Math 217 NAME ____

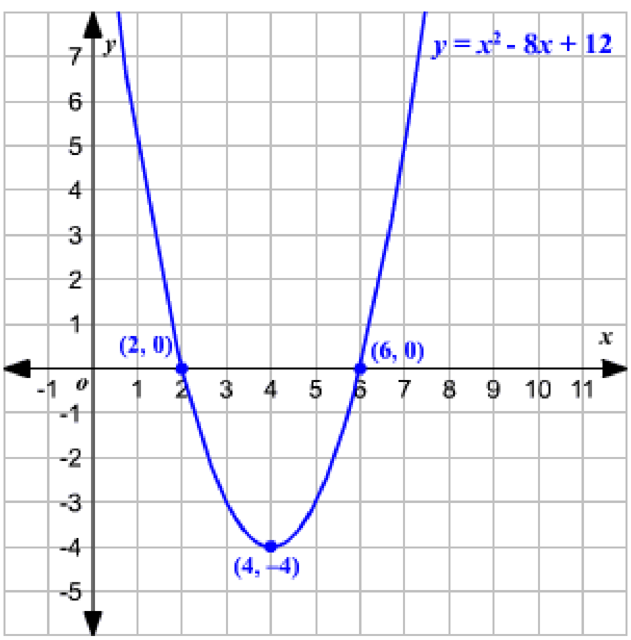
Introduction to Quadratic Functions

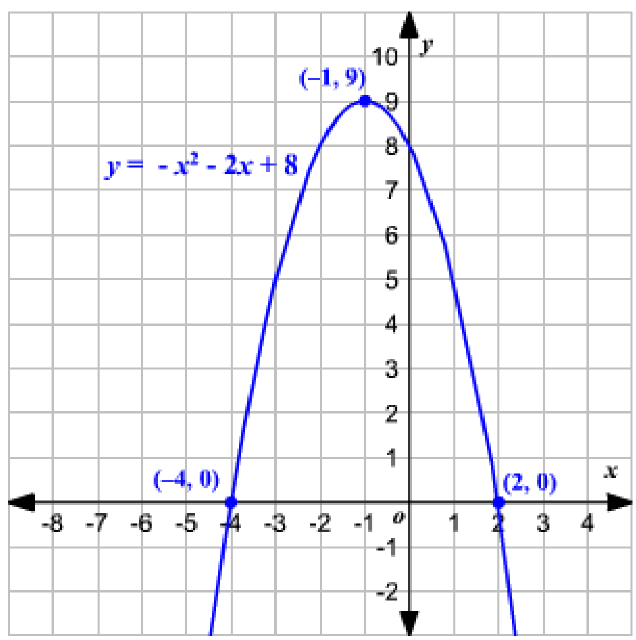
A **quadratic function** has the form $f(x) = ax^2 + bx + c$, where a, b and c are real numbers, with $a \neq 0$.

For each of the following quadratic functions, identify a, b and c.

Function	a	b	c
A. $f(x) = x^2 + 2x - 3$			
B. $f(x) = -2x^2 - 4x + 6$			
C. $f(x) = x^2 - 6x + 9$			
D. $f(x) = -4x^2 + 9$			
E. $f(x) = x^2 - 5x$			

The graph of a quadratic function is a **parabola**. Below are two examples.





Important features of the parabola:

1. **<u>Opening direction</u>**: The parabola corresponding to the graph of a quadratic function either opens **up** or **down**. In the equation $f(x) = ax^2 + bx + c$,

    - If a is **positive**, then the parabola opens **up**
    - If a is **negative**, then the parabola opens **down**

    In the examples above, the **firstparabola opens up** and the **second parabola opens down**.

2. **<u>Vertex</u>**: The vertex of the parabola is the **point on the graph** where the graph **changes direction**. It is usually referred to with ordered pair $(\boldsymbol{h}, \boldsymbol{k})$.

    In the examples above, the **vertex** of the **first** graph is $(\mathbf{4}, -\mathbf{4})$ and the **vertex** of the **second** graph is $(-\mathbf{1}, \mathbf{9})$.

