# Introduction to Mechanics
# Banked Turns
# Non-uniform Circular Motion


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# Last time

- more friction examples
- springs
- circular motion

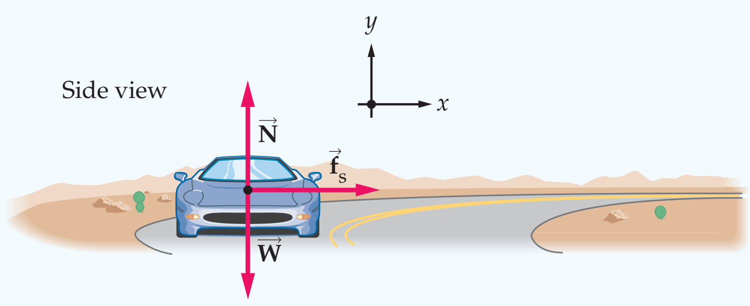
# Overview

- circular motion
  - banked turns
  - non-uniform circular motion and tangential acceleration

# Circular motion example

Last lecture we did an example with a car making a turn on a horizontal road surface...

Sketch:



# A Banked Turn

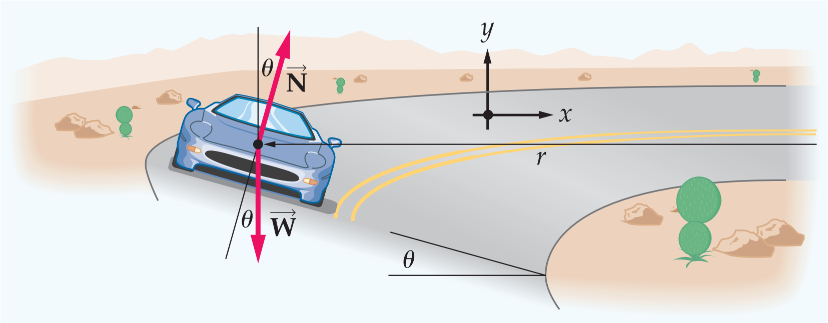
Curved roadways are often not flat. The are often **banked**, that is sloped at an angle to the horizontal.


This is so that a component of the normal force on the car can help provide some or all of the centripetal force.

[0]Photo from Walker, "Physics".

# A Banked Turn

A turn has a radius $r$. What should the angle $\theta$ be so that a car traveling at speed $v$ can turn the corner without relying on friction?



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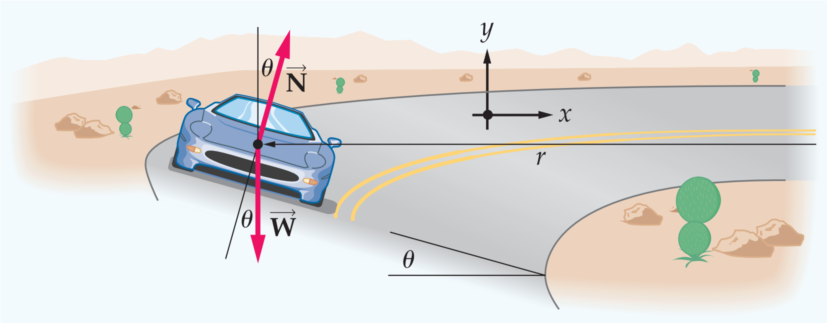
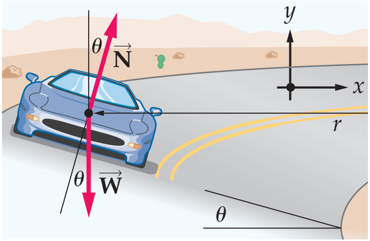



Hint: consider what the net force vector must be in this case.

