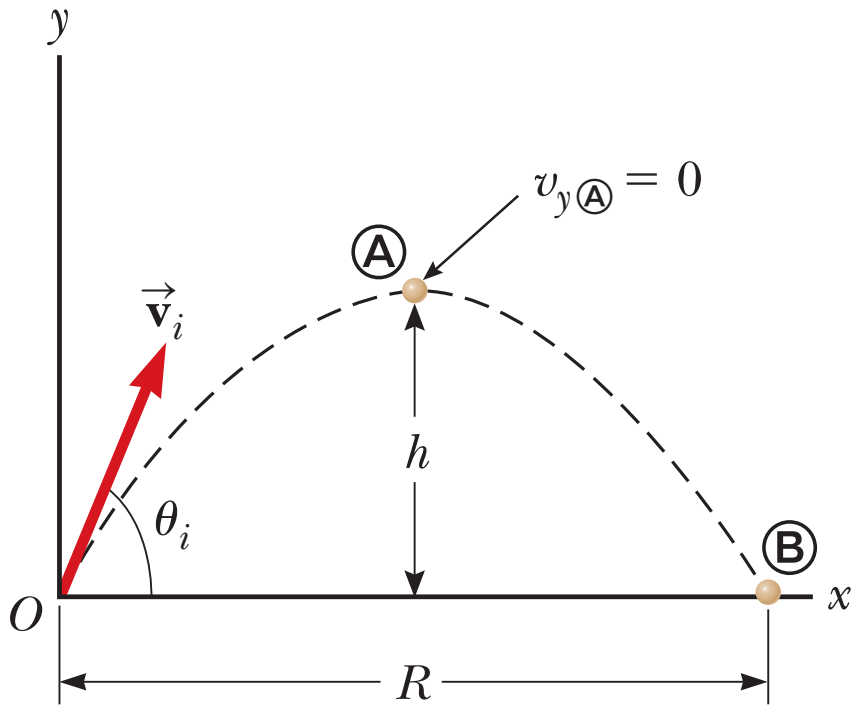
$$\begin{aligned} \Delta y &= v_0 \sin\theta t - \frac{1}{2} g t^2 \\ &= (13.5\text{m/s})\sin(54.0^\circ)(1.764\ \text{s}) \\ &\quad -\frac{1}{2}(9.81\ \text{m/s}^2)(1.764\ \text{s})^2 \\ &= \underline{4.00\ \text{m}} \end{aligned}$$

Time of Flight of a Projectile

time of flight

The time from launch to when projectile hits the ground.

How can we find the time of flight of a projectile?



Assuming that it lands at the same height it is launched from.

Time of Flight of a Projectile

Notice that just when striking the ground, $\Delta y = 0$.

$$\begin{aligned} \Delta y &= v_{0y} t + \frac{1}{2} a_y t^2 \\ 0 &= v_0 \sin\theta t - \frac{1}{2} g t^2 \end{aligned}$$

Now cancel a factor of t. Warning! This will remove one solution to this equation in t. What is it?

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$$\frac{1}{2} g t = v_0 \sin\theta$$

$$t_{\text{flight}} = \frac{2 v_0 \sin\theta}{g}$$

Time of Flight Example, #32

A soccer ball is kicked with a speed of 9.50 m/s at an angle of 25.0° above the horizontal. If the ball lands at the same level from which it was kicked, how long was it in the air?

[1] Walker, "Physics", page 106.

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Hypothesis: about 2 seconds.

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$$t_{\text{flight}} = \frac{2v_0 \sin\theta}{g}$$

