Introduction to Linear Functions: Part I

A **linear function** is a relation between two variables, x and y, known respectively as independent and dependent variable, which can be written in the form $y = a + bx$, where a and b are constant real numbers.

1. For each of the following linear functions, identify a and b.

Equation	a	b
$y = -1 + 2x$		
$y = 7.5 - 1.2x$		
$y = 4 - \frac{1}{2}x$		
$y = \frac{3}{4}x$		
$y = -5$		
$y = -5 - \frac{2}{3}x$		

Linear functions can be represented as equations, as done above. But they can also be expressed as a graph. Let's graph a couple of the linear functions given above.

A simple way to graph functions is by making an input-output table and plotting the corresponding points. You can pick any values for x to do this. We generally pick convenient values near 0.

2. Fill out the following tables by substituting the given values of x into the equation for the linear function.

$y = -1 + 2x$

x	y
-2	
-1	
0	
1	
2	

$y = 4 - \frac{1}{2}x$

x	y
-4	
-2	
0	
2	
4	

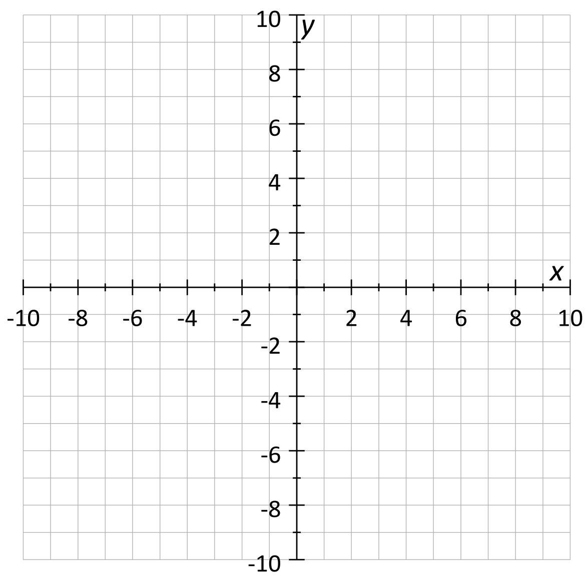
3. Before we graph, look at your tables carefully for patterns. What do you notice?

4. In the first table above, as x increases by 1, what happens to y?

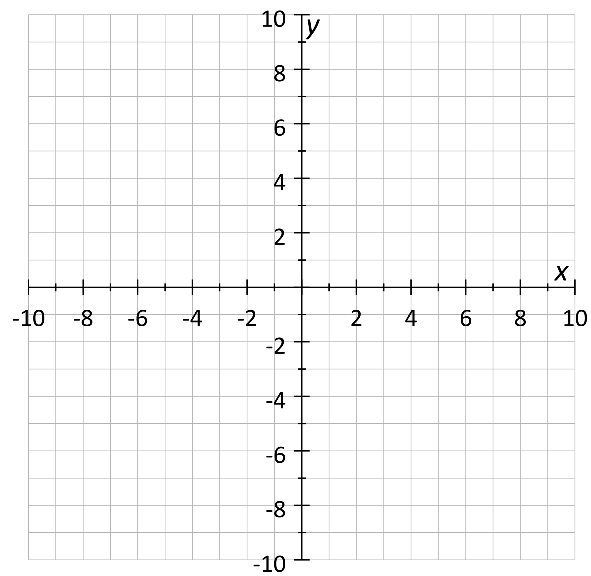
5. In the second table above, as x increases by 1, what happens to y?

