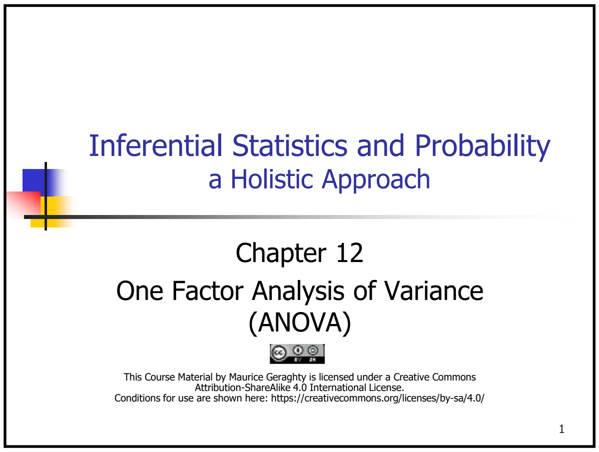
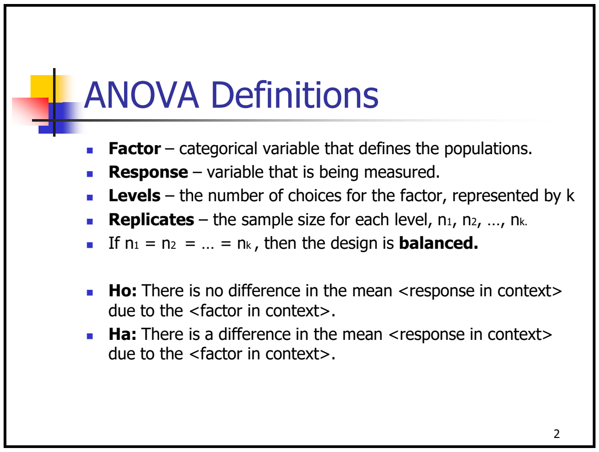


Inferential Statistics and Probability
a Holistic Approach

Chapter 12

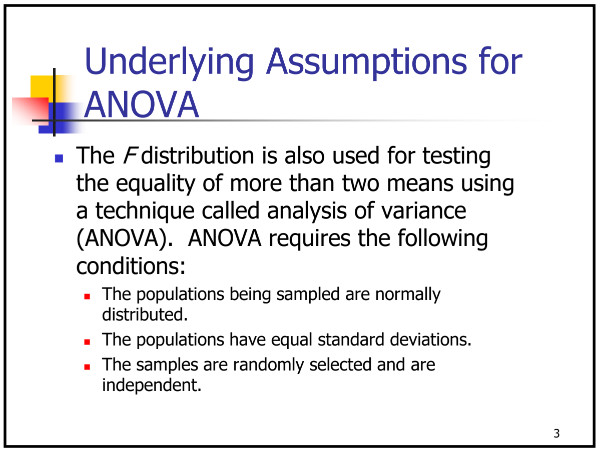
One Factor Analysis of Variance (ANOVA)



ANOVA Definitions

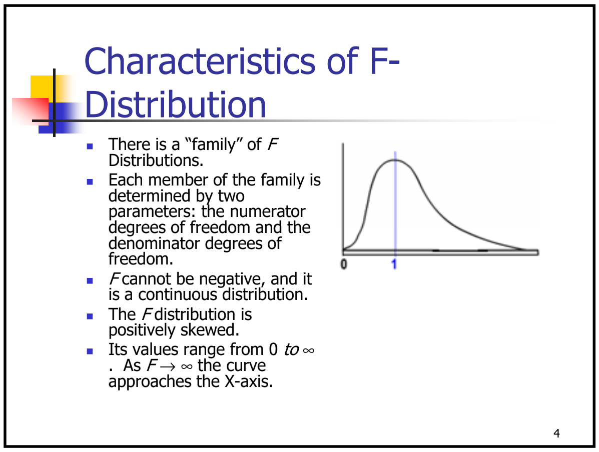
- **Factor** – categorical variable that defines the populations.
- **Response** – variable that is being measured.
- **Levels** – the number of choices for the factor, represented by k
- **Replicates** – the sample size for each level, n_1, n_2, …, n_k.
- If $n_1 = n_2 = … = n_k$, then the design is **balanced.**

- **Ho:** There is no difference in the mean <response in context> due to the <factor in context>.
- **Ha:** There is a difference in the mean <response in context> due to the <factor in context>.



