# Introduction to Mechanics
# Circular Motion

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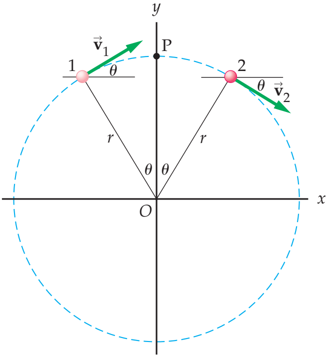
# Last time

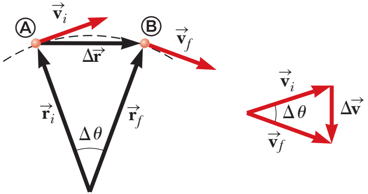
- more friction examples
- springs
- circular motion

# Overview

- circular motion
    - acceleration
    - force and circular motion
    - motion in a vertical circle

# Uniform Circular Motion Acceleration




The net force is directed towards the center of the circle, just as the **change** in velocity (it's acceleration!) is directed towards the center.

# Uniform Circular Motion Acceleration

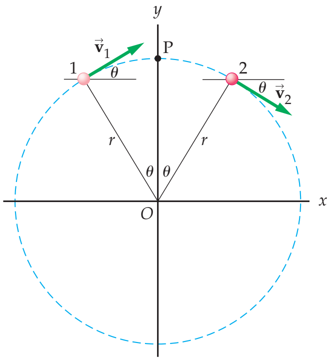
How large is the acceleration of the object?

It should depend on:

- the **speed** of the object - in this case, a higher the speed means a larger acceleration
- the **radius** of the path - the tighter the turn, the smaller the radius, the larger the acceleration

# Uniform Circular Motion Acceleration

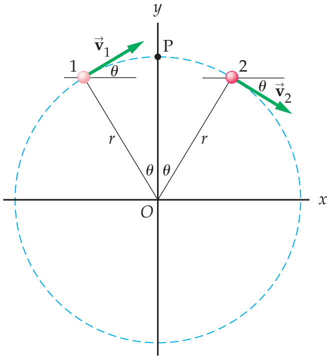
For points 1 and 2, the $x$-component of the velocity is the same, but the $y$-component changes sign.




$$\vec{\mathbf{a}}_{\text{avg}} = \frac{\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1}{\Delta t} = \left( \frac{v_{2,y} - v_{1,y}}{\Delta t} \right) \hat{\mathbf{j}} = \frac{-2v \sin\theta}{\Delta t} \hat{\mathbf{j}}$$

# Uniform Circular Motion Acceleration

How much time does it take to go from 1 to 2? Depends on the speed of the particle...




Let $s$ be the distance the particle travels.

$$\Delta t = \frac{s}{v} = \frac{2r\theta}{v}$$

# Uniform Circular Motion Acceleration

All together:

$$\vec{\mathbf{a}}_{\text{avg}} = \frac{-2v\sin\theta}{(2r\theta)/v}\,\hat{\mathbf{j}} = \frac{-v^2}{r}\left(\frac{\sin\theta}{\theta}\right)\hat{\mathbf{j}}$$

# Uniform Circular Motion Acceleration

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This is the average acceleration over time $\Delta t$. Could we figure out the instantaneous velocity?

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This is the average acceleration over time $\Delta t$. Could we figure out the instantaneous velocity?

For shorter and shorter windows of time $\theta \to 0$.

As $\theta \to 0$, $\sin\theta \to \theta$, so $\left(\frac{\sin\theta}{\theta}\right) \to 1$.

$$\vec{\mathbf{a}} = \frac{-v^2}{r}\,\hat{\mathbf{j}}$$

# Uniform Circular Motion Acceleration

The direction of this acceleration is also always changing.

The easiest way to describe how it points using vectors is to make a vector defined to point out from the origin through the object.

This is the **radial** direction.

We can always write:

$$\vec{\mathbf{a}} = \frac{-v^2}{r}\,\hat{\mathbf{r}}$$

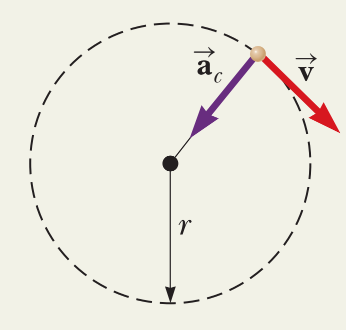
where the minus sign means that the acceleration points in towards the center of the circle, rather than outward.

