Introduction to Mechanics
Friction

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Last time

- objects accelerated together
- the introduced the Atwood machine

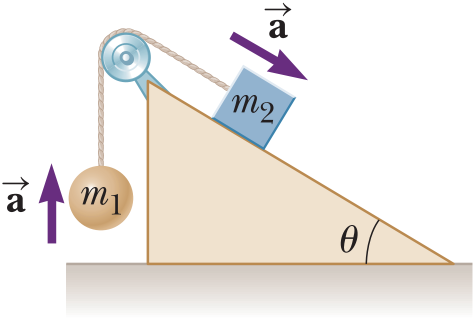
Overview

- Atwood machine variant
- friction (kinetic and static)
- solving problems with friction

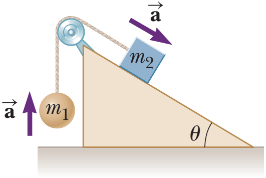
Pulley with an Incline

Let's change up our Atwood machine apparatus so that one of the masses is on a slanted surface with no friction. Assume $m_2 \sin\theta > m_1$, so the blocks slide as shown.

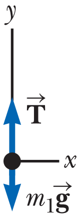
Try to solve this one yourself! $a = ?$, $T = ?$

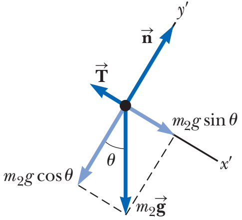


Pulley with an Incline



We can still consider each object separately:





Acceleration? Tension?

Pulley with an Incline

We must have $a_y = a_{x'} = a$.

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Object 1:

<u>x-direction</u>: no forces w/ components in $x \Rightarrow F_{\text{net},x} = 0, a_x = 0$.

<u>y-direction</u>:

$$\begin{aligned} F_{\text{net,y}} &= m_1 a_y \\ T - m_1 g &= m_1 a \end{aligned} \tag{1}$$

Pulley with an Incline

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<u>y-direction</u>:

$$\begin{aligned} F_{\text{net,y}} &= m_1 a_y \\ T - m_1 g &= m_1 a \qquad (1) \end{aligned}$$

Object 2:
<u>x'-direction</u>:

$$\begin{aligned} F_{\text{net},x'} &= m_2 a_{x'} \\ m_2 g \sin\theta - T &= m_2 a \qquad (2) \end{aligned}$$

<u>y'-direction</u>: $a_{y'} = 0$.

Pulley with an Incline

$$\begin{aligned} T - m_1 g &= m_1 a \qquad (1) \\ m_2 g \sin\theta - T &= m_2 a \qquad (2) \end{aligned}$$

Add eq (1) and (2):

$$\begin{aligned} m_2 g \sin\theta - m_1 g &= (m_1 + m_2) a \\ a &= \frac{(m_2 \sin\theta - m_1) g}{m_1 + m_2} \end{aligned}$$

Pulley with an Incline

$$T - m_1 g = m_1 a \tag{1}$$
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Putting a into (1):

$$\begin{aligned} m_1 \frac{(m_2 \sin\theta - m_1) g}{m_1 + m_2} &= T - m_1 g \\ T &= \frac{m_1 m_2 (\sin\theta + 1) g}{m_1 + m_2} \end{aligned}$$

Pulley with an Incline

$$
\begin{aligned}
T - m_1 g &= m_1 a \qquad (1) \\
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Putting a into (1):

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\end{aligned}
$$

Does this agree with what we had for the Atwood machine when $\theta = 90^\circ$?

Question

You push on a heavy crate and moves it across the floor. However, even as you push it does not accelerate and if you stop pushing, the box stops moving. Why?

Some Types of Forces: Friction

Friction is a **resistive force** that occurs when two surfaces are in contact.

Friction opposes the motion of one surface relative the other.

[1] Figure from boundless.com

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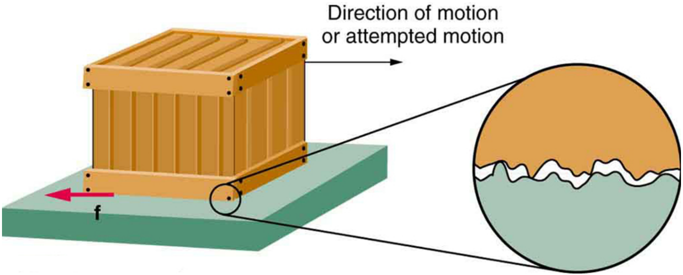


Tiny defects in the surfaces of the floor and the crate catch on one another as the crate is pushed.