Underlying Assumptions for ANOVA

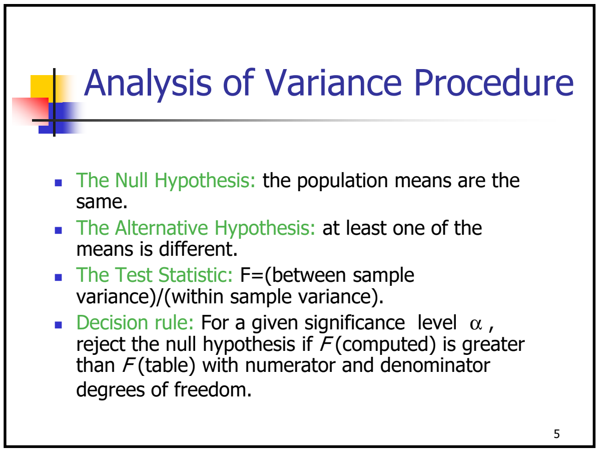
- The *F* distribution is also used for testing the equality of more than two means using a technique called analysis of variance (ANOVA). ANOVA requires the following conditions:
  - The populations being sampled are normally distributed.
  - The populations have equal standard deviations.
  - The samples are randomly selected and are independent.



Characteristics of F-Distribution

- There is a "family" of *F* Distributions.
- Each member of the family is determined by two parameters: the numerator degrees of freedom and the denominator degrees of freedom.
- *F* cannot be negative, and it is a continuous distribution.
- The *F* distribution is positively skewed.
- Its values range from 0 *to* ∞ . As $F \rightarrow \infty$ the curve approaches the X-axis.

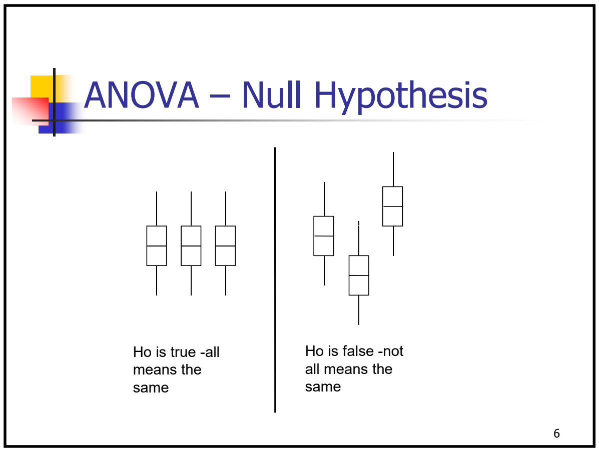
4



Analysis of Variance Procedure

- The Null Hypothesis: the population means are the same.
- The Alternative Hypothesis: at least one of the means is different.
- The Test Statistic: F=(between sample variance)/(within sample variance).
- Decision rule: For a given significance level α , reject the null hypothesis if *F* (computed) is greater than *F* (table) with numerator and denominator degrees of freedom.