6. Graph the points in each table above in the grids given below. Draw a straight line through the points you graphed.

$y = -1 + 2x$



$y = 4 - \frac{1}{2}x$



7. Recall from the table at the beginning of the lesson that

Equation	a	b
$y = -1 + 2x$	-1	2
$y = 4 - \frac{1}{2}x$	4	$-\frac{1}{2}$

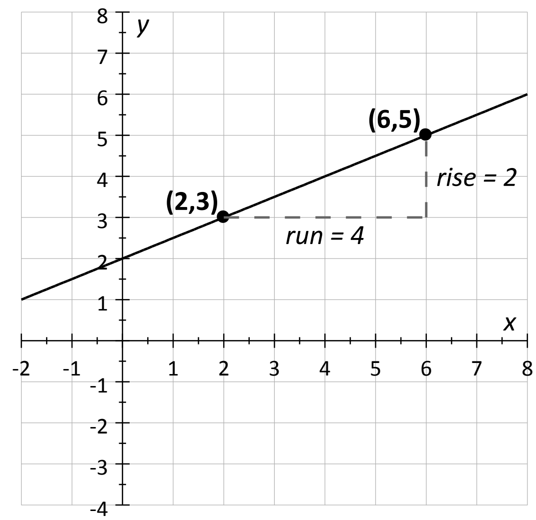
Look at the value of a for each equation. How does this value show up in the graph of the function?

You need to know: The ***y*-intercept** of a graph is the value of y where the graph touches the y-axis. This occurs when $x = 0$. In the graph of $y = a + bx$, the y-intercept is a.

8. Look at the value of b for each equation. How does this value show up in the graph of the function? HINT: Look also at your answer to #3, #4 and #5.

You need to know: In the graph of $y = a + bx$, the number b represents the steepness of the line called the **slope**. The slope of a line is the rate of change in y with respect to x.

The **slope** of a line can be thought of in many different ways. One way, is as a ratio of “rise” over “run”. Looking at the diagram below, to get from one point (2, 3) to another point on the line (6, 5) we have a rise of 2 units and a run of 4 units.



This means that for this line, the slope would be:

$$slope = \frac{rise}{run} = \frac{2}{4} = \frac{1}{2}$$

Any two points determine a straight line and its slope.

The **slope** of the line that contains the points (x_1, y_1) and (x_2, y_2) is:

$$slope = \frac{rise}{run} = \frac{change\ in\ y}{change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_2 \neq x_1.$$

9. For the line above, what are the points, (x_1, y_1), and (x_2, y_2)?

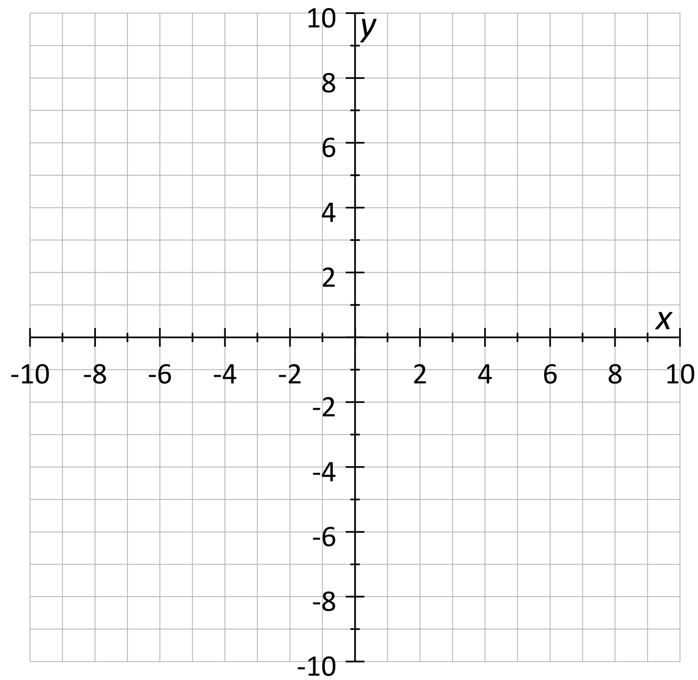
(x_1, y_1) = __________ (x_2, y_2) = __________

10. Use the information in the previous question and the formula given above to find the slope of the line. Did you get the same answer as we got before?

11. Looking at the graph above, you may also notice that the point (0, 2) is on the line. What would you get for the slope if you used the point (0, 2) and one of the other points given, say (6, 5) to find the slope. Would you get the same answer? Check to confirm.