(Air resistance is another resistive force.)

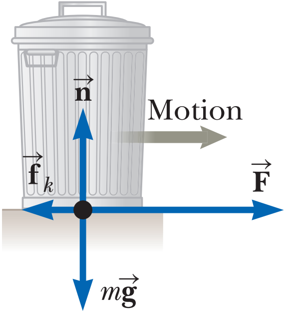
[1]Figure from boundless.com

Friction

There are actually two types of friction:

- kinetic (moving)
- static (stationary)

Kinetic Friction



kinetic friction $\propto$ normal force

$$f_k = \mu_k N$$

μ_k is the coefficient of **kinetic** friction

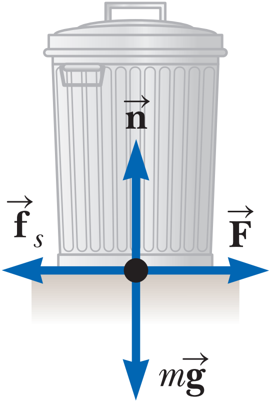
Some types of forces

Kinetic Friction

The kinetic friction force always acts to oppose motion of the surfaces relative to each other. That means the kinetic friction, $\vec{\mathbf{f}}_k$, always points opposite to the velocity vector.

Friction

Static Friction

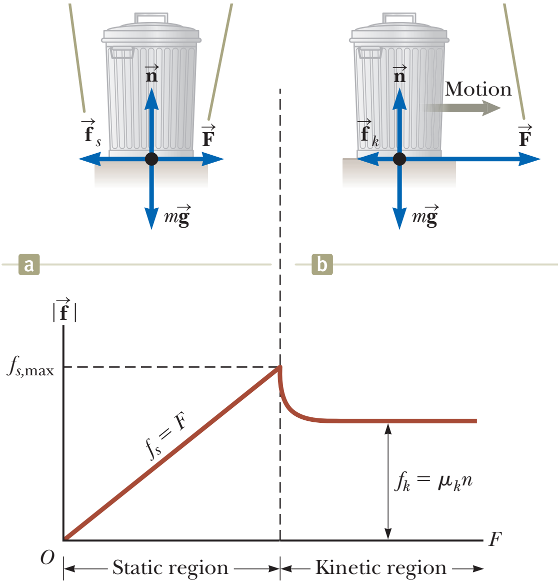


max. static friction $\propto$ normal force

$$f_s \leqslant \mu_s N$$

μ_s is the coefficient of **static** friction

Friction



$\vec{\mathbf{n}}$
$\vec{\mathbf{f}}_s$
$\vec{\mathbf{F}}$
$m\vec{\mathbf{g}}$
$\vec{\mathbf{n}}$
Motion
$\vec{\mathbf{f}}_k$
$\vec{\mathbf{F}}$
$m\vec{\mathbf{g}}$
a
b
$|\vec{\mathbf{f}}|$
$f_{s,\max}$
$f_s = F$
$f_k = \mu_k n$
F
O
Static region
Kinetic region

Friction Example 1

For waxed wood on wet snow, $\mu_s = 0.14$ and $\mu_k = 0.1$. You pull horizontally on a sled of mass 10 kg that is at rest initially. You exert a force of 5 N on the sled. What is the magnitude of the static frictional force that acts on the sled?

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There is no acceleration in the y direction.

$$\begin{aligned} F_{\text{net},y} &= m\cancel{a_y}^{\,0} \\ N - W &= 0 \\ N &= mg \end{aligned}$$

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$$\begin{aligned} f_{s,\mathrm{max}} &= \mu_s N \\ &= \mu_s m g \\ &= (0.14)(10\ \mathrm{kg})g \\ &= 13.7\ \mathrm{N} \end{aligned}$$

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$13.7\text{ N} > 5\text{ N}$, so the sled will remain at rest.

If the sled is at rest and remains at rest, it does not accelerate.
$\vec{\mathbf{F}}_{\text{net}} = m\cancelto{0}{\vec{\mathbf{a}}}$.

