Introduction to Mechanics
Springs
Circular Motion

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Last time

- friction examples

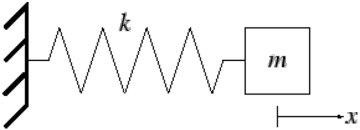
Overview

- elastic forces / springs
- circular motion
    - centripetal acceleration in circular motion

Some types of forces

Elastic Forces

Springs exert forces as they are being compressed or extended. They have a natural length, at which they remain if there are no external forces acting.



Hooke's Law gives

$$\vec{\mathbf{F}}_{\text{spring}} = -k\,\vec{\mathbf{x}}$$

where k is a constant. $\vec{\mathbf{x}}$ is the amount of displacement of one end of a spring from its natural length. (The amount of compression or extension.

[1]Figure from CCRMA Stanford Univ.

Elasticity

The force that the spring exerts to restore itself to its original length is proportional to how much it is compressed or stretched.

This is called Hooke's Law:

$$\vec{\mathbf{F}} = -k\,\vec{\mathbf{x}}$$

where x is the distance that the spring is stretched or compressed by and k is a constant that depends on the spring itself. (The "spring constant").

If a very large force is put on the spring eventually it will break: it will not return to its original shape. The *elastic limit* is the maximum distance the spring can be stretched so that it still returns to its original shape.

Spring example

If a 2 kg painting is hung from a spring, the spring stretches 10 cm. What if instead a 4 kg painting is hung from the spring? How far will it stretch?

(A) 10 cm

(B) 20 cm

(C) 30 cm

(D) None of the above.

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Spring example

If a 2 kg painting is hung from a spring, the spring stretches 10 cm. What if instead a 4 kg painting is hung from the spring? How far will it stretch?

We don't know the spring constant, but we can work it out from the information about the first 2 kg painting. The force on the spring is just the weight of the painting.

$$k = \frac{F_g}{x} = \frac{(2 \text{ kg})g}{0.1 \text{ m}} = 196.2 \text{ N/m}$$

$$x = \frac{F}{k} = \frac{(4 \text{ kg})g}{(196.2 \text{ N/m})} = 0.2 \text{ m}$$

If you put on twice the force, you stretch the spring twice as far!

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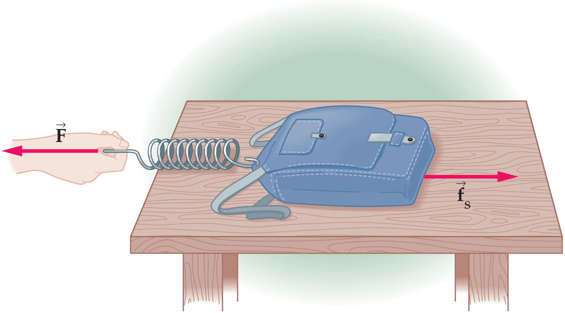
Spring-Friction Example

A backpack full of books weighing 52.0 N rests on a table in a physics laboratory classroom. A spring with a force constant of 150 N/m is attached to the backpack and pulled horizontally, as shown. (a) If the spring is pulled until it stretches 2.00 cm and the pack remains at rest, what is the force of friction exerted on the backpack by the table? (b) Does your answer to part (a) change if the mass of the backpack is doubled? Explain.



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[1] Walker, page 179, #24.

Spring-Friction Example

(a) If the spring is pulled until it stretches 2.00 cm and the pack remains at rest, what is the force of friction exerted on the backpack by the table?

Sketch a free-body diagram for the backpack.

Hypothesis:

Spring-Friction Example

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Sketch a free-body diagram for the backpack.

Hypothesis: Will point to the right, units, Newtons. Magnitude will equal kx, since $\vec{\mathbf{F}}_{\text{net}} = 0$. Much less than the weight, perhaps about 5 N.

Spring-Friction Example

(a) If the spring is pulled until it stretches 2.00 cm and the pack remains at rest, what is the force of friction exerted on the backpack by the table?

