Math 10 - Homework 5 ANSWERS

Additional Problems:

1. The average number of years of post secondary education of employees in an industry is 1.5. A company claims that this average is higher for its employees. A random sample of 16 of its employees has an mean of 2.1 years of post secondary education with a standard deviation of 0.6 years.

   a. Find a 95% confidence interval for the mean number years of post secondary education for the company's employees. How does this compare with the industry value?

   \[ 2.1 \pm 2.131(0.6)/\sqrt{16} \rightarrow 2.1 \pm 0.32 \rightarrow (1.78, 2.42) \]  
   The company is well above the industry average.

2. When polling companies report a margin of error, they are referring to a 95% confidence interval. Go to the website www.pollingreport.com and verify the stated margins of error for 2 polls.

   Many answers possible

Constructing Confidence Intervals In Exercises 3 and 4 you are given the sample mean and the sample standard deviation. Assume the random variable is normally distributed and use a t-distribution to construct a 95% confidence interval for the population mean \( \mu \). What is the margin of error of the confidence interval?

3. Repair Costs: Microwaves In a random sample of five microwave ovens, the mean repair cost was $75.00 and the standard deviation was $12.50.

   \[ 75 \pm 2.776(12.50)/\sqrt{5} \rightarrow 75 \pm 15.5 \rightarrow ($59.50, $90.50) \)  
   \( t \) with 4 df

4. Repair Costs: Computers In a random sample of seven computers, the mean repair cost was $100.00 and the standard deviation was $42.50.

   \[ 100 \pm 2.447(42.50)/\sqrt{7} \rightarrow 100 \pm 39.3 \rightarrow ($60.70, $139.30) \)  
   \( t \) with 6 df

5. You did some research on repair costs of microwave ovens and found that the standard deviation is \( \sigma = $15. \) Repeat Exercise 3, using a normal distribution with the appropriate calculations for a standard deviation that is known. Compare the results.

   \[ 75 \pm 1.96 \cdot \frac{15.00}{\sqrt{5}} = $75.00 \pm 13.15 = ($61.85, $88.15) \]  
   \( z \) creates narrower confidence interval

6. Mini-Soccer Balls A soccer ball manufacturer wants to estimate the mean circumference of mini-soccer balls within 0.15 inch. Assume that the population of circumferences is normally distributed.

   a. Determine the minimum sample size required to construct a 99% confidence interval for the population mean. Assume the population standard deviation is 0.20 inch.

      \[ n = \left( \frac{2.576 \cdot 0.20}{0.15} \right)^2 = 12 \]

   b. Repeat part (a) using a standard deviation of 0.10 inch. Which standard deviation requires a larger sample size? Explain.

      \[ n = \left( \frac{2.576 \cdot 0.10}{0.15} \right)^2 = 3 \]  
      larger standard deviation increases sample size.
(c) Repeat part (a) using a confidence level of 95%. Which level of confidence requires a larger sample size? Explain.
\[ n = \left( \frac{1.96 \cdot 0.20}{0.15} \right)^2 = 7 \text{ Increase in confidence increase sample size.} \]

7. If all other quantities remain the same, how does the indicated change affect the minimum sample size requirement (Increase, Decrease or No Change)?

(a) Increase in the level of confidence - Increase

(b) Increase in the error tolerance - Decrease

(c) Increase in the standard deviation - Increase

8. **Stressful Travel:** In a survey of 3224 U.S. adults, 1515 said flying is the most stressful form of travel. Construct a 95% confidence interval for the proportion of all adults who say flying is the most stressful form of travel.
\[ 0.470 \pm 1.96\sqrt{\frac{(0.470)(0.530)}{3224}} \rightarrow 0.470 \pm 0.017 \rightarrow (0.453, 0.487) \]

9. **Accidents and Alcohol:** A study of 2008 traffic fatalities found that 800 of the fatalities were alcohol related. Find a 99% confidence interval for the population proportion and explain what it means.
\[ 0.398 \pm 2.576\sqrt{\frac{(0.398)(0.602)}{2008}} \rightarrow 0.398 \pm 0.028 \rightarrow (0.370, 0.426) \text{ We are 99% sure that percentage of alcohol related accidents is between 37.0% and 42.6%} \]