    When you're not given the graph, but given the formula for the quadratic function, $f(x) = ax^2 + bx + c$, the vertex is given by

    $$h = -\frac{b}{2a} \text{and} k = f\left(-\frac{b}{2a}\right)$$

    Let's do this for the first example above.

    Example: $f(x) = x^2 - 8x + 12$

    Note that $a = 1$, $b = -8$ and $c = 12$. Substituting into the formulas above,

    $$h = -\frac{b}{2a} = -\frac{-8}{2(1)} = \frac{8}{2} = 4 \text{and}$$

    $$k = f\left(-\frac{b}{2a}\right) = f(4) = (4)^2 - 8(4) + 12 = 16 - 32 + 12 = -4$$

    Hence, the vertex occurs at $(h, k) = (4, -4)$, which can be easily verified by looking at the graph.

3. **$\boldsymbol{y}$-intercept**: The y-intercept is the value of y where the graph touches the y-axis, which occurs when $x = 0$. The y-intercept can be found from the function formula by substituting $x = 0$.

    For the **first** example above, **$\boldsymbol{y}$-intercept = 12**, and for the **second** example, **$\boldsymbol{y}$-intercept= 8**.

    The graph of a quadratic function can have only one y-intercept.

4. **$\boldsymbol{x}$-intercept(s)**: The x-intercept(s) are the value(s) of x where the graph touches the x-axis, which occurs when $y = 0$.

    In the examples above, you can simply read the x-intercepts off from the graphs:

    - In the **first** example, the **$\boldsymbol{x}$-intercepts** are $\mathbf{2}$ and $\mathbf{6}$
    - In the **second** example, the **$\boldsymbol{x}$-intercepts** are $-\mathbf{4}$ and $\mathbf{2}$

When you're not given the graph, but given the formula for the quadratic function, $f(x) = ax^2 + bx + c$, you may solve for the x-intercepts by setting $y = f(x) = 0$ and solving for x. That is, set $ax^2 + bx + c = 0$. The values of x that satisfy this equation are given by the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Let's do this for the first example above.

Example: $f(x) = x^2 - 8x + 12$

Set $x^2 - 8x + 12 = 0$. Note that $a = 1$, $b = -8$ and $c = 12$. Substituting into the quadratic formula, we have:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(12)}}{2(1)}$$

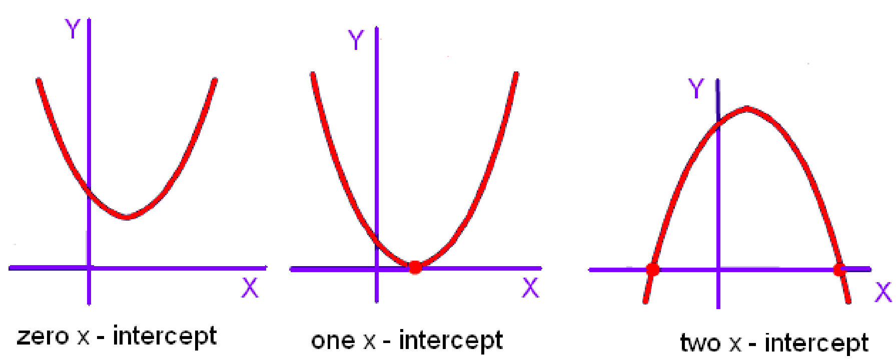
$$x = \frac{8 \pm \sqrt{64 - 48}}{2} = \frac{8 \pm \sqrt{16}}{2} = \frac{8 \pm 4}{2}$$

So,

$$x = \frac{8+4}{2} = 6 \text{or} x = \frac{8-4}{2} = 2$$

which are precisely the x-intercepts we see in the graph.

Important note: The graph of a quadratic function, which is a parabola, may have **zero**, **one** or **two**x-intercepts.