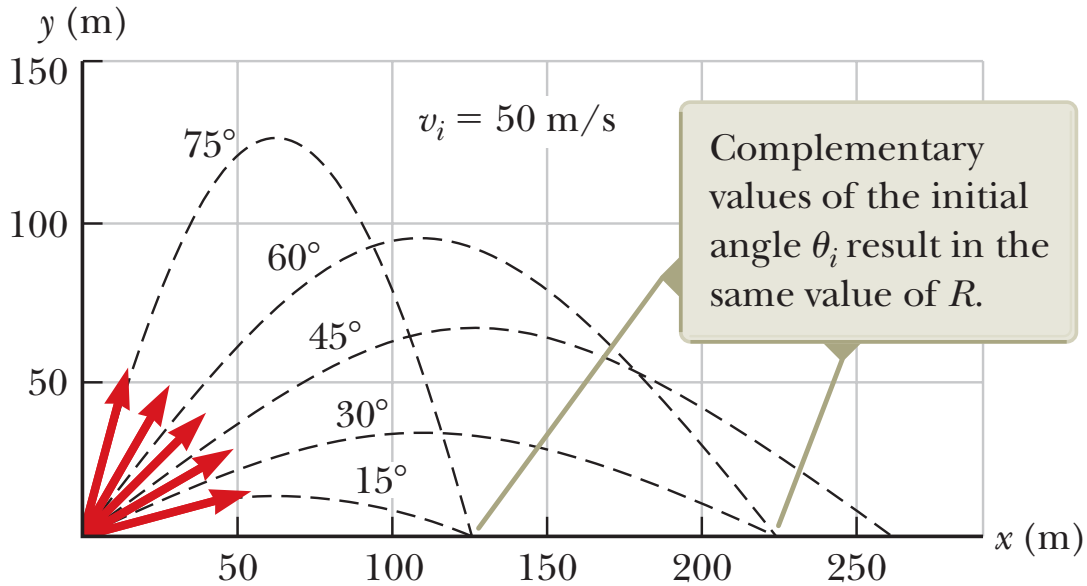
("ball lands at the same level from which it was kicked")

$$\begin{aligned} t_{\text{flight}} &= \frac{2(9.50 \text{ m/s})\sin(25.0^\circ)}{9.8 \text{ m/s}^2} \\ &= \underline{0.819 \text{ s}} \end{aligned}$$

Reasonable?: Less than half of my guess, but the angle it is kicked at is quite shallow, so the answer makes sense.

Time of Flight of a Projectile

Quick Quiz 4.3[1] Rank the launch angles for the five paths in the figure with respect to time of flight from the shortest time of flight to the longest. (Assume the magnitude v_i remains the same.)



A 15°, 30°, 45°, 60°, 75°

B 45°, 30°, 60°, 15°, 75°

C 15°, 75°, 30°, 60°, 45°

D 75°, 60°, 45°, 30°, 15°

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[1] Page 86, Serway & Jewett

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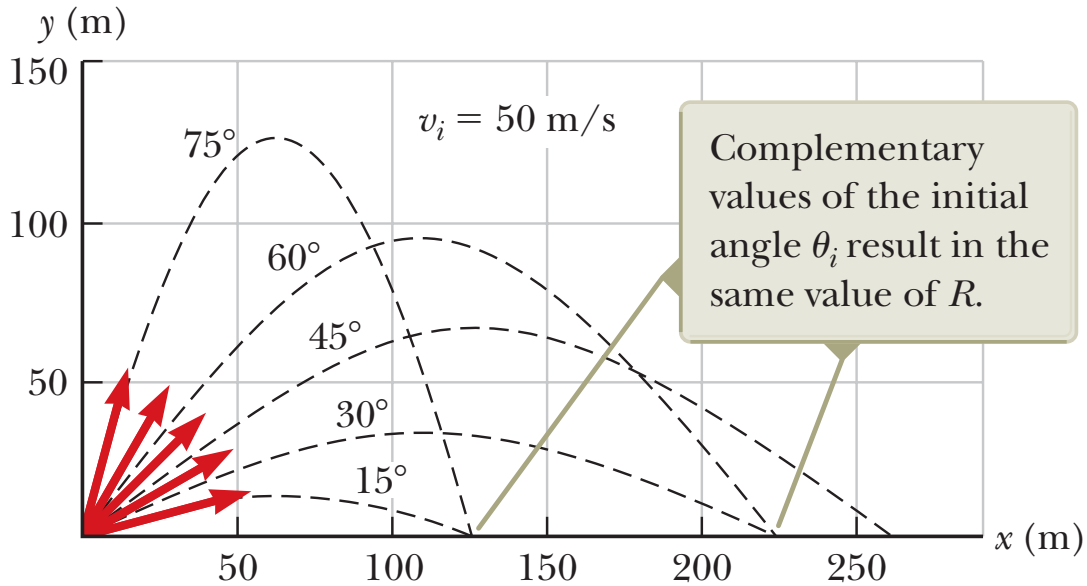