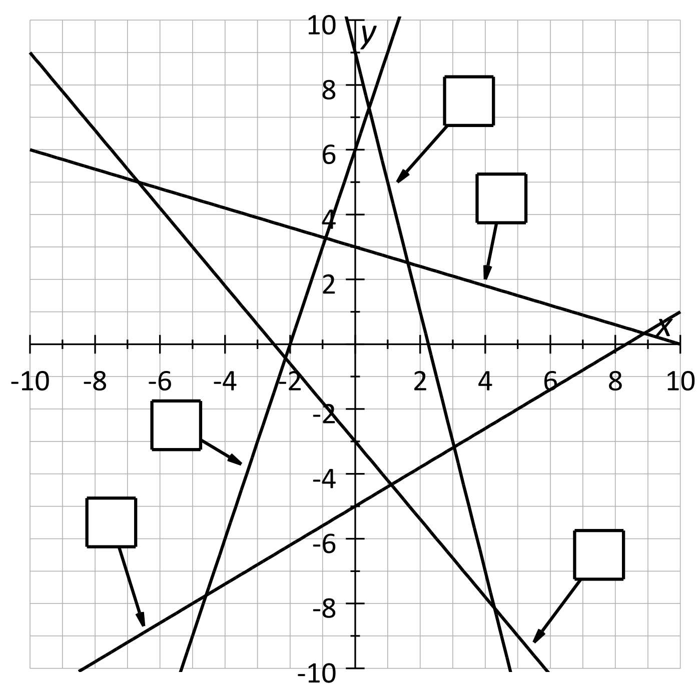
12. Find the slope of the line that passes through the points $(-1,0)$ and $(2,2)$.

Plot the two points to the right. Use the slope of the line to identify and label three other points on the line.



13. Identify the slope of each of the lines given to the right.

HINT: For each line, choose two convenient points and use the slope formula.



You need to know: Notice that the slope is sometimes positive and sometimes negative.

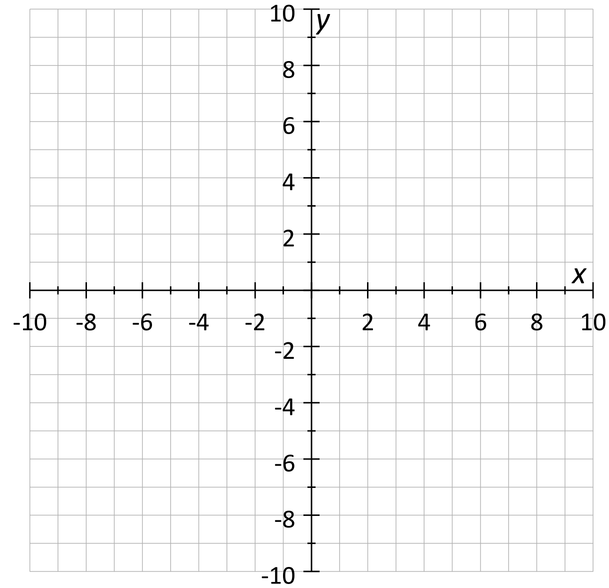
- **Positive** slope corresponds to lines that are **increasing** from left to right.
- **Negative** slope corresponds to lines that are **decreasing** from left to right.

Now let's use the information you just learned about a and b to graph $y = a + bx$.

14. Using the information above, state the slope and the y-intercept for the line given by $y = 5 + \frac{2}{3}x$.

    slope = ____________ y-intercept = ____________

    Once we have this information, we can use it to graph the line corresponding to this equation. Plot the y-intercept on the coordinate axes to the right.



Remember that slope is a ratio of *rise* over *run*. Notice that the slope of our line is b = 2/3. This means that we can think of the rise as 2 and the run as 3. Starting from the y-intercept, move up 2 units and to the right 3 units. Mark another point at this location. This point is also on the line we are graphing. Now that you have two points on the line, draw the line that contains them, extending beyond the two points. Put arrows at both ends of the line.