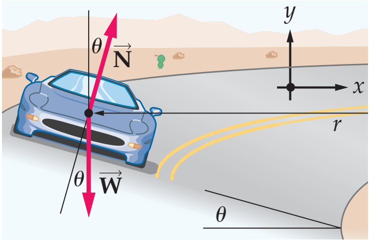
# A Banked Turn




$y$-direction (vertical):

$$
\begin{aligned}
F_{y,\text{net}} &= 0 \\
N_y + W_y &= 0
\end{aligned}
$$

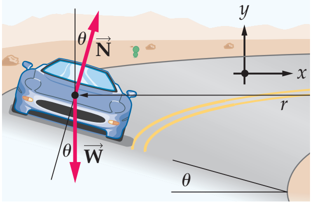
# A Banked Turn




$y$-direction (vertical):

$$\begin{aligned} F_{y,\text{net}} &= 0 \\ N_y + W_y &= 0 \\ N\cos\theta - W &= 0 \\ N\cos\theta &= W \\ N &= \frac{mg}{\cos\theta} \end{aligned}$$

# A Banked Turn

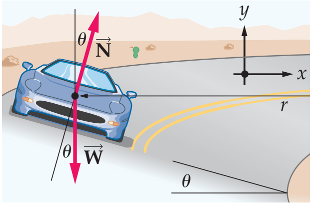



$x$-direction (horizontal):

$$F_{x,\text{net}} = m\,a_{cp}$$

$$N_x = \frac{mv^2}{r}$$

# A Banked Turn

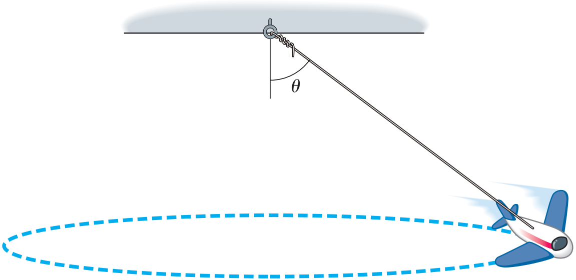



$x$-direction (horizontal):

$$
\begin{aligned}
F_{x,\text{net}} &= m\, a_{cp} \\
N_x &= \frac{mv^2}{r} \\
N \sin\theta &= \frac{mv^2}{r} \\
\frac{mg}{\cos\theta}\sin\theta &= \frac{mv^2}{r} \\
\tan\theta &= \frac{v^2}{rg} \Rightarrow \underline{\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)}
\end{aligned}
$$

# Banked Turn Related Problems

This situation is called a “conical pendulum”. But notice, it is actually a banked-turn-style problem in disguise!

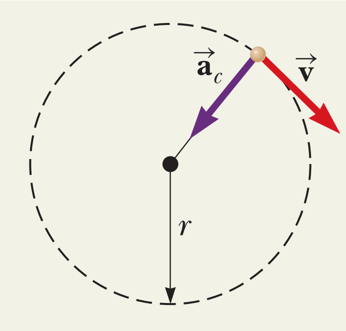



The role that was played by the normal force in the banked turn problem is now played by the tension in the string.

[1] See prob 85, Ch 6.

# Uniform Circular Motion

The velocity vector points along a tangent to the circle

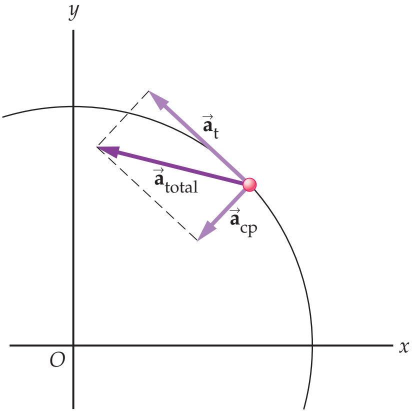



For uniform circular motion:

- the radius is constant
- the speed is constant
- the *magnitude* of the acceleration is constant

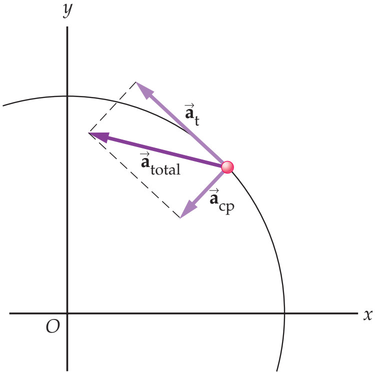
# Non-uniform Circular Motion

A particle can speed up or slow down while following a circular arc. It it does this it must have a component of its acceleration along the direction of its velocity.