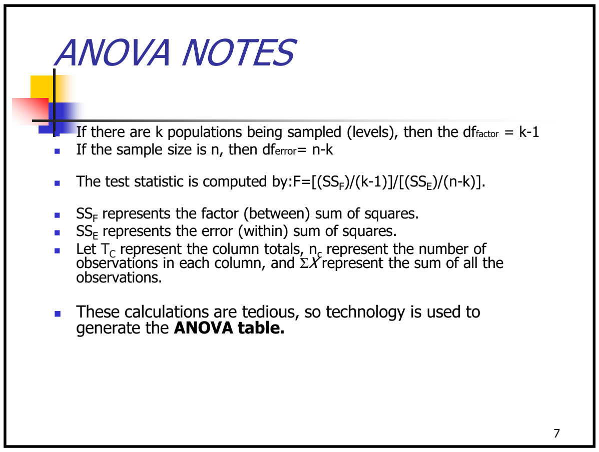
5



ANOVA – Null Hypothesis

Ho is true -all means the same
Ho is false -not all means the same

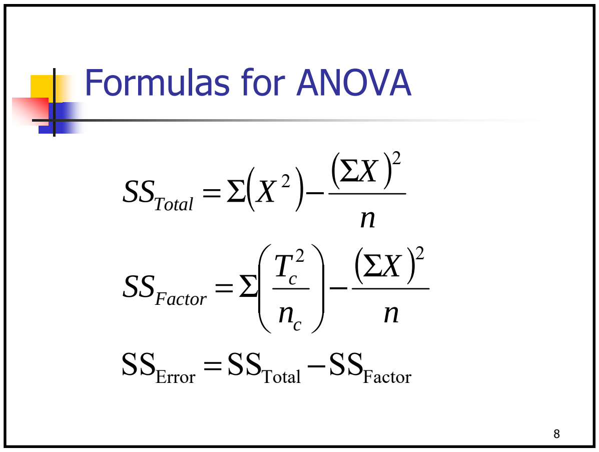
6



ANOVA NOTES

- If there are k populations being sampled (levels), then the df$_{factor}$ = k-1
- If the sample size is n, then df$_{error}$= n-k
- The test statistic is computed by:F=[(SS$_F$)/(k-1)]/[(SS$_E$)/(n-k)].
- SS$_F$ represents the factor (between) sum of squares.
- SS$_E$ represents the error (within) sum of squares.
- Let T$_C$ represent the column totals, n$_c$ represent the number of observations in each column, and ΣX represent the sum of all the observations.
- These calculations are tedious, so technology is used to generate the **ANOVA table.**

7



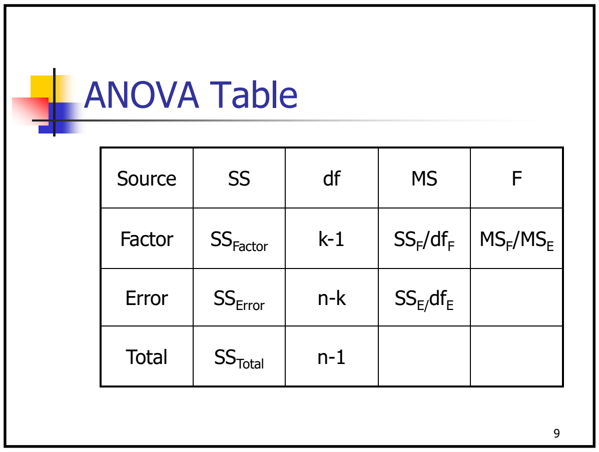
Formulas for ANOVA

$$SS_{Total} = \Sigma\left(X^2\right) - \frac{(\Sigma X)^2}{n}$$

$$SS_{Factor} = \Sigma\left(\frac{T_c^2}{n_c}\right) - \frac{(\Sigma X)^2}{n}$$

$$\mathrm{SS_{Error}} = \mathrm{SS_{Total}} - \mathrm{SS_{Factor}}$$

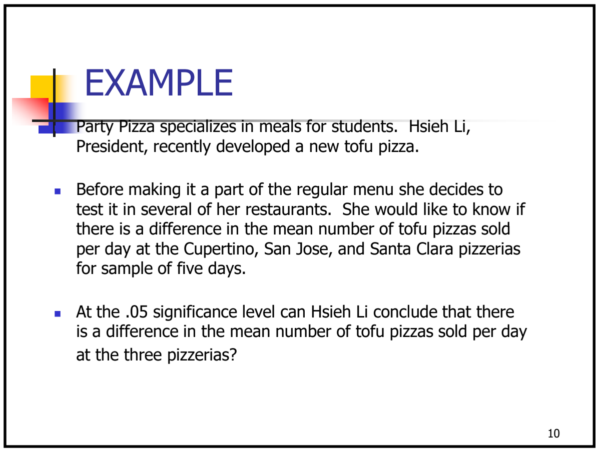
8



ANOVA Table

Source	SS	df	MS	F
Factor	SS$_{Factor}$	k-1	SS$_F$/df$_F$	MS$_F$/MS$_E$
Error	SS$_{Error}$	n-k	SS$_{E/}$df$_E$	
Total	SS$_{Total}$	n-1		

9

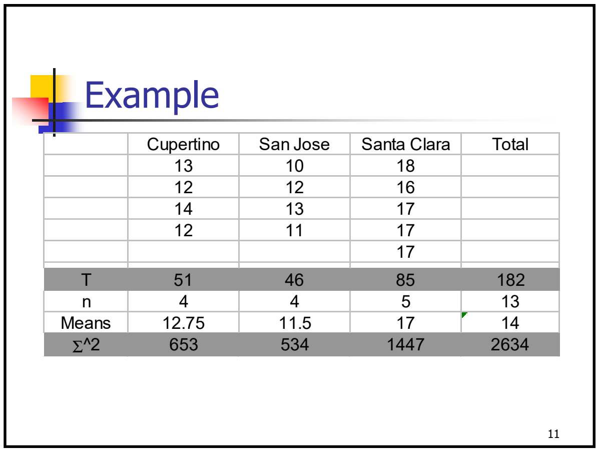


EXAMPLE

Party Pizza specializes in meals for students. Hsieh Li, President, recently developed a new tofu pizza.