10. **Happy at Work?** In a survey of 1003 U.S. adults, 662 would be happy spending the rest of their career with their current employer. Construct a 90% confidence interval for the proportion who would be happy staying with their current employer. Does this result surprise you?
\[ 0.660 \pm 1.645\sqrt{\frac{(0.660)(0.340)}{1003}} \rightarrow 0.660 \pm 0.025 \rightarrow (0.635, 0.685) \text{ I thought it would be lower.} \]

11. **Computer Repairs** You wish to estimate, with 95% confidence and within 3.5% of the true population, the proportion of computers that need repairs or have problems by the time the product is three years old
   a. No preliminary estimate is available. Find the minimum sample size needed.
   \[ n = (.5)(.5)(1.96 / 0.035)^2 = 784 \]

   b. Find the minimum sample size needed, using a prior study that found that 19% of computers needed repairs or had problems by the time the product was three years old.
   \[ n = (.19)(.81)(1.96 / 0.035)^2 = 483 \]

   c. Compare the results from parts (a) and (b). much smaller sample size needed if \( p \) is close to .19
12. Lawn Mower A lawn mower manufacturer is trying to determine the standard deviation of the life of one of its lawn mower models. To do this, it randomly selects 12 lawn mowers that were sold several years ago and finds that the sample standard deviation is 3.25 years. Use a 99% level of confidence.

\[ df = 11 \quad \frac{\alpha}{2} = .005 \quad \chi^2_L = 2.603 \quad \chi^2_R = 26.757 \]

99% CI for \( \sigma \): \[ \left( \frac{(11)(3.25^2)}{26.757}, \frac{(11)(3.25^2)}{2.603} \right) = (2.08, 6.68) \]

13. Monthly Income The monthly incomes of 20 randomly selected individuals who have recently graduated with a bachelor's degree in social science have a sample standard deviation of $107. Use a 95% level of confidence.

\[ df = 19 \quad \frac{\alpha}{2} = .025 \quad \chi^2_L = 8.907 \quad \chi^2_R = 32.852 \]

95% CI for \( \sigma \): \[ \left( \frac{(19)(107^2)}{32.852}, \frac{(19)(107^2)}{8.907} \right) = (81.37, 156.28) \]

Read the attached article on the CBS News poll regarding the birth control pill.

a. What would the point estimator be for the proportion of adults who believe the pill has made women’s lives better.

\[ \hat{p} = 56\% = 0.56 \]

b. What is the sample size for this study?

\[ n = 591 \]

c. What is the margin of error for this poll as reported in the article. Assuming a 95% level of confidence, verify this poll by calculation.

The reported margin of error is plus or minus 4% (0.04).

Calculated confidence interval: \[ 0.56 \pm 1.96 \cdot \sqrt{\frac{(0.56)(1 - 0.56)}{591}} = 0.56 \pm 0.040 = (0.52, 0.60) \]

So the reported margin of error is verified.
15. What are the two types of hypotheses used in a hypothesis test? How are they related?

**Ho:** Null Hypotheses – A statement about a population parameter that is assumed to be true for the purposes of testing

**Ha:** Alternative Hypothesis - A statement about a population parameter that is assumed to be true if the Null Hypothesis is rejected during testing.

These two Hypotheses are complements of each other.

16. Describe the two types of error possible in a hypothesis test decision.

Type I error: Rejecting a true Ho
Type II error: Failing to reject a false Ho

**True or False?**
In Exercises 17-22, determine whether each statement is true or false. If it is false, rewrite it as a true statement.

