$$\boxed{f(x) = \frac{x^3}{x^2+1}}$$

A) $\boxed{\text{DOMAIN } (-\infty, \infty)}$

B) $f(0) = 0$ $(0,0)$ is the only intercept

C) $f(-x) = \frac{(-x)^3}{(-x)^2+1} = \frac{-x^3}{x^2+1} = -f(x)$

$\boxed{\text{so } f(x) \text{ is ODD, symmetric w.r.t. origin.}}$

D) $x^2+1 > 0$, so $\boxed{\text{NO VERTICAL ASYMPTOTES}}$

$\lim_{x\to\infty} \frac{x^3}{x^2+1} = \infty$ $\lim_{x\to-\infty} \frac{x^3}{x^2+1} = -\infty$ $\boxed{\text{No horizontal Asymptotes}}$

SLANT ASYMPTOTE

If $\lim_{x\to\infty} [f(x) - mx+b] = 0$, then $y = mx+b$ is a SLANT Asymptote

$$f(x) = \frac{x^3}{x^2+1} = \frac{x^3+(x-x)}{x^2+1}$$

$$= \frac{x^3+x}{x^2+1} - \frac{x}{x^2+1}$$

$$= x - \frac{x}{x^2+1}$$

$f(x) - x = \frac{-x}{x^2+1} \to 0$ as $x \to \pm\infty$

$\boxed{\text{So } y = x \text{ is A SLANT Asymptote}}$

E) $f'(x) = \frac{3x^2(x^2+1) - (x^3)(2x)}{(x^2+1)^2} = \frac{3x^4+3x^2-2x^4}{(x^2+1)^2} = \frac{x^2(x^2+3)}{(x^2+1)^2}$

$f'(x) > 0$ for all x except $f'(0) = 0$

$f(x)$ is an increasing function and

F) $\boxed{\text{there are no local extrema}}$

G) $f'(x) = \dfrac{x^4+3x^2}{(x^2+1)^2}$

$$f''(x) = \frac{(4x^3+6x)(x^2+1)^2 - (x^4+3x^2)\,2(x^2+1)(2x)}{(x^2+1)^4}$$

which simplifies to $f''(x) = \dfrac{2x(3-x^2)}{(x^2+1)^3}$

$(2x)(3-x^2) = 0 \qquad x = 0, \pm\sqrt{3}$. ⇐ points of inflection

	$2x$	$3-x^2$	$(x^2+1)^3$	$f''(x)$	$f(x)$
$(-\infty, -\sqrt{3})$	−	−	+	+	concave up
$(-\sqrt{3}, 0)$	−	+	+	−	concave down
$(0, \sqrt{3})$	+	+	+	+	concave up
$(\sqrt{3}, \infty)$	+	−	+	−	concave down

H.)