- Before making it a part of the regular menu she decides to test it in several of her restaurants. She would like to know if there is a difference in the mean number of tofu pizzas sold per day at the Cupertino, San Jose, and Santa Clara pizzerias for sample of five days.
- At the .05 significance level can Hsieh Li conclude that there is a difference in the mean number of tofu pizzas sold per day at the three pizzerias?

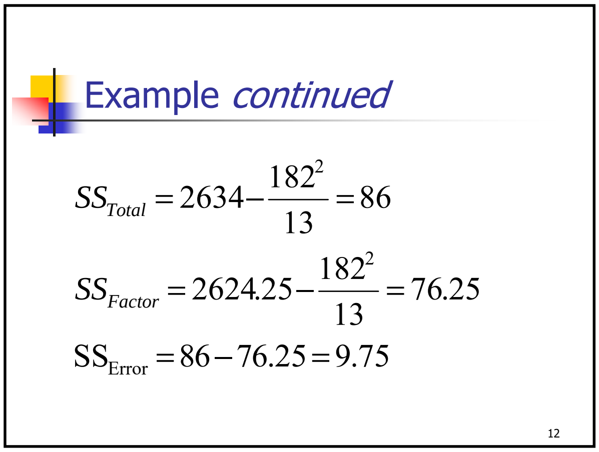
10



Example

	Cupertino	San Jose	Santa Clara	Total
	13	10	18	
	12	12	16	
	14	13	17	
	12	11	17	
			17	
T	51	46	85	182
n	4	4	5	13
Means	12.75	11.5	17	14
Σ^2	653	534	1447	2634

11



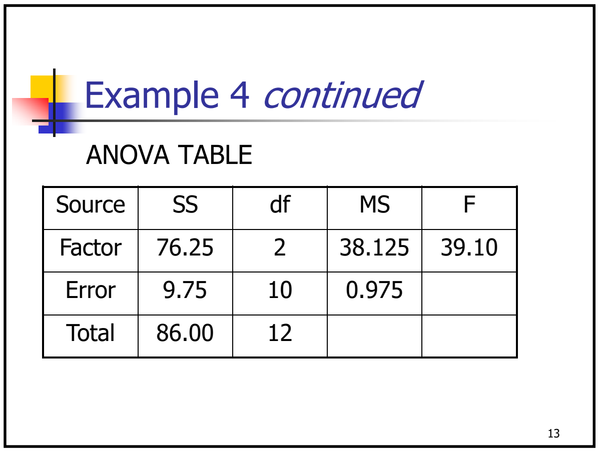
Example *continued*

$$SS_{Total} = 2634 - \frac{182^2}{13} = 86$$

$$SS_{Factor} = 2624.25 - \frac{182^2}{13} = 76.25$$

$$SS_{Error} = 86 - 76.25 = 9.75$$

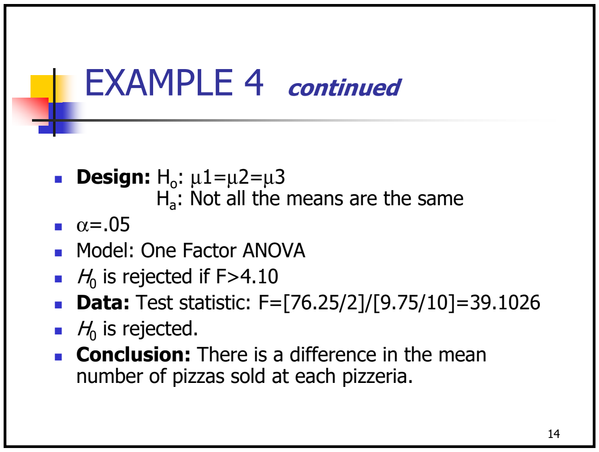
12



Example 4 *continued*

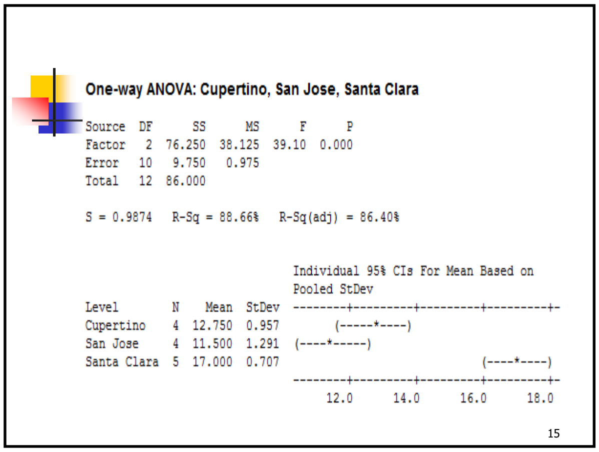
ANOVA TABLE

Source	SS	df	MS	F
Factor	76.25	2	38.125	39.10
Error	9.75	10	0.975	
Total	86.00	12		



