

## ANOVA - Tukey's HSD Test

**Application:** One-way ANOVA – pair-wise comparison of means.

**Requirements:** Model must be balanced, which means that the sample size in each population should be the same. The samples taken in each population are called **replicates**. Each population is called a **treatment**. (Note: There are methods of approximating this model if the design is not balanced, but we will not cover them.)

**Tests:**  $H_o : \mu_i = \mu_j$        $H_a : \mu_i \neq \mu_j$  where the subscripts  $i$  and  $j$  represent two different populations

**Overall significance** level of  $\alpha$ . This means that **all pairwise tests** can be run at the same time with an overall significance level of  $\alpha$ .

**Test Statistic:**  $HSD = q \sqrt{\frac{MSE}{n_c}}$

q = value from attached studentized range table

MSE = Mean Square Error from ANOVA table

$n_c$  = number of replicates per treatment

**Decision:** Reject  $H_o$  if  $|\bar{X}_i - \bar{X}_j| > HSD$

### Example:

Valencia oranges were tested for juiciness at 4 different orchards. Eight oranges were sampled from each orchard, and the total ml of juice per 20 gms of orange was calculated:

<b>Orchard A:</b>	<b>Orchard B:</b>	<b>Orchard C:</b>	<b>Orchard D:</b>
11,13,12,14, 9,13,11,9	10,9,8,10, 11,12,7,8	13,15,14,11, 12,10,16,11	9,7,11,9, 9,11,10,8
SS Total =158.469		SS Between=69.594	

- Test for a difference in Orchards using alpha = .05
- Perform all the pairwise comparisons using Tukey's Test and an overall risk level of 5%.

One factor ANOVA

Mean	n	Std. Dev	
11.5	8	1.85	Orchard A
9.4	8	1.69	Orchard B
12.8	8	2.12	Orchard C
9.3	8	1.39	Orchard D
10.7	32	2.26	Total

ANOVA table

Source	SS	df	MS	F	p-value
Treatment	69.594	3	23.198	7.31	.0009
Error	88.875	28	3.174		
Total	158.469	31			

Post hoc analysis

p-values for pairwise t-tests

	Orchard D	Orchard B	Orchard A	Orchard C
	9.3	9.4	11.5	12.8
Orchard D	9.3			
Orchard B	9.4	.8894		
Orchard A	11.5	.0175	.0241	
Orchard C	12.8	.0005	.0007	.1715

Tukey simultaneous comparison t-values (d.f. = 28)

	Orchard D	Orchard B	Orchard A	Orchard C
	9.3	9.4	11.5	12.8
Orchard D	9.3			
Orchard B	9.4	0.14		
Orchard A	11.5	2.53	2.39	
Orchard C	12.8	3.93	3.79	1.40

critical values for experimentwise error rate:

0.05	2.73
0.01	3.42