# Circular Motion

## Centripetal acceleration

The acceleration of an object that follows a circular arc of radius, $r$, at constant speed $v$. Its magnitude is

$$a_{cp} = \frac{v^2}{r}$$



# Circular Motion

## Centripetal acceleration

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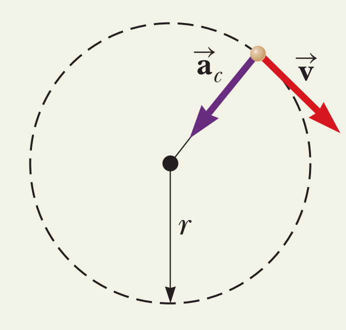
# Uniform Circular Motion

The velocity vector points along a tangent to the circle




For uniform circular motion:

- the radius is constant
- the speed is constant
- the *magnitude* of the acceleration is constant

[1]Figures from Serway & Jewett.

# Uniform Circular Motion and Net Force

If an object moves in a **uniform circle**, its velocity must always be changing. $\Rightarrow$ It is accelerating.

$$a = a_{cp} = \frac{v^2}{r}$$

What is the net force on the object?

# Uniform Circular Motion and Net Force

If an object moves in a **uniform circle**, its velocity must always be changing. $\Rightarrow$ It is accelerating.

$$a = a_{cp} = \frac{v^2}{r}$$

What is the net force on the object?

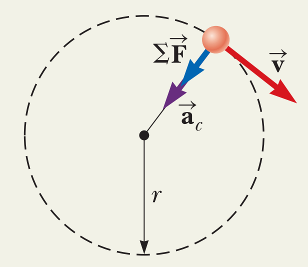
$$F_{\text{net}} = ma_{cp} = \frac{mv^2}{r}$$

# Uniform Circular Motion - Now with Force

Centripetal force:

$$F_{\text{net}} = \frac{mv^2}{r}$$

Directed toward the center of the turn.




[1]Figures from Serway & Jewett.

# Uniform Circular Motion

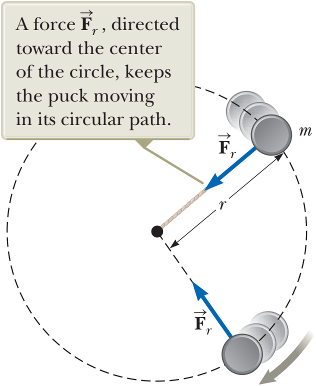
$$F_{\text{net}} = \frac{mv^2}{r}$$

As a vector:

$$\vec{\mathbf{F}}_{\text{net}} = -\frac{mv^2}{r}\hat{\mathbf{r}}$$

# Centripetal Net Force

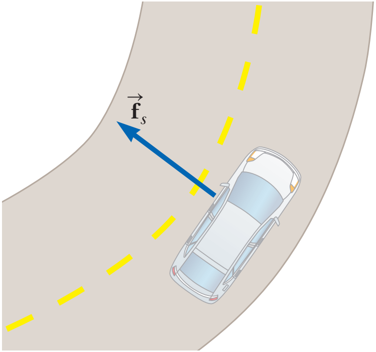
Something must provide this force:




It could be tension in a rope.

# Centripetal Net Force

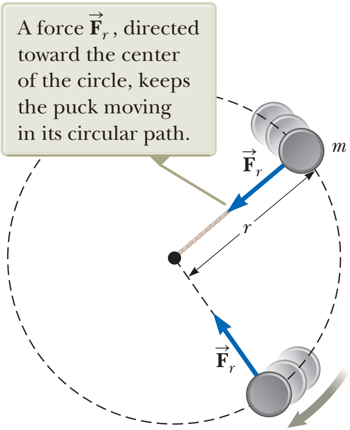
Something must provide this force:




It could be friction.

# Centripetal Net Force

Consider the example of a string constraining the motion of a puck:



# Centripetal Net Force

**Question.** What will the puck do if the string breaks?

**(A)** Fly radially outward.

**(B)** Continue along the circle.

**(C)** Move tangentially to the circle.

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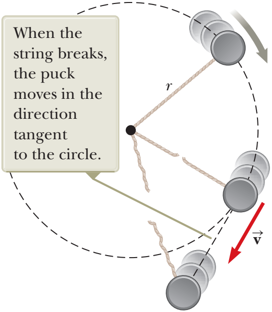

# Summary

- uniform circular motion acceleration
- forces and circular motion
- motion in a vertical circle

**Final Exam**, Thursday, Mar 26, by Canvas & Zoom, be ready at 9am.

## Homework

- Quiz 7 (take home quiz, due tomorrow, 1pm)
- Forces and Motion worksheet (due Thurs, 10am)

Walker Physics:

- Ch 6, onward from page 177. Problems: 55, 59, 61, 63, 105, 110 (vertical circle)