Strategy: use $\vec{\mathbf{F}}_{\text{net}} = 0$, analyze horizontal direction.

$$\begin{aligned}
F_{\text{net},x} &= m\cancelto{0}{a_x} \\
f_s - F &= 0 \\
f_s &= F \\
f_s &= kx \\
&= (150\ \text{N})(0.02\ \text{m}) \\
&= \underline{3.00\ \text{N}}
\end{aligned}$$

Spring-Friction Example

(b) Does your answer to part (a) change if the mass of the backpack is doubled? Explain.

Spring-Friction Example

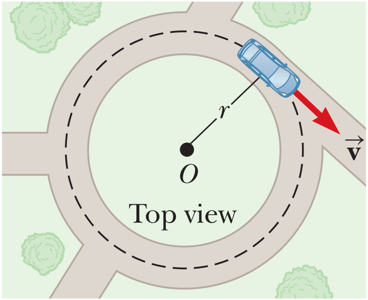
(b) Does your answer to part (a) change if the mass of the backpack is doubled? Explain.

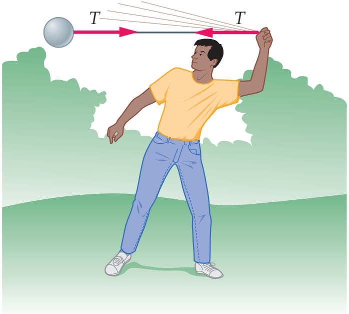
The solution did not use the mass of the backpack, so no, the answer will not change if the mass is doubled.

The static friction force is always just big enough to counteract the applied force on an object (unless the applied force exceeds the max static friction force).

Circular motion

Objects that move along an arc of a circle are said to be undergoing circular motion.



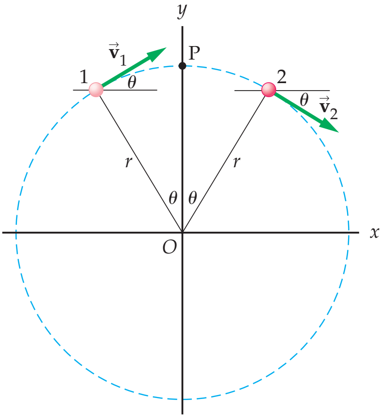


It is possible that such an object moves with constant speed. But does it move with constant velocity?

[1]Left Figure: from Serway & Jewett, 9th ed. Right Figure: from Walker.

Circular motion

Does it move with constant velocity? No!



The direction of the object's velocity is changing.

Circular motion

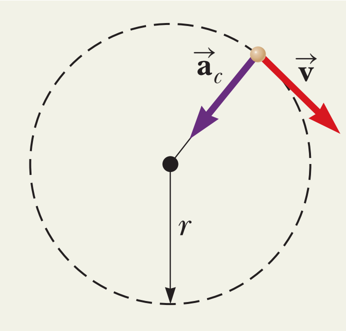
Newton's first law tells us that an object in motion will continue with a constant velocity unless acted upon by a net force.

What does that tell us about an object moving in a circle?

It must be experiencing a non-zero net force.

Uniform Circular Motion

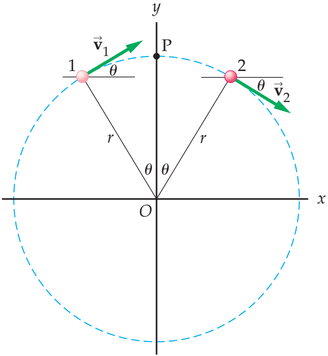
The velocity vector points along a tangent to the circle

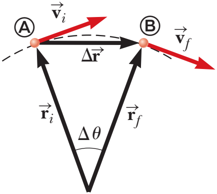


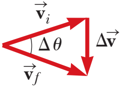
For uniform circular motion:

- the radius is constant
- the speed is constant
- the *magnitude* of the acceleration is constant

Circular Motion







The net force is directed towards the center of the circle, just as the **change** in velocity (it's acceleration!) is directed towards the center.