A 15°, 30°, 45°, 60°, 75° ←

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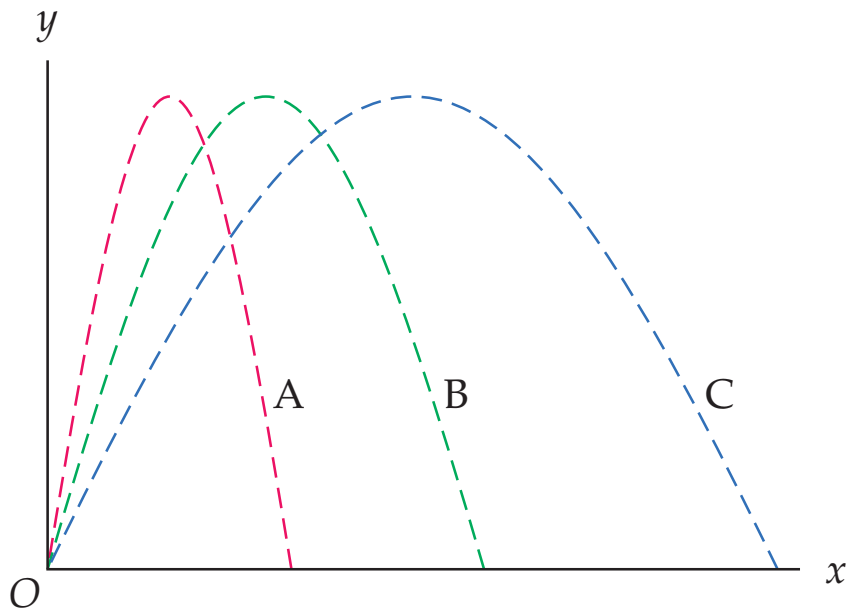
Height and initial speed conceptual question

Three projectiles (A, B, and C) are launched with different initial speeds so that they reach the same maximum height, as shown. List the projectiles in order of increasing time of flight.



(A) A, B, C

(B) C, B, A

(C) B, C, A

(D) all the same

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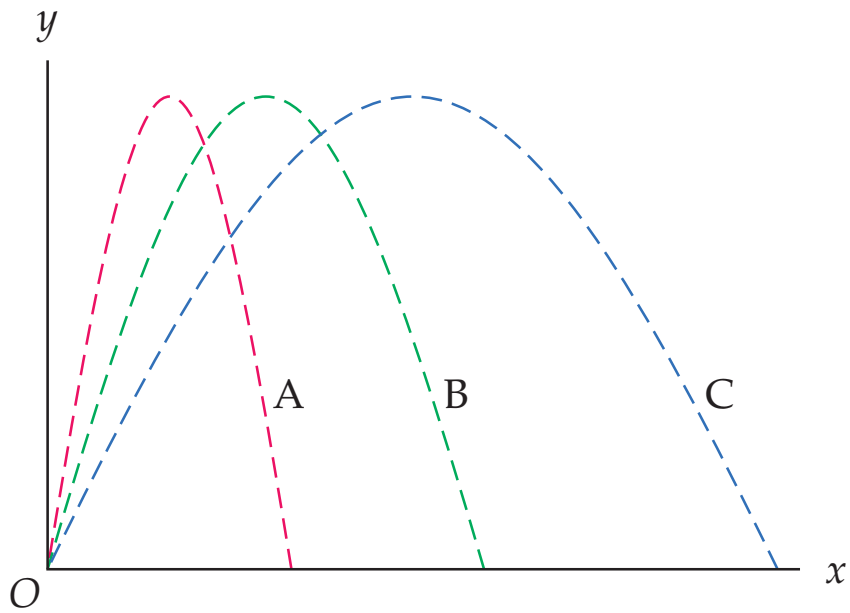