[1]Figure from Walker, "Physics".

# Non-uniform Circular Motion




The centripetal acceleration $a_{cp}$ is toward the center of the circle and changes the direction of the velocity.

The tangential acceleration $a_t$ is tangent to the circle and causes a change of speed.

# Radial and Tangential Accelerations

**41.** M A train slows down as it rounds a sharp horizontal turn, going from 90.0 km/h to 50.0 km/h in the 15.0 s it takes to round the bend. The radius of the curve is 150 m. Compute the acceleration at the moment the train speed reaches 50.0 km/h. Assume the train continues to slow down at this time at the same rate.

[1] Page 93, Serway & Jewett.

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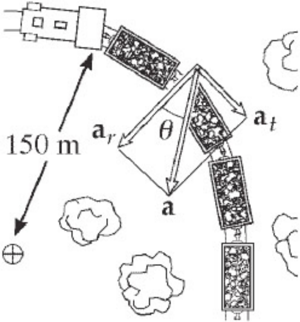
Sketch:

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$$v_i = 90.0 \text{ km/h} = \left(\frac{90.0}{3.6}\right) \text{ m/s} = 25.0 \text{ m/s}$$

$$v_f = 50.0 \text{ km/h} = \left(\frac{50.0}{3.6}\right) \text{ m/s} = 13.9 \text{ m/s}$$

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M

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Tangential accel. corresponds to changing speed: $a_{t,\text{avg}} = \frac{\Delta v}{\Delta t}$

Centripetal accel. corresponds to changing direction: $a_{cp} = \frac{v^2}{r}$

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$a_t = -0.741\ \text{m/s}^2$ ; $a_r = -1.29\ \text{m/s}^2$ (calling outward positive)

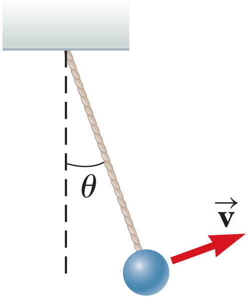
$\vec{\mathbf{a}} = 1.48\ \text{m/s}^2$ inward at an angle $29.9^\circ$

backward from the direction of travel

[1] Page 93, Serway & Jewett.

# Radial and Tangential Accelerations

One end of a cord is fixed and a small 0.500-kg object is attached to the other end, where it swings in a section of a vertical circle of radius 2.00 m as shown. When $\theta = 20.0°$, the speed of the object is 8.00 m/s. At this instant, find (a) the tension in the string, (b) the tangential and radial components of acceleration.

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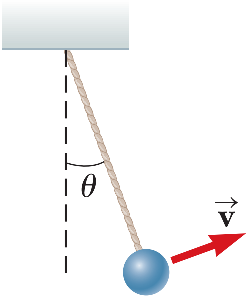


(a) $T = 20.6$ N ; (b) $a_c = 32.0$ m/s$^2$, $a_t = 3.35$ m/s$^2$

[1]Serway & Jewett, 9th ed., ch 6, prob 18.

# Discuss Quiz and/or Test Problems?

Questions from the quizzes or test?

# Summary

- banked turns
- non-uniform circ. motion and tangential acceleration

## Canvas Quiz/Survey due Thursday night, not posted yet, will take ~5 mins, get credit for it as a quiz!

## Final Exam, Thursday, Mar 26, by Canvas & Zoom, be ready at 9am.

## Homework

- Forces and Motion worksheet (due Thursday, **10am**)

Walker Physics:

- Ch 6, onward from page 177. Problems: 85 & 107 (banked turns)