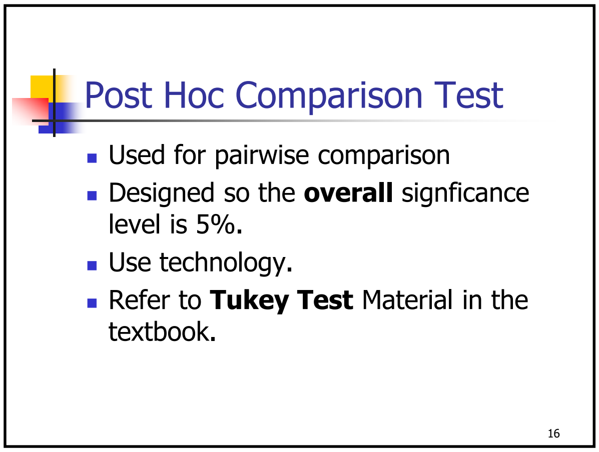
EXAMPLE 4 ***continued***

- **Design:** H_o: $\mu1=\mu2=\mu3$
  H_a: Not all the means are the same
- α=.05
- Model: One Factor ANOVA
- H_0 is rejected if F>4.10
- **Data:** Test statistic: F=[76.25/2]/[9.75/10]=39.1026
- H_0 is rejected.
- **Conclusion:** There is a difference in the mean number of pizzas sold at each pizzeria.



One-way ANOVA: Cupertino, San Jose, Santa Clara

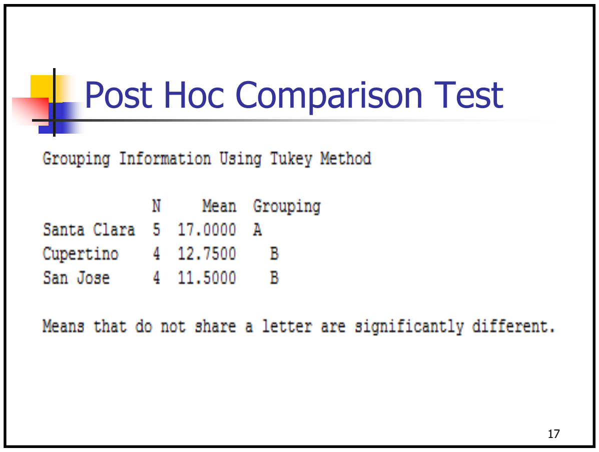
```
Source  DF      SS      MS      F      P
Factor   2  76.250  38.125  39.10  0.000
Error   10   9.750   0.975
Total   12  86.000

S = 0.9874   R-Sq = 88.66%   R-Sq(adj) = 86.40%


                               Individual 95% CIs For Mean Based on
                               Pooled StDev
Level        N    Mean  StDev  --------+---------+---------+---------+-
Cupertino    4  12.750  0.957       (-----*----)
San Jose     4  11.500  1.291  (----*-----)
Santa Clara  5  17.000  0.707                           (----*----)
                               --------+---------+---------+---------+-
                                    12.0      14.0      16.0      18.0
```



Post Hoc Comparison Test

- Used for pairwise comparison
- Designed so the **overall** signficance level is 5%.
- Use technology.
- Refer to **Tukey Test** Material in the textbook.

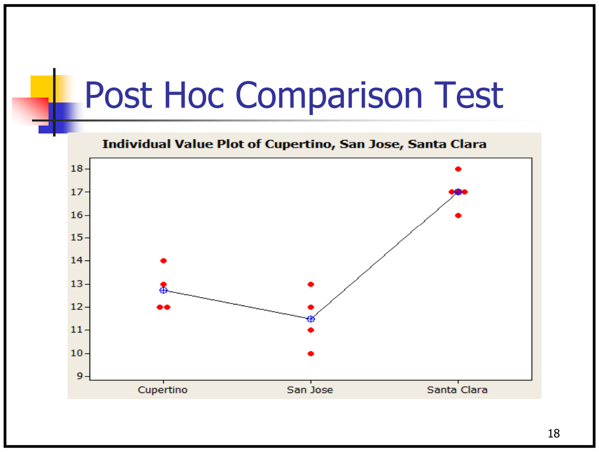
16



Post Hoc Comparison Test

```
Grouping Information Using Tukey Method

             N     Mean  Grouping
Santa Clara  5  17.0000  A
Cupertino    4  12.7500    B
San Jose     4  11.5000    B

Means that do not share a letter are significantly different.
```