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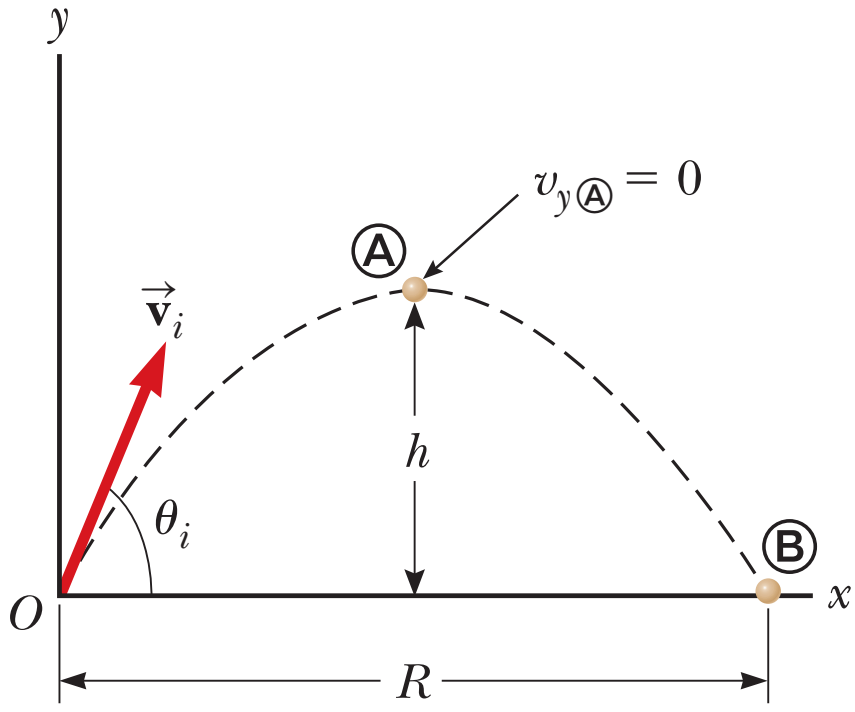
Range of a Projectile

range

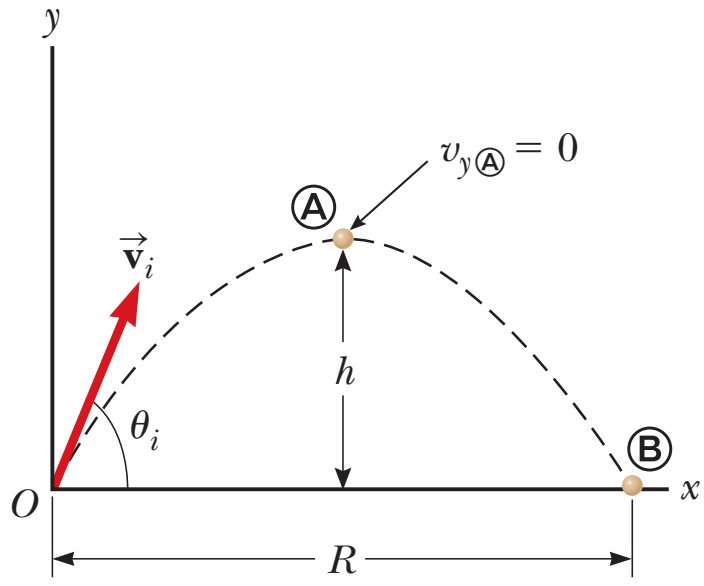
The distance in the horizontal direction that a projectile covers before hitting the ground.

How can we find the range of a projectile?



Assuming that it lands at the same height it is launched from.

Range of a Projectile

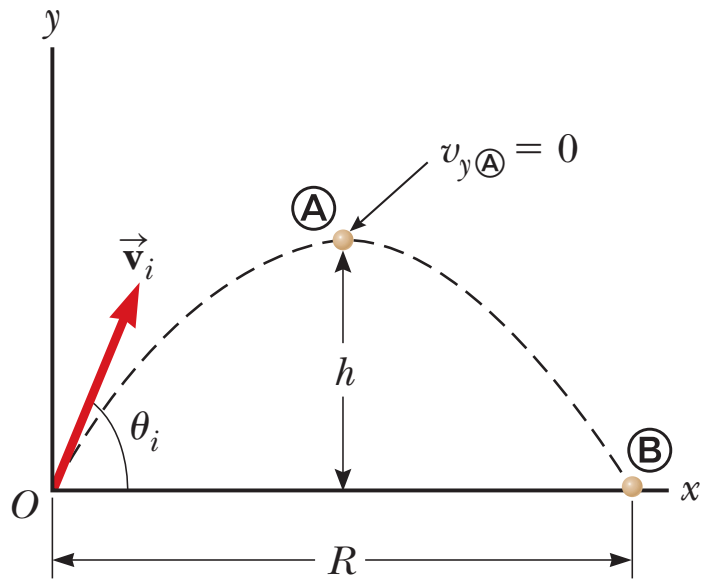


There is no acceleration in the x-direction! $(a_x = 0)$

$$\Delta x = v_x t$$

We just need t. But t is the **time of flight**!

