

## Math 10 MPS - Homework 7 - answers

1. The geyser Old Faithful in Yellowstone National Park is claimed to erupt for on average for about three minutes. Thirty-six observations of eruptions of the Old Faithful were recorded (time in minutes)

1.8	1.98	2.37	3.78	4.3	4.53
1.82	2.03	2.82	3.83	4.3	4.55
1.88	2.05	3.13	3.87	4.43	4.6
1.9	2.13	3.27	3.88	4.43	4.6
1.92	2.3	3.65	4.1	4.47	4.63
1.93	2.35	3.7	4.27	4.47	6.13

Sample mean = 3.394 minutes. Sample standard deviation = 1.168 minutes

Test the hypothesis that the mean length of time for an eruption is 3 minutes answering ALL the following questions:

### A. General Question

- a. Why do you think this test is being conducted?

**We are trying to test the claim that Old Faithful erupts for three minutes.**

### B. Design

- a. State the null and alternative hypotheses

**$H_0: \mu = 3$     $H_a: \mu \neq 3$**

- b. What is the appropriate test statistic/model?

**t -Test of mean vs. Hypothesized Value, population standard deviation unknown.**

$$t = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} \quad df = 35$$

- c. What is significance level of the test?

**Not given, so I will choose  $\alpha = 5\%$**

- d. What is the decision rule?

**Critical value method: Reject  $H_0$  if  $t < -1.96$  or  $t > 1.96$**

### C. Power Analysis

Suppose you wanted to conduct a Power analysis if the population mean under  $H_a$  was actually 3.3 minutes. Use the online Power calculator to answer the following questions.

- a. Determine the Power of the test using the online power calculator. **0.3326**
- b. Determine Beta.  **$1 - 0.3326 = 0.6674$**
- c. Determine the sample size needed if you wanted to conduct the test in Question with 95% power.  
 **$n = 199$**

The screenshot shows a web-based power calculator interface with the following fields and values:

- sigma**: Value 1.168
- True  $|\mu - \mu_0|$** : Value .3
- n**: Value 36
- power**: Value .3226
- alpha**: 0.05
- Two-tailed**:

On the right side of the interface, the calculated results are displayed:

- n**: 199
- power**: .9501

D. Conduct the test

- a. Are there any unusual observations that question the integrity of the data or the assumptions of the model?  
**The value 6.13 appears to be an outlier and increases the standard deviation.**
- b. Is the decision to reject or fail to reject  $H_0$ ?

$$t = \frac{3.394 - 3}{1.168 / \sqrt{36}} = 2.02 \quad \text{Reject } H_0$$

- E. Conclusions - State a one paragraph conclusion that is consistent with the decision using language that is clearly understood in the context of the problem. Address any potential problems with the sampling methods and address any further research you would conduct.

**The mean length of time for eruptions of Old Faithful is not three minutes, it appears to be longer. This conclusion is based on 36 measurements taken of Old Faithful.**

**One measurement of 6.13 seems highly unusual and does not seem to fit the data. I would review the data to make sure this result was recorded correctly.**

2. 15 I-pod users were asked how many songs were on their I-pod. Here are the summary statistics of that study:

$$\bar{X} = 650 \quad s = 200$$

Can you support the claim that the population standard deviation is under 300? Conduct the test with  $\alpha = 5\%$ . Show all steps.

**$H_0: \sigma \geq 300$     $H_a: \sigma < 300$**

**Chi square test of standard deviation vs. Hypothesized Value**

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \quad df = 14 \quad \alpha = .05$$

**Reject  $H_0$  if  $\chi^2 < 6.571$  (lower tailed test)**

1.  $\chi^2 = \frac{(14)200^2}{300^2} = 6.22 \rightarrow \text{Reject } H_0$

**The standard deviation is under 300.**

3. Define the following terms – **All answers are in glossary of my Online text.**

4. The drawing shown diagrams a hypothesis test for population mean design under the Null Hypothesis (top drawing) and a specific Alternative Hypothesis (bottom drawing). The sample size for the test is 200.

a. State the Null and Alternative Hypotheses

$$H_0: \mu \geq 8 \quad H_a: \mu < 8$$

b. What are the values of  $\mu_0$  and  $\mu_a$  in this problem?

$$\mu_0 = 8 \quad H_a: \mu_a = 4$$

c. What is the significance level of the test?  $\alpha = .10$

d. What is the Power of the test when the population mean = 4?

$$\text{Power} = .91$$

e. Determine the probability associated with Type I error.

$$\alpha = .10$$

f. Determine the probability associated with Type II error.

$$\beta = .09$$

g. Under the Null Hypothesis, what is the probability the sample mean will be over 6?

$$1 - \alpha = .90$$

h. If the significance level was set at 5%, would the power increase, decrease or stay the same?

**Power would decrease**

i. If the test was conducted, and the p-value was .085, would the decision be Reject or Fail to Reject the Null Hypothesis? **Reject  $H_0$  because  $.085 < .10$**

j. If the sample size was changed to 100, would the shaded on area on the bottom ( $H_a$ ) graph increase, decrease or stay the same? **Increase: Beta would increase when sample size decreases.**