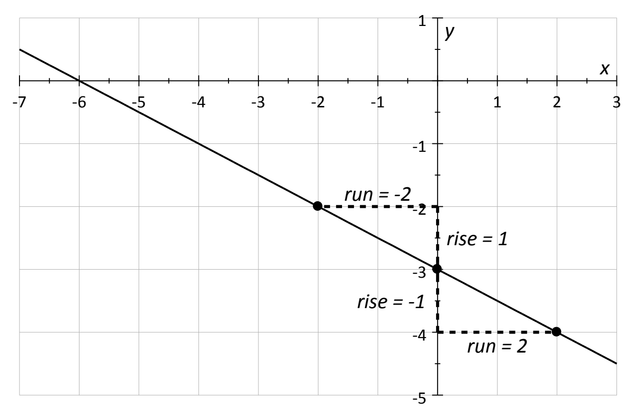
You have just graphed this line using the **slope-intercept method**.

15. Let's look at the line given by $y = -3 - \frac{1}{2}x$. State the slope and y-intercept for the line above.

    slope = ____________ y-intercept = ____________

    This line is different from the last one in that the slope is negative. We can think of the slope in a few different ways:

    $$-\frac{1}{2} = \frac{-1}{2} = \frac{1}{-2}.$$

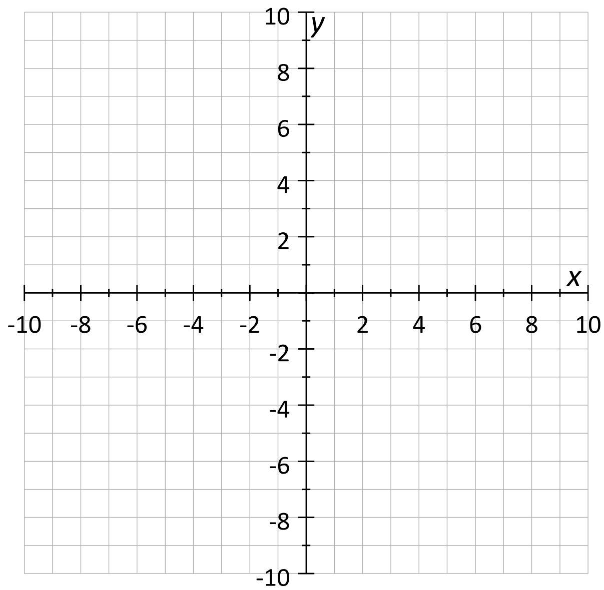


The first way is how we often write negative fractions. The second and third ways are easier when graphing lines. Notice that a slope of $\frac{-1}{2}$ means you have a *rise* of -1 (or you are going down one unit) and a *run* of 2 (meaning you are moving to the right 2 units). Notice from the graph that this gives another point on the line.

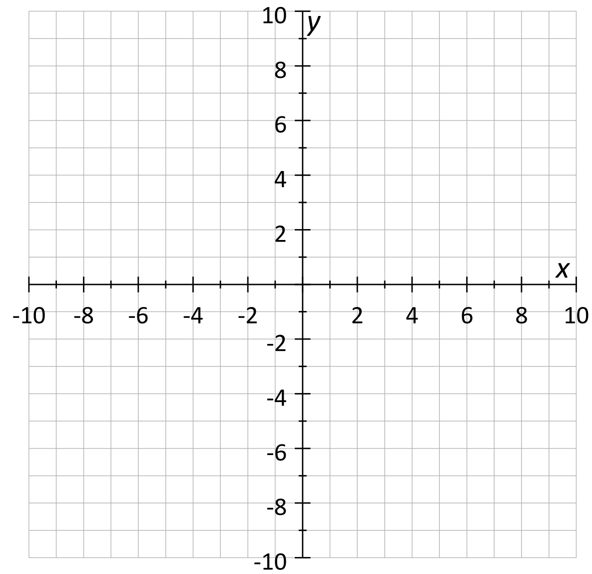
If we express the slope as $\frac{1}{-2}$, then we have a *rise* of 1 (move *up* one unit) and a *run* of -2 (move *left* two units). This provides another point on the line. Although these two interpretations of a negative slope give two different points, they are on the same line, so it does not matter what method we use.