Range of a Projectile



$$\Delta x = v_x t$$

Assuming lands at same as is launched.

$$R = v_i \cos\theta \left(\frac{2v_0 \sin\theta}{g} \right)$$

$$R = \frac{2v_0^2 \sin\theta \cos\theta}{g}$$

$$R = \frac{v_0^2 \sin(2\theta)}{g}$$

Range of a Projectile

A long jumper leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s. How far does he jump in the horizontal direction?

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Draw a Sketch.
Hypothesis: he'll be in the air for less than a second. Less than 11 m, more than 2 m.

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$$\begin{aligned} R &= \frac{v_0^2 \sin(2\theta)}{g} \\ &= \frac{(11.0 \text{ m/s})^2 \sin(2 \times 20.0^\circ)}{9.81 \text{ m/s}^2} \\ &= \underline{7.93 \text{ m}} \end{aligned}$$

Reasonable?: Yes, the answer is in the range I predicted.

Maximizing Range

$$R = \frac{v_0^2 \sin(2\theta)}{g}$$

What angle maximizes the range of the projectile?

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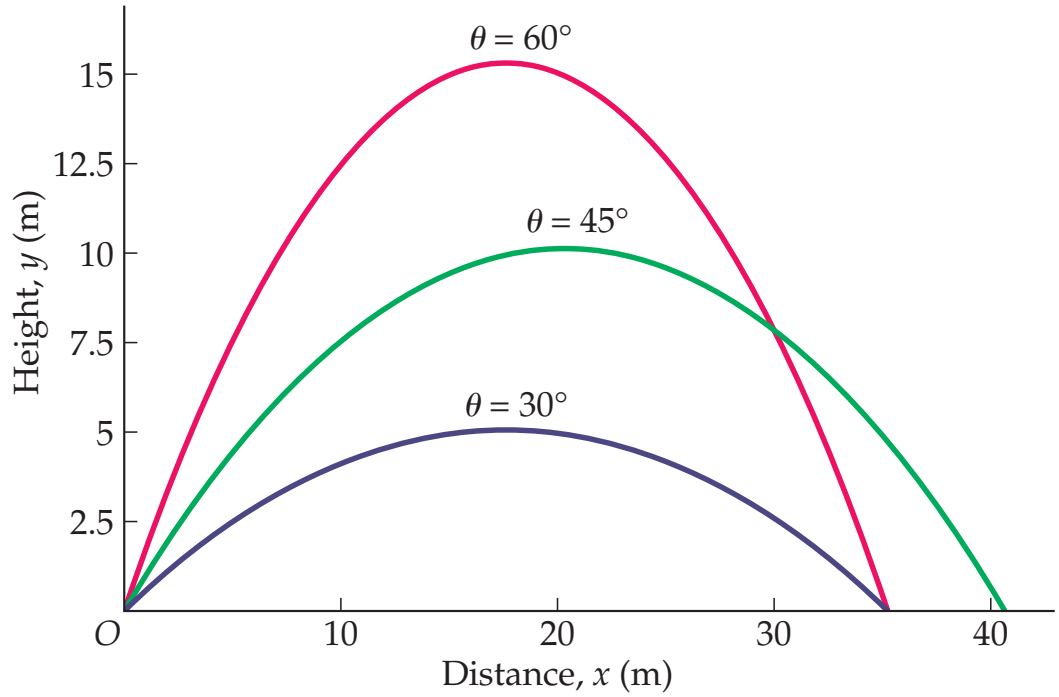
