Math 10 - Homework Ch 8 Answers

1. A researcher wanted to estimate the MPG for municipal busses in a large city. 100 busses were samples and the sample mean price was 9.8 MPG. For the following questions, assume the population standard deviation is 1.2 MPG.

    a. Find a 95% confidence interval for the population mean. Explain what this interval means in the context of the problem.

    $$9.8 \pm 1.96(1.2)/\sqrt{100} \rightarrow 9.8 \pm .235 \rightarrow (9.565, 10.035)$$
    We are 95% confident that mean MPG for busses is between 9.565 and 10.035 MPG

    b. Find a 99% confidence interval for the population mean. Why does the confidence interval have a larger margin of error?

    $$9.8 \pm 2.576(1.2)/\sqrt{100} \rightarrow 9.8 \pm .31 \rightarrow (9.50, 10.11)$$
    To be more confident, we need a bigger target, more margin of error.

    c. Suppose you wanted to redo the sampling or the 95% confidence interval in order to get a margin of error of plus or minus 0.1 MPG. Determine the necessary sample size.

    $$n = \left[\frac{(1.96)(1.2)}{0.1} \right]^2 = 554$$

2. The average number of years of post secondary education of employees in an industry is 1.5. A company claims that this *average* is higher for its employees. A random sample of 16 of its employees has an *mean* of 2.1 years of post secondary education with a *standard deviation* of 0.6 years.

    a. Find a 95% confidence interval for the mean number years of post secondary education for the company's employees. How does this compare with the industry value?

    $$2.1 \pm 2.131(0.6)/\sqrt{16} \rightarrow 2.1 \pm .32 \rightarrow (1.78, 2.42)$$ **The company is well above the industry average.**

3. When polling companies report a margin of error, they are referring to a 95% confidence interval. Go to the website www.pollingreport.com and verify the stated margins of error for 2 polls.

    Many answers possible

Constructing Confidence Intervals In Exercises 3 and 4 you are given the sample mean and the sample standard deviation. Assume the random variable is normally distributed and use a t-distribution to construct a 95% confidence interval for the population mean μ. What is the margin of error of the confidence interval?

4. *Repair Costs: Microwaves* In a random sample of five microwave ovens, the mean repair cost was $75.00 and the standard deviation was $12.50.

    $$75 \pm 2.776(12.50)/\sqrt{5} \rightarrow 75 \pm 15.5 \rightarrow (\$59.50, \$90.50)$$ **(t with 4 df)**

5. **Repair Costs: Computers** In a random sample of seven computers, the mean repair cost was $100.00 and the standard deviation was $42.50.

$$100 \pm 2.447(42.50)/\sqrt{7} \rightarrow 100 \pm 39.3 \rightarrow (\$60.70, \$139.30)$$ **(t with 6 df)**

6. You did some research on repair costs of microwave ovens and found that the standard deviation is $\sigma = \$15$. Repeat Exercise 3, using a **normal distribution** with the appropriate calculations for a standard deviation that is known. Compare the results.

$$75 \pm 1.96(12.50)/\sqrt{5} \rightarrow 75 \pm 11.0 \rightarrow (\$64.00, \$86.00)$$ **(z creates narrower confidence interval)**

7. You did some research on repair costs of computers and found that the standard deviation is $\sigma = \$50$. Redo Exercise 4, using a **normal distribution** with the appropriate calculations for a standard deviation that is known. Compare the results.

$$100 \pm 1.96(42.50)/\sqrt{7} \rightarrow 100 \pm 31.5 \rightarrow (\$68.50, \$131.50)$$ **(z creates narrower confidence interval)**

8. **Mini-Soccer Balls** A soccer ball manufacturer wants to estimate the mean circumference of mini-soccer balls within 0.15 inch. Assume that the population of circumferences is normally distributed.