16. Graph the following two lines using the slope-intercept method.

$y = -2 + \frac{4}{3}x$



$y = 3 - \frac{2}{5}x$



17. Consider the linear function represented in the table to the right.

x	y
-2	13
-1	11
0	9
1	7
2	5

a. State the y-intercept.

b. State the slope.

c. Find the equation of the line that the points on the table pass through in the form $y = a + bx$.

18. Find the equation of the line corresponding to each of the tables below.

a.

x	y
2	-12
3	-9
4	-6
5	-3
6	0

$y =$ ______________________________

b.

x	y
-2	8
0	5
2	2
4	-1
6	-4

$y =$ ______________________________

19. We wish to find the equation of the line that passes through the points $(0,5)$ and $(3,7)$.

    a. Find the y-intercept.

    b. Find the slope.

    c. Use your answers to parts a and b to write down the equation of the desired line in the form $y = a + bx$.

20. We wish to find the equation of the line that passes through the points $(-3,2)$ and $(2,-8)$.

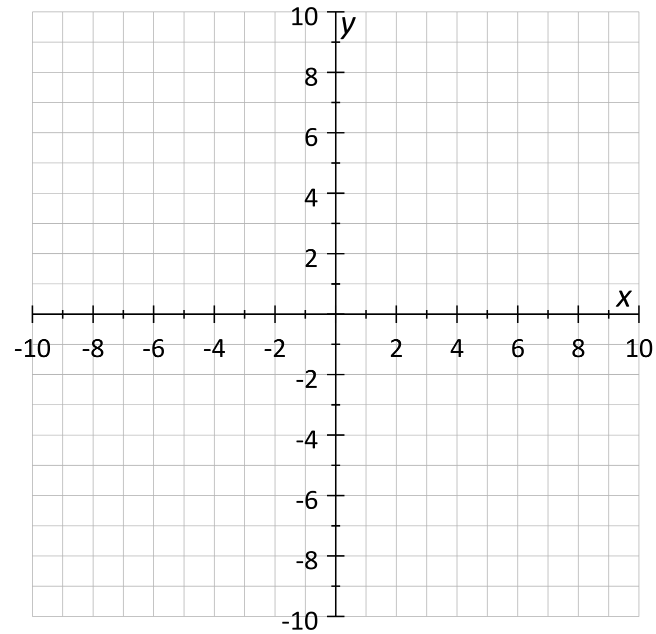
    d. Find the slope.

    b. Substitute the slope and one of the points given into the equation $y = a + bx$. Now solve for a, the y-intercept.

    c. Use your answers to parts a and b to write down the equation of the desired line in the form $y = a + bx$.

21. Consider the line that passes through the points $(4,-2)$ and $(8,3)$.
    a. Find the equation of the line.

b. Plot the two points given. Use your graph to verify that your slope and y-intercept are correct.



c. Use your graph to estimate the value of y when $x = 7$.

d. Now find the value of y when $x = 7$ using the equation you came up with in part a.

e. Use your graph to estimate the value of x when $y = -5$.

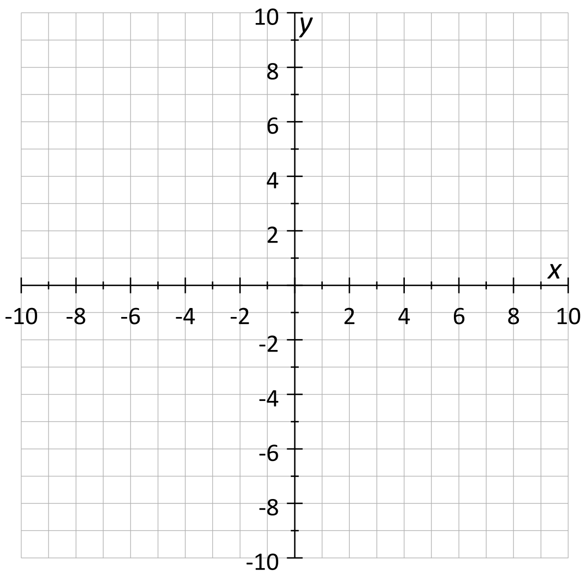
f. Now find the value of x when $y = -5$ using the equation you came up with in part a. HINT: Substitute $y = -5$ into the equation of the line and solve for x.

g. Find the value of x when $y = 20$.

