Introduction to Mechanics
Trajectory Equation

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Last time

- maximum height of a projectile
- time of flight of a projectile
- range of a projectile
- trajectory equation

Overview

- trajectory equation
- another projectile motion example

Projectile Trajectory

Suppose we want to know the height of a projectile (relative to its launch point) in terms of its x coordinate. Suppose it is launched at an angle θ above the horizontal, with initial velocity v_0.

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$$x = v_0 \cos\theta t \Rightarrow t = \frac{x}{v_0 \cos\theta}$$

y-direction:

$$y = v_0 \sin\theta t - \frac{1}{2} g t^2$$

Substituting for t gives:

$$y = (\tan\theta) x - \frac{g}{2 v_0^2 \cos^2\theta} x^2$$

Projectile Trajectory

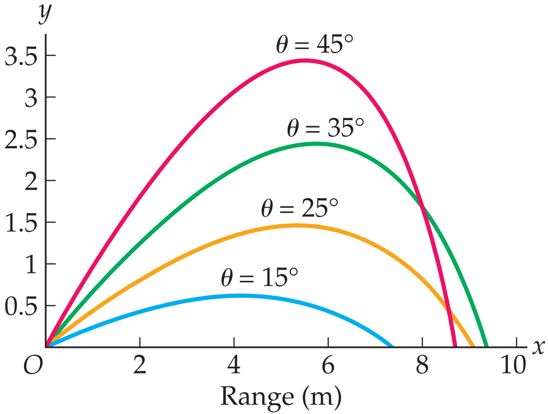
$$y = (\tan\theta)x - \left(\frac{g}{2v_0^2\cos^2\theta}\right)x^2$$

This is why projectiles trace out a parabola with respect to horizontal position, as well as with respect to time.

What Happens with Air Resistance?

The projectile's path without air resistance is a symmetrical parabola.

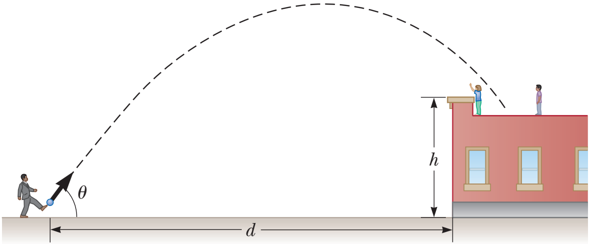
With air resistance, this is no longer the case.



[1]Figure from Walker, "Physics", page 97.

Projectile Motion Example

25. A playground is on the flat roof of a city school, 6.00 m above the street below (Fig. P4.25). The vertical wall of the building is $h = 7.00$ m high, forming a 1-m-high railing around the playground. A ball has fallen to the street below, and a passerby returns it by launching it at an angle of $\theta = 53.0°$ above the horizontal at a point $d = 24.0$ m from the base of the building wall. The ball takes 2.20 s to reach a point vertically above the wall. (a) Find the speed at which the ball was launched. (b) Find the vertical distance by which the ball clears the wall. (c) Find the horizontal distance from the wall to the point on the roof where the ball lands.



[1]Serway & Jewett, "Physics for Scientists & Engineers", 9th ed, #25, p103.

Projectile Motion Example

Part (a)
Hypothesis: about 12 m/s (faster than a normal person can run)

Given: $\Delta x = 24.0$ m, $t = 2.20$ s, $\theta = 53.0^\circ$
Asked for: v_0

Projectile Motion Example

Part (a)
Hypothesis: about 12 m/s (faster than a normal person can run)

Given: $\Delta x = 24.0$ m, $t = 2.20$ s, $\theta = 53.0^\circ$
Asked for: v_0

Strategy: we know $\Delta x = v_{0x} t$ and $v_{0x} = v_0 \cos\theta$.

Rearranging:

$$\begin{aligned} \Delta x &= v_0 \cos\theta\, t \\ v_0 &= \frac{\Delta x}{\cos\theta\, t} \\ &= \frac{(24.0 \text{ m})}{\cos(53^\circ)(2.20 \text{ s})} \\ &= \underline{18.1 \text{ m/s}} \end{aligned}$$

Projectile Motion Example

Part (b)
Hypothesis: about 2 m

Given: $\Delta x = 24.0$ m, $t = 2.20$ s, $\theta = 53.0^\circ$, $v_0 = 18.1$ m/s, $h = 7.00$ m
Asked for: height above the wall, $\Delta y - h$

Projectile Motion Example

Part (b)
Hypothesis: about 2 m

Given: $\Delta x = 24.0$ m, $t = 2.20$ s, $\theta = 53.0^\circ$, $v_0 = 18.1$ m/s, $h = 7.00$ m
Asked for: height above the wall, $\Delta y - h$

Strategy: there are a couple of ways to solve it. One way:

$$\Delta y = v_{0y} t - \frac{1}{2} g t^2$$

$$\begin{aligned}
\Delta y - h &= v_0 \sin\theta\, t - \frac{1}{2} g t^2 - h \\
&= (18.1 \text{ m/s}) \sin(53^\circ)(2.20 \text{ s}) - \frac{1}{2}(9.81)(2.20)^2 - 7\text{m} \\
&= \underline{1.13 \text{ m}}
\end{aligned}$$

Projectile Motion Example

Part (c)
Hypothesis: again, about 2 m

Given: $\theta = 53.0^\circ$, $v_0 =$ m/s, $\Delta y = 6.00$ m
Asked for: distance behind the wall, $\Delta x - d$

Projectile Motion Example

Part (c)
Hypothesis: again, about 2 m

Given: $\theta = 53.0^\circ$, $v_0 =$ m/s, $\Delta y = 6.00$ m
Asked for: distance behind the wall, $\Delta x - d$

Strategy: Could find t, then Δx. Or, use trajectory equation:

$$y = (\tan\theta)x - \frac{g}{2v_i^2\cos^2\theta}x^2$$

Projectile Motion Example

Solve

$$\frac{g}{2v_i^2 \cos^2 \theta} x^2 - (\tan \theta) x + y = 0$$

for x.

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$$x = \frac{\tan \theta \pm \sqrt{\tan^2 \theta - 4 \left(\frac{g}{2v_i^2 \cos^2 \theta} \right) y}}{2 \left(\frac{g}{2v_i^2 \cos^2 \theta} \right)}$$

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Putting in the numbers in the question:

$$x = 26.79 \text{ m} \quad \text{or} \quad x = 5.44 \text{ m}$$

Want the solution that is larger than 24 m, since ball makes it onto roof.

Distance behind wall: 26.79 m - 24 m = $\underline{2.79 \text{ m}}$.

Summary

- trajectory equation for a projectile
- relative motion example

Test 2 Monday, Feb 24.

Homework

Walker Physics:

- Ch 4, onward from page 100. Con. Ques: 7, 9; Problems: 1, 40 & 41, 43, 71, 77, 87, 67 (projectile in disguise)
- Read ahead in Ch 5.