Circular Motion

How large is the acceleration of the object?

It should depend on:

Circular Motion

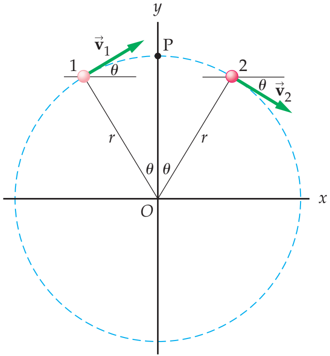
How large is the acceleration of the object?

It should depend on:

- the **speed** of the object - in this case, a higher the speed means a larger acceleration
- the **radius** of the path - the tighter the turn, the smaller the radius, the larger the acceleration

Circular Motion

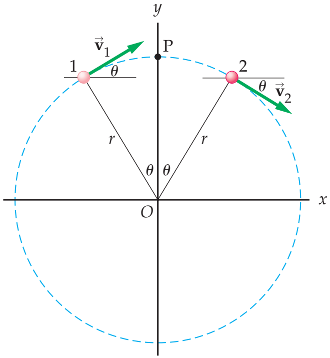
For points 1 and 2, the x-component of the velocity is the same, but the y-component changes sign.



$$\vec{\mathbf{a}}_{\text{avg}} = \frac{\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1}{\Delta t} = \left(\frac{v_{2,y} - v_{1,y}}{\Delta t}\right)\hat{\mathbf{j}} = \frac{-2v\sin\theta}{\Delta t}\hat{\mathbf{j}}$$

Circular Motion

How much time does it take to go from 1 to 2? Depends on the speed of the particle...



Let s be the distance the particle travels.

$$\Delta t = \frac{s}{v} = \frac{2r\theta}{v}$$

Circular Motion

All together:

$$\vec{\mathbf{a}}_{\text{avg}} = \frac{-2v\sin\theta}{(2r\theta)/v}\,\hat{\mathbf{j}} = \frac{-v^2}{r}\left(\frac{\sin\theta}{\theta}\right)\hat{\mathbf{j}}$$

Circular Motion

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This is the average acceleration over time Δt. Could we figure out the instantaneous velocity?

Circular Motion

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For shorter and shorter windows of time $\theta \to 0$.

As $\theta \to 0$, $\sin\theta \to \theta$, so $\left(\frac{\sin\theta}{\theta}\right) \to 1$.

$$\vec{\mathbf{a}} = \frac{-v^2}{r}\,\hat{\mathbf{j}}$$

Circular Motion

The direction of this acceleration is also always changing.

The easiest way to describe how it points using vectors is to make a vector defined to point out from the origin through the object.

This is the **radial** direction.

We can always write:

$$\vec{\mathbf{a}} = \frac{-v^2}{r}\,\hat{\mathbf{r}}$$

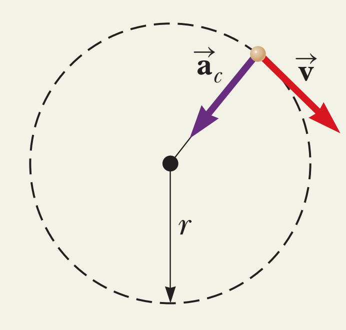
where the minus sign means that the acceleration points in towards the center of the circle, rather than outward.

Circular Motion

Centripetal acceleration

The acceleration of an object that follows a circular arc of radius, r, at constant speed v. Its magnitude is

$$a_{cp} = \frac{v^2}{r}$$



Summary

- springs
- circular motion, centripetal acceleration

Quiz Monday.

Homework

- Forces and Motion worksheet

Walker Physics:

- Ch 6, onward from page 177. Problem: 19, 21, 25, 73, 101 (springs)
- Ch 6, onward from page 177. Problems: 53 (circ motion)