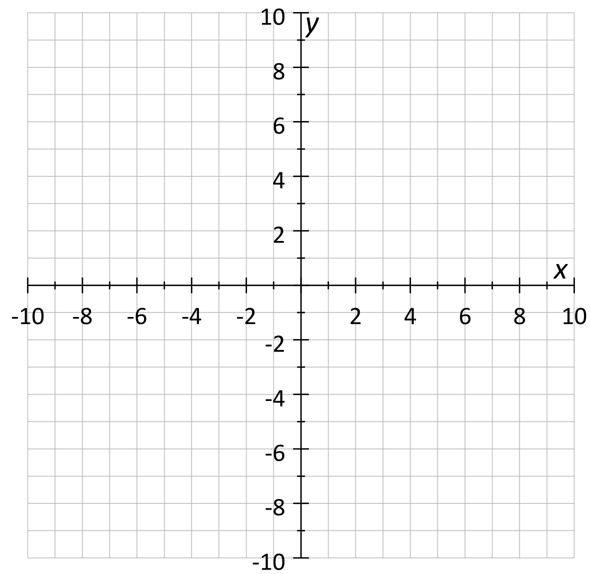
Take It Home: Introduction to Linear Functions: Part I

1. Graph the each of the following lines using the slope-intercept method.

$y = 5 - \frac{2}{3}x$



$y = -2 + 3x$



2. Find the equation of the line from which the following table was built.

x	y
-3	-5
0	-1
3	3
6	7
9	11

3. Find the equation of the line that passes through the points $(0,3)$ and $(3,7)$.

4. Find the equation of the line that passes through the points $(6,3)$ and $(5,5)$.

5. Consider the linear function with equation $y = -5 + 2x$.

   a. Find the value of y when $x = 8$.

   b. Find the value of x when $y = 15$.

6. Consider the linear function with equation $y = 12.5 - 0.7x$.

   a. Find the value of y when $x = 15$.

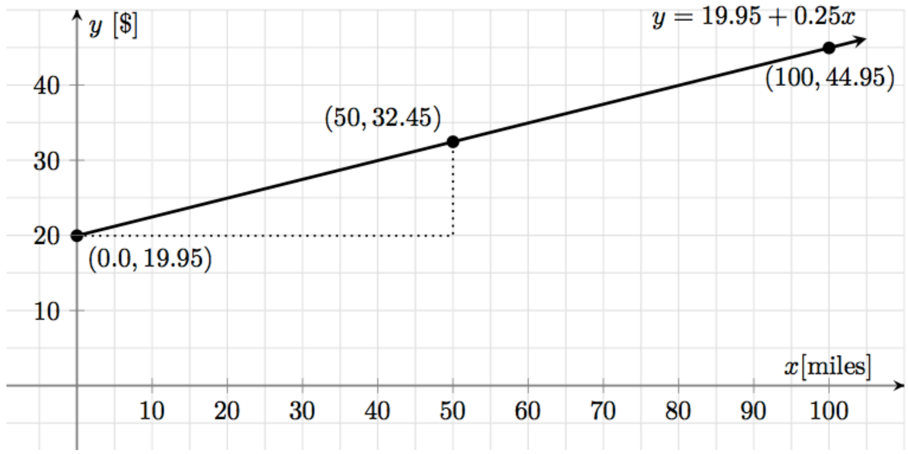
   b. Find the value of x when $y = 8$.

Introduction to Linear Functions: Part II

In this lesson, we focus on linear Functions in real life.