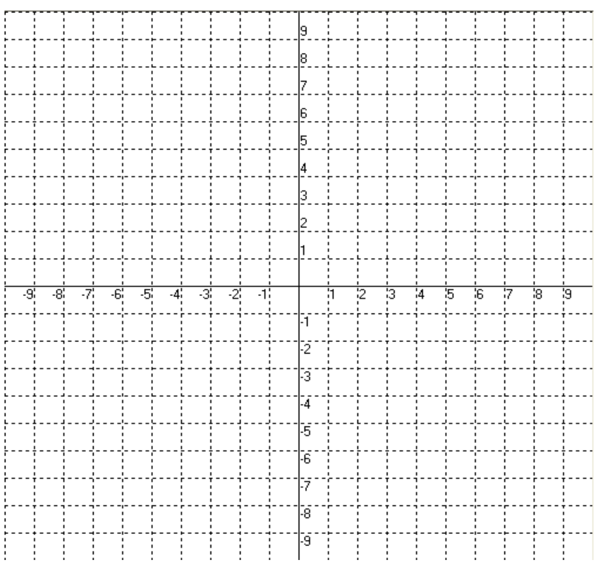
$b^2 - 4ac$is negative $\quad$ $b^2 - 4ac = 0$ $\quad$ $b^2 - 4ac$ is positive

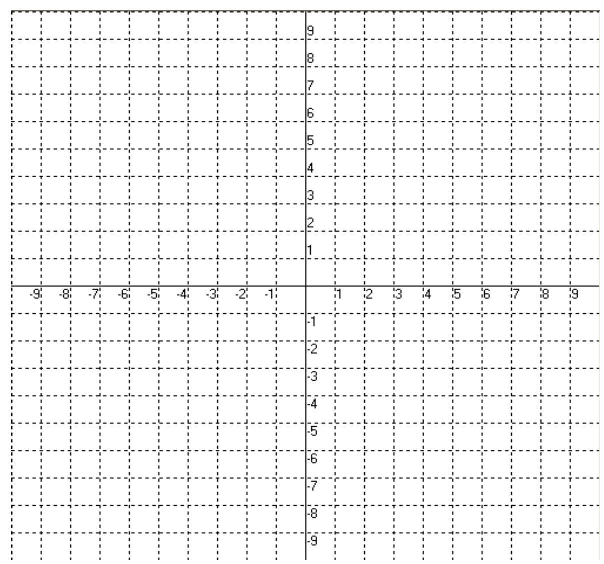
TRY THESE:

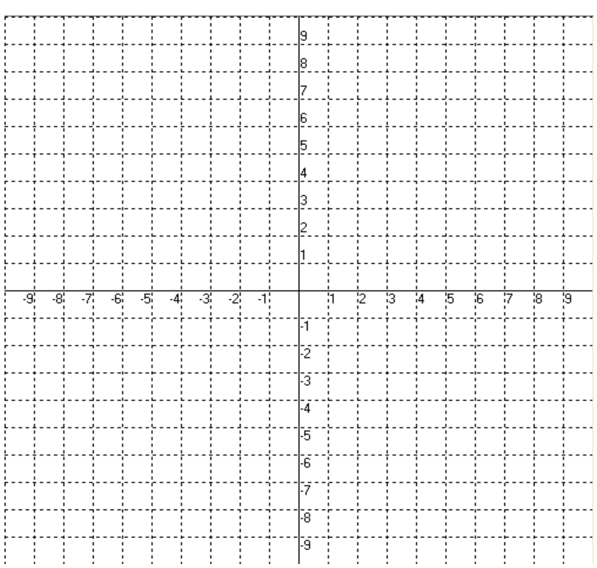
1. Fill out the table below. Do the work on scratch paper.

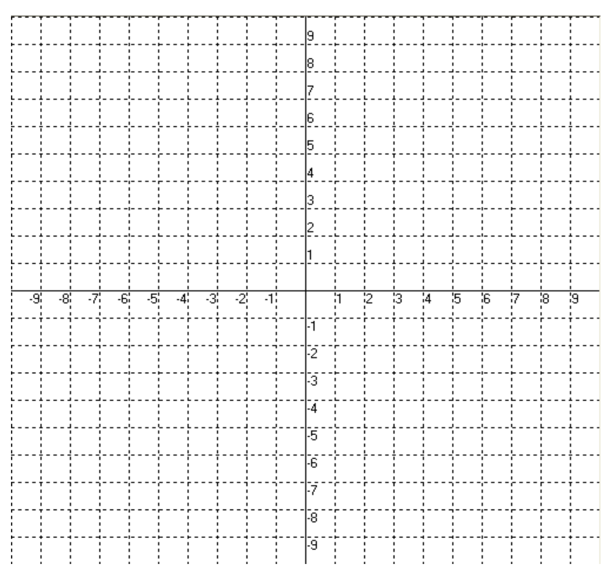
Function	Opening Direction	Vertex	y-intercept	x-intercept(s)
A. $f(x) = x^2 + 2x - 3$				
B. $f(x) = -2x^2 - 4x + 6$				
C. $f(x) = x^2 - 6x + 9$				
D. $f(x) = -4x^2 + 9$				
E. $f(x) = x^2 - 5x$				