17. In a hypothesis test, you assume the alternative hypothesis is true. **False, you assume the Null Hypothesis is true.**

18. A statistical hypothesis is a statement about a sample. **False, it is a statement about a population parameter.**

19. If you decide to reject the null hypothesis, you can support the alternative hypothesis. **True**

20. The level of significance is the maximum probability you allow for rejecting a null hypothesis when it is actually true. **True**

21. A large P-value in a test will favor a rejection of the null hypothesis. **False, a small p-value supports rejecting the null hypothesis.**

22. If you want to support a claim, write it as your null hypothesis. **False, to support a claim write it as the alternative hypothesis.**

**Stating Hypotheses**
In Exercises 23-28, use the given statement to represent a claim. Write its complement and state which is Ho and which is Ha.

23. Ha: \( p > .65 \)  \( \text{Ho: } p \leq .65 \)

24. Ho: \( \mu \leq 128 \)  \( \text{Ha: } \mu > 128 \)

25. Ha: \( \sigma^2 \neq 5 \)  \( \text{Ho: } \sigma^2 = 5 \)

26. Ho: \( \mu = 1.2 \)  \( \text{Ha: } \mu \neq 1.2 \)

27. Ho: \( p \geq 0.45 \)  \( \text{Ha: } p < 0.45 \)

28. Ha: \( \sigma < 0.21 \)  \( \text{Ho: } \sigma \geq 0.21 \)
Think about the context of the claim. Determine whether you want to support or reject the claim.

a. State the null and alternative hypotheses in words.

b. Write the null and alternative hypotheses in appropriate symbols

c. Describe in words Type I error (the consequence of rejecting a true null hypothesis.)

d. Describe in words Type II error (the consequence of failing to reject a false null hypothesis.)

29. You represent a chemical company that is being sued for paint damage to automobiles. You want to support the claim that the mean repair cost per automobile is not $650. How would you write the null and alternative hypotheses?

Ho: $\mu=650$ (cost is $650)$    Ha: $\mu \neq 650$ (Cost is not $650$)

Type I Error – Claim cost is not $650$, when it actually is $650$
Type II Error – Cost is not $650$, but fail to reject the claim that is $650$.

30. You are on a research team that is investigating the mean temperature of adult humans. The commonly accepted claim is that the mean temperature is about 98.6°F. You want to show that this claim is false. How would you write the null and alternative hypotheses?

Ho: $\mu=98.6$ (Normal Temp is 98.6°F)   Ha: $\mu \neq 98.6$ (Normal Temp is not 98.6°F)

Type I Error – Claim normal temperature is not 98.6F, when it actually is 98.6F
Type II Error – Normal Temperature is not 98.6F, but fail to detect that.

31. A light bulb manufacturer claims that the mean life of a certain type of light bulb is at least 750 hours. You are skeptical of this claim and want to refute it.

Ho: $\mu \geq 750$ (Bulbs last at least 750 hours)  Ha: $\mu < 750$ (Bulbs last less than 750 hours)

Type I Error – Incorrectly claim light bulbs last less than 750 hours
Type II Error – Fail to detect that light bulbs last less than 750 hours

32. As stated by a company’s shipping department, the number of shipping errors per million shipments has a standard deviation that is less than 3. Can you support this claim?

Ho: $\sigma \geq 3$ (Standard Deviation of shipping errors is at 3)  Ha: $\sigma < 3$ (Standard Deviation of errors is under 3)

Type I Error – Incorrectly claim Std Dev of Shipping errors is under 3
Type II Error – Fail to detect that Std Dev of errors is under 3

33. A research organization reports that 33% of the residents in Ann Arbor, Michigan are college students. You want to reject this claim.

Ho: $p=0.33$ (33% of residents are college students)   Ha: $p \neq 0.33$ (It’s not true 33% of residents are students)

Type I Error – Incorrectly claim percentage of residents who are college students is not 33%
Type II Error – Fail to detect that residents who are college students is not 33%

34. The results of a recent study show that the proportion of people in the western United States who use seat belts when riding in a car or truck is under 84%. You want to support this claim.

Ho: $p \geq 0.84$ (At least 84% of west people use seat belts)   Ha: $p < 0.84$ (Less than 84% use seat belts)

Type I Error – Incorrectly claim less than 84% of people in west use seat belts
Type II Error – Fail to detect that less than 84% of people in west use seat belts