Example 1: You rent a moving truck. It costs $19.95, plus $0.25 per each mile driven. You are interested in the total cost. Presented in tabular and graphical forms, we have:

Miles driven	Total cost
0	$19.95
1	$20.20
2	$20.45
3	$20.70
4	$20.95
50	$32.45
100	$44.95



Let $x =$ the number of miles driven, and $y =$ the total cost of rental

Notice in the table above that:

- $a = y$-intercept = $19.95, which is the value of y when $x = 0$
- $b =$ slope = $0.25/mile
  This is true since the total cost goes up by $0.25 (rise) when the number of miles increase by 1 (run)

Presented as a formula, the relationship between the number of miles and the total cost is given by

$$y = 19.95 + 0.25x$$

Let's do a unit analysis on the equation above. The table below shows the units for each quantity.

y	($)
19.95	($)
0.25	($/mile)
x	(miles)

Looking at just the units in the equation $y = 19.95 + 0.25x$, we have

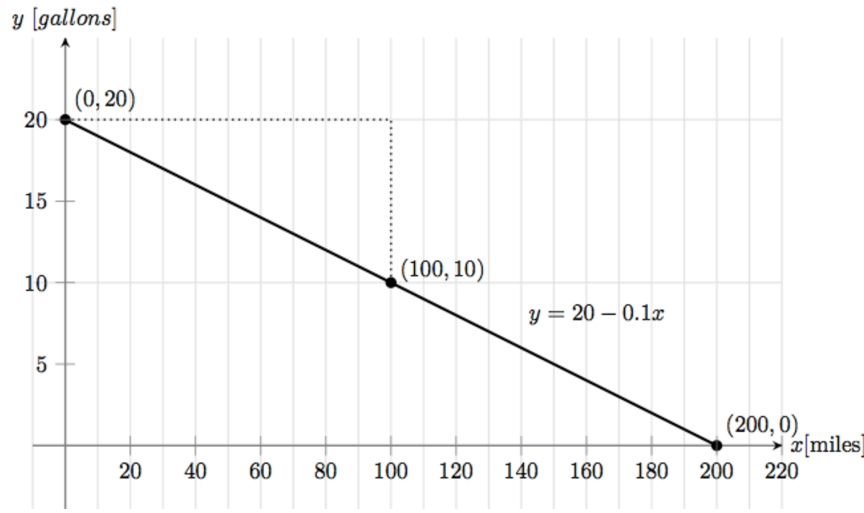
$$\begin{array}{ccccccc} y & = & 19.95 & + & 0.25 & \cdot & x \\ (\$) & = & (\$) & + & \left(\frac{\$}{\text{mile}}\right) & \cdot & (\text{miles}) \end{array}$$

Notice that the units make sense in that we are adding $s to get $s.

Additionally, notice that the units of the slope are always equal to the units of y divided by the units of x.

Example 2: Suppose you have an SUV with a 20-gallon tank that is full. You drive around town, getting 10 miles per gallon. You are interested in the amount of fuel left in the tank as you drive around.
Presented in tabular and graphical forms, we have:

Miles driven	Gallons remaining
0	20
10	19
20	18
30	17
40	16
50	15
100	10



Let $x =$ the number of miles driven, and $y =$ number of gallons of fuel remaining

Notice in the table above that:

- $a = y$-intercept = 20 gallons, which is the value of y when $x = 0$
- $b =$ slope = $\frac{rise}{run} = \frac{-1\ gallon}{10\ miles} = -0.1\frac{gal}{mile}$

Presented as a formula, the relationship between the number of miles driven and the amount of fuel remaining is given by

$$y = 20 - 0.1x$$

Doing unit analysis on the equation, we have

$$\begin{array}{ccccccc} y & = & 20 & - & 0.1 & \cdot & x \\ (\text{gal}) & = & (\text{gal}) & - & \left(\frac{\text{gal}}{\text{mile}}\right) & \cdot & (\text{miles}) \end{array}$$

7. At a community college, students pay \$50 flat, plus \$25 per unit. Let $x =$ the number of units, and $y =$ total amount paid.

   a. State the units of x: ________________ State the units of y: ________________

   b. Find the slope. Include units.

   c. Find the formula for the linear function relating x and y in the form $y = a + bx$.

   d. Perform a unit analysis on the equation in part c.

