Math 10 – Confidence Intervals for proportion and standard deviation Answers (gw13)

Confidence interval for μ when σ is unknown	$\bar{X} \pm t\frac{s}{\sqrt{n}} \quad df = n-1$
Confidence interval for proportion p	$\hat{p} \pm Z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
Determine sample size for given Margin of Error E	$p(1-p)\left(\frac{Z}{E}\right)^2$ use p=0.5 when no estimate for p

1. ***Microwave Ovens:*** In a random sample of 5 microwave ovens, the mean repair cost was \$75.00 and the sample standard deviation was \$12.50. Assume the random variable is normally distributed and use a **t-distribution** to construct a 95% confidence interval for the population mean μ. What is the margin of error of the confidence interval?

   Use t with df=4

   $$75 \pm 2.776 \cdot \frac{12.50}{\sqrt{5}} = \$75.00 \pm \$15.51 = (\$59.49, \$90.51)$$

   Margin of Error is \$15.51

2. ***Stressful Travel:*** In a survey of 3,224 U.S. adults, 1515 said flying is the most stressful form of travel. Construct and interpret a 95% confidence interval for the proportion of all adults who say flying is the most stressful form of travel.

   $$\hat{p} = \frac{1515}{3224} = 0.470$$

   $$0.470 \pm 1.96 \cdot \sqrt{\frac{(0.470)(1-0.470)}{3224}} = 0.470 \pm 0.016 = (0.454, 0.486)$$

3. ***Computer Repairs*** You wish to estimate, with 95% confidence and within 3.5% of the true population, the proportion of computers that need repairs or have problems by the time the product is three years old

   a. No preliminary estimate is available. Find the minimum sample size needed.

   $$n = (.5)(1-.5)\left(\frac{1.96}{.035}\right)^2 = 784$$

   b. Find the minimum sample size needed, using a prior study that found that 19% of computers needed repairs or had problems by the time the product was three years old.

   $$n = (.19)(1-.19)\left(\frac{1.96}{.035}\right)^2 = 483$$

   c. Compare the results from parts (a) and (b).

   Using p =.5 requires a larger sample to get the needed margin of error.

4. Here is an excerpt from an article by CBS News regarding racial tensions in the United States:

Poll: One year after Charlottesville, majority of Americans see racial tensions on the rise
One year after the deadly rally in Charlottesville, Virginia, most Americans feel racial tensions have grown over the past year. A new CBS News poll conducted by YouGov finds 61 percent of Americans say that racial tensions have increased over the past year. Majorities of whites, blacks and Hispanics feel this way, but blacks are especially likely to think so: 78 percent feel tensions have increased.

The CBS News 2018 Battleground Tracker is a series of panel studies in the U.S., This national poll was conducted by YouGov using a nationally representative sample of 2,238 U.S. adults between August 8-10, 2018. The margin of error based upon the entire sample is approximately 2.5 percent.

a. What would the point estimator be for the proportion of American adults who feel racial tensions have grown over the past year?
p = 0.61

b. What is the sample size for this poll?
n=2238

c. What is the **margin of error** for this poll as reported in the article. Assuming a 95% level of confidence, **verify** this poll by calculation.

The reported margin of error is plus or minus 2.5% (0.025).

Calculated confidence interval: $0.61 \pm 1.96 \cdot \sqrt{\frac{(0.61)(1-0.61)}{2238}} = 0.61 \pm 0.020 = (0.59, 0.63)$

So the reported margin of error is higher, probably due to other factors, not sampling error.

Confidence interval for variance σ^2 Take square roots of endpoints for CI for Standard Deviation σ	$\left(\frac{(n-1)s^2}{\chi_R^2}, \frac{(n-1)s^2}{\chi_L^2} \right)$

5. *Monthly Income* The monthly incomes of 20 randomly selected individuals who have recently graduated with a bachelor's degree in social science have a sample standard deviation of $107. Use a 95% level of confidence and find **and interpret** a confidence interval for the population standard deviation.

$df = 19 \qquad CL = 95\% \qquad \chi_L^2 = 8.907 \qquad \chi_R^2 = 32.852$

$95\%\ CI\ \ for\ \sigma: \left(\sqrt{\frac{(19)(107^2)}{32.852}}, \sqrt{\frac{(19)(107^2)}{8.907}} \right) = (81.37, 156.28)$

We are 95% confident that the standard deviation of recent bachelor's degree in sociology recipients

is between 81.37 and 156.28