Introduction to Mechanics
Non-uniform Circular Motion
Introducing Energy

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Last time

- applying the idea of centripetal force
- banked turns
Overview

- non-uniform circular motion and tangential acceleration
- energy and work
Non-uniform Circular Motion

A particle can speed up or slow down while following a circular arc. It must have a component of its acceleration along the direction of its velocity.

\[ \vec{a}_t, \vec{a}_{\text{cp}}, \vec{a}_{\text{total}} \]

\[ 1 \text{ Figure from Walker, “Physics”}. \]
Thus, a centrifuge can produce centripetal accelerations that are many thousand times greater than the acceleration of gravity. In fact, devices referred to as ultracentrifuges can produce accelerations as great as 1 million g. Even in the relatively modest case considered in Exercise 6–1, the forces involved in a centrifuge can be quite significant. For example, if the contents of the test tube have a mass of 12.0 g, the centripetal force that must be exerted by the bottom of the tube is or about 250 lbs!

Finally, an object moving in a circular path may increase or decrease its speed. In such a case, the object has both an acceleration tangential to its path that changes its speed, plus a centripetal acceleration perpendicular to its path, that changes its direction of motion. Such a situation is illustrated in Figure 6–15. The total acceleration of the object is the vector sum of and . We will explore this case more fully in Chapter 10.

\[ a_{cp} = \frac{v^2}{r} \]

EXERCISE 6–1

The centrifuge in Figure 6–14 rotates at a rate that gives the bottom of the test tube a linear speed of 89.3 m/s. If the bottom of the test tube is 8.50 cm from the axis of rotation, what is the centripetal acceleration experienced there?

Solution

Applying the relation yields

\[ a_{cp} = \frac{v^2}{r} \]

In this expression, \( g \) is the acceleration of gravity, \( 9.81 \text{ m/s}^2 \).
A train slows down as it rounds a sharp horizontal turn, going from 90.0 km/h to 50.0 km/h in the 15.0 s it takes to round the bend. The radius of the curve is 150 m. Compute the acceleration at the moment the train speed reaches 50.0 km/h. Assume the train continues to slow down at this time at the same rate.
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Sketch:
A train slows down as it rounds a sharp horizontal turn, going from 90.0 km/h to 50.0 km/h in the 15.0 s it takes to round the bend. The radius of the curve is 150 m. Compute the acceleration at the moment the train speed reaches 50.0 km/h. Assume the train continues to slow down at this time at the same rate.
Radial and Tangential Accelerations

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\[ v_i = 90.0 \text{ km/h} = \left( \frac{90.0}{3.6} \right) \text{ m/s} = 25.0 \text{ m/s} \]

\[ v_f = 50.0 \text{ km/h} = \left( \frac{50.0}{3.6} \right) \text{ m/s} = 13.9 \text{ m/s} \]
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Tangential accel. corresponds to changing speed: \( a_{t,avg} = \frac{\Delta v}{\Delta t} \)

Centripetal accel. corresponds to changing direction: \( a_{cp} = \frac{v^2}{r} \)

\(^1\)Page 93, Serway & Jewett.
Radial and Tangential Accelerations

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Radial and Tangential Accelerations

One end of a cord is fixed and a small 0.500-kg object is attached to the other end, where it swings in a section of a vertical circle of radius 2.00 m as shown. When $\theta = 20.0^\circ$, the speed of the object is 8.00 m/s. At this instant, find (a) the tension in the string, (b) the tangential and radial components of acceleration.

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1Serway & Jewett, 9th ed., ch 6, prob 18.
Energy

Energy is a difficult concept to define, but it is very important for physics.

Energy can take many different forms.

Knowing the amount of energy a system has can tell us about what states or configurations we can find the system in.
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One way that energy is often described is that it represents the ability of a system to do work.

We need to know what work is!
Work

Work is an amount of energy.

The amount of work done on an object depends on the applied force and the displacement of the object as the force acts.
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The amount of work done on an object depends on the applied force and the displacement of the object as the force acts.

If the force is in the same direction as the displacement,

\[ W = Fd \]
Work

\[ W = Fd \]

Units of Work?

They have a special name: Joules, symbol J.

1 joule = 1 J = 1 Nm

Work is not a vector. Work is a scalar.
Example

What is the work done in lifting a 3.0-kg book 0.50 m?
Example

What is the work done in lifting a 3.0-kg book 0.50 m?

To lift a book with constant velocity, requires $F_{\text{app}} = mg$.
What if the force is not in the direction of the displacement?

We need to extend our definition of work.
For a constant applied force, Work is defined as:

\[ W = F \cdot d = Fd \cos \theta \]
In this expression:

\[ W = \mathbf{F} \cdot \mathbf{d} = Fd \cos \theta \]

we use something called the dot product of two vectors \( \mathbf{A} \) and \( \mathbf{B} \):

\[ \mathbf{A} \cdot \mathbf{B} = AB \cos \theta \]
Units of Work

Work can be positive or negative!

\[ W = Fd \cos \theta > 0 \quad W = Fd \cos \theta = 0 \quad W = Fd \cos \theta < 0 \]

positive work \quad \text{zero work} \quad \text{negative work}

For work done on a system:

- Positive \( \Rightarrow \) energy is transferred to the system.
- Negative \( \Rightarrow \) energy is transferred from the system.
Work done by individual forces

If there are several forces acting on a system, each one can have an associated work.
Work done by individual forces

\[ W_n = 0 \quad W_{mg} = 0 \quad W_F = Fd \cos \theta \]

In other words, we can ask what is the work done on the system by each force separately.
The **net work** is the sum of all the individual works.

\[ W_{\text{net}} = \sum_i W_i \]

where \( W_i = F_i d \cos \theta \) is the work done by the force \( F_i \).

If the system can be modeled as a particle (the only case we consider in this course):

\[ W_{\text{net}} = F_{\text{net}} d \cos \theta \]

assuming the net force is constant.
A car coasts down a hill that makes an angle \( \phi \) to the horizontal.

The work done by the weight (\( mg \) force) is

(A) positive
(B) negative
(C) zero
(D) cannot be determined
A car coasts down a hill that makes an angle $\phi$ to the horizontal.

The work done by the normal force, $\vec{N}$, is

(A) positive
(B) negative
(C) zero
(D) cannot be determined
A car coasts down a hill that makes an angle \( \phi \) to the horizontal.

The work done by the air resistance (\( \mathbf{F}_{\text{air}} \) force) is

(A) positive  
(B) negative  
(C) zero  
(D) cannot be determined
A car coasts down a hill that makes an angle $\phi$ to the horizontal.

The net (or total) work done by all forces on the car is

(A) positive

(B) negative

(C) zero

(D) cannot be determined
Summary

- non-uniform circ. motion and tangential acceleration
- energy and work

Homework

Walker Physics:

- Ch 6, onward from page 177. Problems: 29, 105, 109, 110
- Ch 7, onward from page 210. Questions: 1, 3, 5; Problems: 5, 7