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$\vec{\mathbf{F}}_{\text{net}} = m \cancelto{0}{\vec{\mathbf{a}}}$.

$$\begin{aligned} F_{\text{net},x} = 0 &= 5 \text{ N} - f_s \\ f_s &= \underline{5 \text{ N}} \text{ (directed opposite to the pulling force)} \end{aligned}$$

Friction Example 2

For waxed wood on wet snow, $\mu_s = 0.14$ and $\mu_k = 0.1$. You pull horizontally on a sled of mass 10 kg that is at rest initially. How much force do you need to apply to get the sled moving? If you continue to apply that force, what will the magnitude of sled's acceleration be once it is moving?

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Sketch.

Hypothesis: 13.7 N, should be the max static friction force we just worked out; 1 m/s^2

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To get the sled moving $F_{app} \geqslant f_{s,\max}$

$$\begin{aligned} f_{s,\max} &= \mu_s N \\ &= (0.14)(10\text{ kg})g \\ &= \underline{13.7\text{ N}} \end{aligned}$$

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$$\begin{aligned} F_{\text{net},x} &= ma_x \\ F_{app} - F_{kf} &= 13.72 - \mu_k n \\ &= 13.7 - (0.1)(10\text{ kg})g \\ &= 3.92\text{ N} \end{aligned}$$

$$a = \frac{F}{m} = \frac{3.92\text{ N}}{10\text{ kg}} = \underline{0.39\text{ ms}^{-2}}$$

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Reasonable?: Yes for the force. The acceleration was a bit less than my guess, but same order of magnitude.

Friction Example 6-2

A trained sea lion slides from rest with constant acceleration down a 3.0-m-long ramp into a pool of water. If the ramp is inclined at an angle of 23° above the horizontal and the coefficient of kinetic friction between the sea lion and the ramp is 0.26, how long does it take for the sea lion to make a splash in the pool?

[1] Walker, "Physics".

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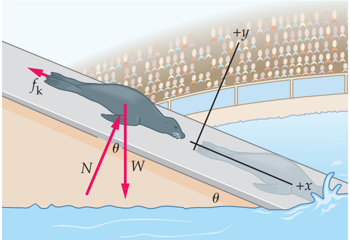

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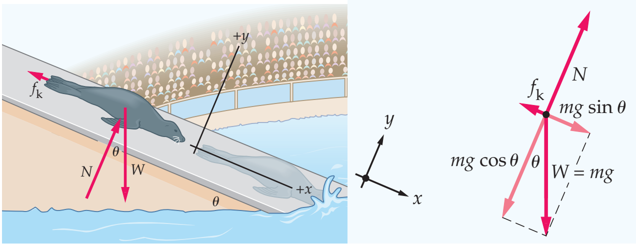

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$\underline{y \text{ direction}}$:

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$\underline{x \text{ direction}}$:

$$F_{\text{net},x} = mg\sin\theta - f_k = ma$$

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\begin{aligned}
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mg\sin\theta - \mu_k N &= ma \\
mg\sin\theta - \mu_k(mg\cos\theta) &= ma \\
a &= g(\sin\theta - \mu_k\cos\theta)
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<u>x direction</u>:

$$\begin{aligned} F_{\text{net},x} = mg\sin\theta - f_k &= ma \\ mg\sin\theta - \mu_k N &= ma \\ mg\sin\theta - \mu_k(mg\cos\theta) &= ma \\ a &= g(\sin\theta - \mu_k\cos\theta) \\ a &= 1.5\ \text{m/s}^2 \end{aligned}$$

Friction Example 6-2

Given: $\Delta x = 3$ m, $a = 1.5$ m/s^2, $v_0 = 0$ m/s.
Asked for: t.

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$$\Delta x = v_0 t + \frac{1}{2} a t^2$$

$$\underline{t = 2.0 \text{ s}}$$

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$$\Delta x = v_0 t + \frac{1}{2} a t^2$$

$$\underline{t = 2.0 \text{ s}}$$

Reasonable?: Less than half my guess, but 23° is a pretty steep slope, so the answer is plausible.

Summary

- Atwood machine variant
- friction
- practice with friction

Quiz Monday.

Homework

Walker Physics:

- Ch 6, onwards from page 177. Questions: 3, 15; Problems: 1, 3, 7, 11, 13, 15, 87 (friction)