8. The number of US households that paid bills online in 2006 was 45 million and has increased by 8.7 million per year since then. Let $t =$ the number of years since 2006, and $h =$ number of US households (in millions) that paid bills online.

a. State the units of t: ________________ State the units of h: ________________

b. Find the slope. Include units.

c. Find the formula for the linear function relating t and h in the form $h = a + bt$.

d. Perform a unit analysis on the equation in part c.

9. Atmospheric pressure at sea level is 1 atmosphere (atm). As altitude, distance above sea level, increases, pressure decreases by approximately 0.0303 atm for every 1 foot increase. Let $x =$ the altitude, and $y =$ atmospheric pressure.

a. State the units of x: ________________ State the units of y: ________________

b. Find the slope. Include units.

c. Find the formula for the linear function relating x and y in the form $y = a + bx$.

d. Perform a unit analysis on the equation in part c.

10. The number of city and county ordinances that restrict outdoor smoking has increased (approximately) linearly from 30 ordinances in 1999 to 1124 ordinances in 2007. Let n be the number or ordinances at t years since 1999. NOTE that in 1999, $t = 0$, and in 2007, $t = 8$.

    a. Find the two ordered pairs (t, n) given in the description.

    b. Find the slope. Include units.

    c. Find the n-intercept. Include units.

    d. Find the formula for the linear function relating t and n in the form $n = a + bt$.

    e. Perform a unit analysis on the equation in part d.

11. For Pacific albacore tuna, the mass and mercury concentration are (approximately) linearly related. A 4-kilogram tuna has an average mercury concentration of 0.10 part per million. A 10-kilogram tuna has an average mercury concentration of 0.19 part per million. Let c be the average mercury concentration (in parts per million) of a Pacific albacore tuna whose mass is m kilograms.

    a. Find the two ordered pairs (c, m) given in the description.

    b. Find the slope. Include units.

    c. Find the w-intercept. Include units.

    d. Find the formula for the linear function relating c and m in the form $m = a + bc$.

    e. Perform a unit analysis on the equation in part d.

Math 217 NAME ______________________________________

Take It Home: Introduction to Linear Functions: Part II

1. A person earns a starting salary of \$34000 at a company. Each year, she receives a \$3000 raise. Let s be the salary after she has worked at the company for t years.

   a. State the units of t: ________________ State the units of s: ________________

   b. Find the slope. Include units.

   c. Find the formula for the linear function relating t and s in the form $s = a + bt$.

   d. Perform a unit analysis on the equation in part c.

2. A calling card has \$50 on it. When using the calling card, you pay \$0.20 every minute. Let R be the amount remaining on the calling card after talking for t minutes.

   a. State the units of t: ________________ State the units of R: ________________

   b. Find the slope. Include units.

   c. Find the formula for the linear function relating t and A in the form $R = a + bt$.

   d. Perform a unit analysis on the equation in part c.

3. In the US, we measure temperature in degrees Fahrenheit (°F). In the most of the rest of the world, temperature is measured in degrees Celcius (°C). Water freezes at 32°F, which is also 0°C. Each increase of 9°F corresponds to an increase of 5°C. Let $c =$ temperature in degrees Celcius, and $f =$ temperature in degrees Fahrenheit.

   a. State the units of c: ________________ State the units of f: ________________

   b. Find the slope. Include units.

   c. Find the formula for the linear function relating c and f in the form $f = a + bc$.

   d. Perform a unit analysis on the equation in part c.

4. The number of inmates younger than 18 held in state prisons has decreased (approximately) linearly from 3147 inmates in 2001 to 2266 inmates in 2005. Let n be the number of inmates younger than 18 held in state prisons t years from 2000.

   a. Find the two ordered pairs (t, n) given in the description.

   b. Find the slope. Include units.

   c. Find the n-intercept. Include units.

   d. Find the formula for the linear function relating t and n in the form $n = a + bt$.

   e. Perform a unit analysis on the equation in part d.