   (a) Determine the minimum sample size required to construct a 99% confidence interval for the population mean. Assume the population standard deviation is 0.20 inch.
   $$n = (2.576 \cdot 0.20/0.15)^2 = 12$$

   (b) Repeat part (a) using a standard deviation of 0.10 inch. Which standard deviation requires a larger sample size? Explain.
   $$n = (2.576 \cdot 0.10/0.15)^2 = 3$$ **larger standard deviation increases sample size.**

   (c) Repeat part (a) using a confidence level of 95%. Which level of confidence requires a larger sample size? Explain.
   $$n = (1.96 \cdot 0.20/0.15)^2 = 7$$ **Increase in confidence increase sample size.**

9. If all other quantities remain the same, how does the indicated change affect the minimum sample size requirement (Increase, Decrease or No Change)?

   (a) Increase in the level of confidence - **Increase**

   (b) Increase in the error tolerance - **Decrease**

   (c) Increase in the standard deviation - **Increase**

10. **Stressful Travel:** In a survey of 3224 U.S. adults, 1515 said flying is the most stressful form of travel. Construct a 95% confidence interval for the proportion of all adults who say flying is the most stressful form of travel.

$$0.470 \pm 1.96\sqrt{(0.470)(0.530)/3224} \rightarrow 0.470 \pm 0.017 \rightarrow (0.453, 0.487)$$

11. **Accidents and Alcohol:** A study of 2008 traffic fatalities found that 800 of the fatalities were alcohol related. Find a 99% confidence interval for the population proportion and explain what it means.

$$0.398 \pm 2.576\sqrt{(0.398)(0.602)/2008} \rightarrow 0.398 \pm 0.028 \rightarrow (0.370,\ 0.426)$$ **We are 99% sure that percentage of alcohol related accidents is between 37.0% and 42.6%**

12. *Happy at Work?* In a survey of 1003 U.S. adults, 662 would be happy spending the rest of their career with their current employer. Construct a 90% confidence interval for the proportion who would be happy staying with their current employer. Does this result surprise you?

$$0.660 \pm 1.645\sqrt{(0.660)(0.340)/1003} \rightarrow 0.660 \pm 0.025 \rightarrow (0.635,\ 0.68.5)$$ **I thought it would be lower.**

13. *Computer Repairs* You wish to estimate, with 95% confidence and within 3.5% of the true population, the proportion of computers that need repairs or have problems by the time the product is three years old
    a. No preliminary estimate is available. Find the minimum sample size needed.
    $$n = (.5)(.5)(1.96/0.035)^2 = 784$$

    b. Find the minimum sample size needed, using a prior study that found that 19% of computers needed repairs or had problems by the time the product was three years old.
    $$n = (.19)(.81)(1.96/0.035)^2 = 483$$

    c. Compare the results from parts (a) and (b).
    much smaller sample size needed if p is close to .19

14. Here is an excerpt from an article by CBS News regarding racial tensions in the United States:

    Poll: One year after Charlottesville, majority of Americans see racial tensions on the rise
    One year after the deadly rally in Charlottesville, Virginia, most Americans feel racial tensions have grown over the past year. A new CBS News poll conducted by YouGov finds 61 percent of Americans say that racial tensions have increased over the past year. Majorities of whites, blacks and Hispanics feel this way, but blacks are especially likely to think so: 78 percent feel tensions have increased.

    The CBS News 2018 Battleground Tracker is a series of panel studies in the U.S., This national poll was conducted by YouGov using a nationally representative sample of 2,238 U.S. adults between August 8-10, 2018. The margin of error based upon the entire sample is approximately 2.5 percent.

    d. What would the point estimator be for the proportion of American adults who feel racial tensions have grown over the past year?
    p = 0.61

    e. What is the sample size for this poll?
    n=2238

    f. What is the **margin of error** for this poll as reported in the article. Assuming a 95% level of confidence, **verify** this poll by calculation.

    The reported margin of error is plus or minus 2.5% (0.025).

    Calculated confidence interval: $$0.61 \pm 1.96 \cdot \sqrt{\frac{(0.61)(1-0.61)}{2238}} = 0.61 \pm 0.020 = (0.59,\ 0.63)$$

    So the reported margin of error is higher, probably due to other factors, not sampling error.