17



Post Hoc Comparison Test

18

Example – Oranges & Orchards

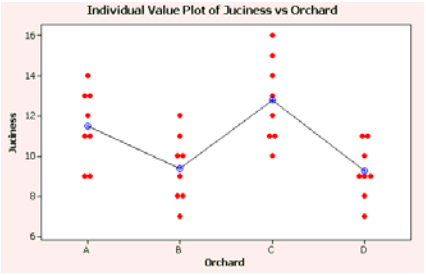
- Valencia oranges were tested for juiciness at 4 different orchards. Eight oranges were sampled from each orchard, and the total ml of juice per 20 gms of orange was calculated.
- Test for a difference in juiciness due to orchards using alpha = .05
- Perform all the pairwise comparisons using Tukey's Test and an overall risk level of 5%.

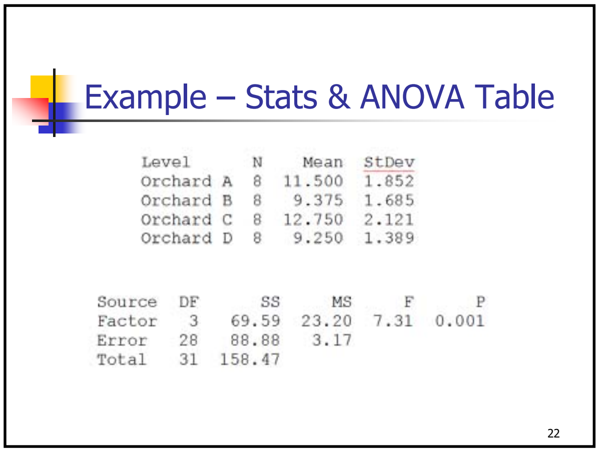
Orchard A:	**Orchard B:**	**Orchard C:**	**Orchard D:**
11,13,12,14, 9,13,11,9	10,9,8,10, 11,12,7,8	13,15,14,11, 12,10,16,11	9,7,11,9, 9,11,10,8

Example - Defintions

- Factor: Orchard (A, B, C or D)
- Response: Juiciness of orange
- Levels: k = 4
- Replicate: $n_A = n_B = n_C = n_D = 8$
- Design: Balanced
- Sample size: n = 8 + 8 + 8 + 8 = 32

Example – Value Plot

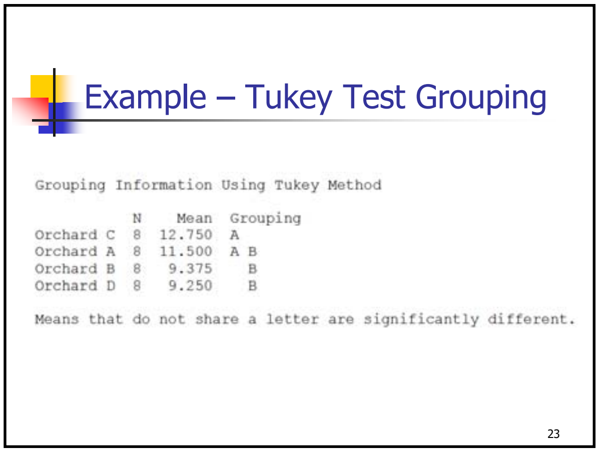




Example – Stats & ANOVA Table

```
Level       N    Mean  StDev
Orchard A   8  11.500  1.852
Orchard B   8   9.375  1.685
Orchard C   8  12.750  2.121
Orchard D   8   9.250  1.389


Source   DF      SS     MS     F      P
Factor    3   69.59  23.20  7.31  0.001
Error    28   88.88   3.17
Total    31  158.47
```

22



Example – Tukey Test Grouping

```
Grouping Information Using Tukey Method

           N    Mean  Grouping
Orchard C  8  12.750  A
Orchard A  8  11.500  A B
Orchard B  8   9.375    B
Orchard D  8   9.250    B

Means that do not share a letter are significantly different.
```

23