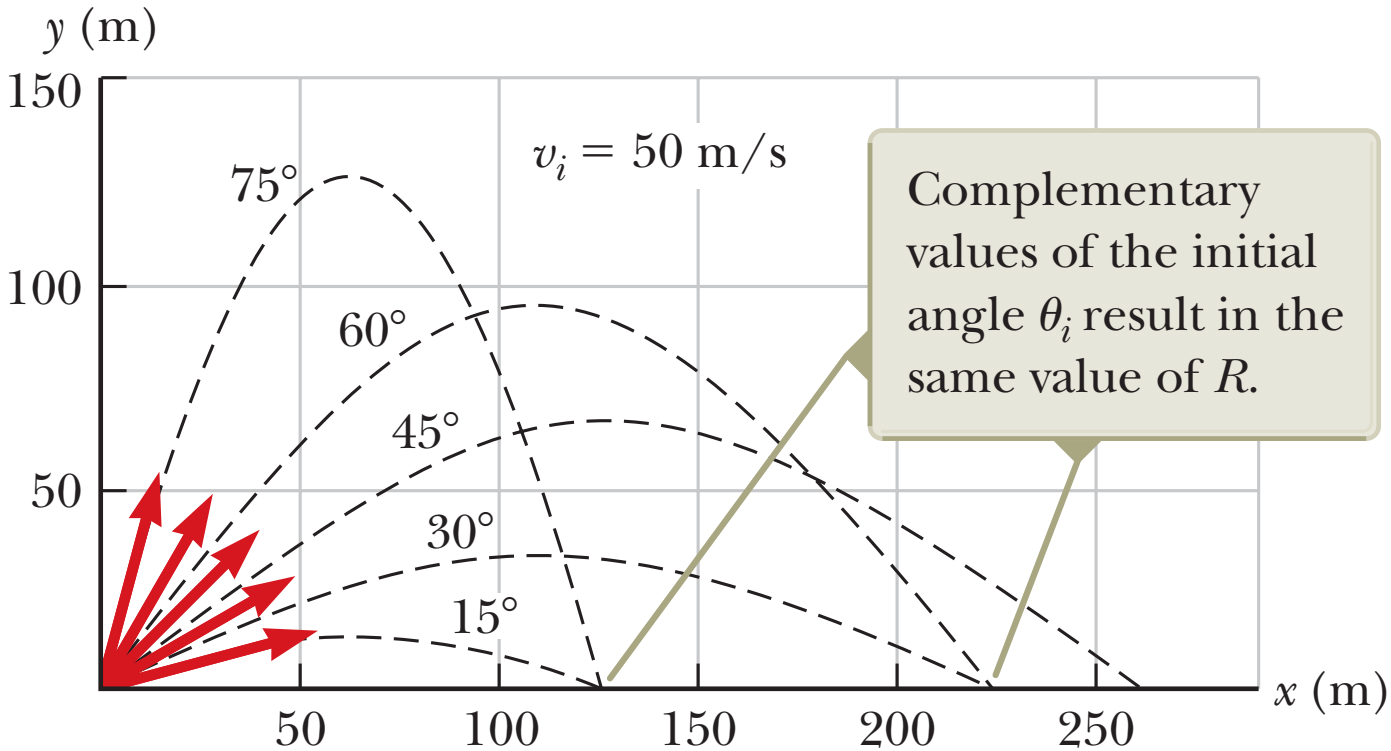


R is maximized when $\sin(2\theta) = 1 \quad \Rightarrow \quad \theta = 45°$

Range of a Projectile

Notice that at angles other than 45°, there are pairs of angles called *complimentary angles* that give the same range.

$$R = \frac{v_0^2 \sin(2\theta)}{g}$$



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Validity of Range Equation

Expression for the range of a projectile:

$$R = \frac{v_0^2 \sin(2\theta)}{g}$$

When is it valid?

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- when the projectile lands at the same height it is launched from!
- when there is no air resistance (we will not deal with air resistance in this course)

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If it's not valid, you also can't assume $\theta = 45^\circ$ maximizes the horizontal distance covered.

Summary

- time of flight
- range of a projectile

Test 2 Monday, Feb 24 (TBC).

Homework

Walker Physics:

- Ch 4, onward from page 100. Problems: 27, 29, 39
- Ch 4, Probs: 37, 47, 49, 55, 83 (these use the range equation)