2. Using the information you found in the table above, sketch the graphs of the functions in A, B, C and D.









3. Orlando kicks a soccer ball with a velocity of v feet per second at a 45 degree angle relative to the ground. If we ignore wind and air resistance, the path of the ball is given by the quadratic equation:

$$y = \square \frac{32}{v^2} \quad .$$

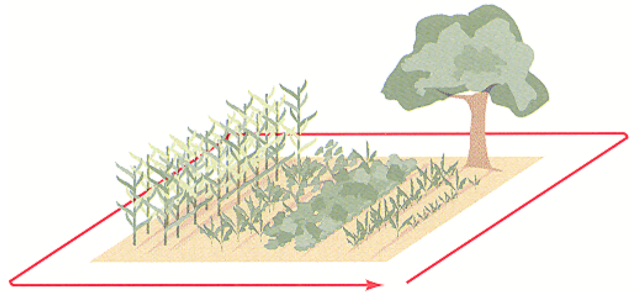
In this model, the input x represents the horizontal distance (in feet) of the ball from the kicker and y represents the height (in feet) above the ground.

a. If Orlando kicks the ball with a velocity of 80 feet per second, write the quadratic function.

b. What is the maximum height the ball reaches above the ground? HINT: The vertex of a parabola that opens down is the highest point on the graph.

c. How far (horizontal distance) will the ball travel when it hits the ground? HINT: The height of the ball is 0 when it hits the ground.

4. Maria has 80 feet of wire-bird netting to use around her vegetable garden to keep out cats. The garden is to be rectangular.



a. Create a table of possible rectangular measurements for the netting. Make up numbers for length or width and use them to fill out the rest of the row. Recall that the area of a rectangle is length times width.

Length	Width	Perimeter	Area
		80	
		80	
		80	
		80	
		80	
		80	

b. Letting x = the length of the rectangular plot, write an expression for the width in terms of x.

c. Write the quadratic equation to determine the area, $A(x)$. Express your final answer as a quadratic function in the form $A(x) = ax^2 + bx + c$.

d. What length and width give the maximum area for the available netting? HINT: Think vertex.

5. You are an assistant to the president of a small commuter airline. You have been asked to develop a strategy for increasing the revenue from your primary route. The current fare for this route is \$160 per person and each flight carries an average of 40 passengers.

Note*:* *Revenue* is the total amount of money collected.

a. What is the average revenue from each flight now?

b. A recent marketing analysis suggests that each \$2 increase in fare will result in one less passenger per flight and each \$2 reduction in fare will produce an additional passenger per flight.

Using this information, fill out the following table.

Number and direction of fare adjustment	**Fare (\$)**	**Number of Passengers**	**Anticipated Revenue**
-3			
-2			
-1	\$158	41	\$6478
0	\$160	40	\$6400
1	\$162	39	\$6318
2			
3			

Let x = the number and direction of fare adjustment (first column)

$f(x)$ = the Fare (second column)

$n(x)$ = the Number of passengers (second column)

c. Determine a formula for 'Fare', $f(x)$, in terms of x. HINT: Your answer should be a linear function.

d. Determine a formula for 'Number of Passengers, $n(x)$, in terms of x. HINT: Your answer should be a linear function.

e. The ‘Anticipated Revenue’ (fourth column above) is simply the product of the ‘Fare’ and ‘Number of Passengers’. Determine a formula for Anticipated Revenue’, $r(x)$, in terms of x. So, $r(x) = f(x) \times n(x)$. Simplify your formula to put it in standard form, $r(x) = ax^2 + bx + c$.

f. Does the graph of $r(x)$ open up or down? How do you know?

g. Determine the y-intercept of your revenue function and state its practical meaning in a complete sentence.

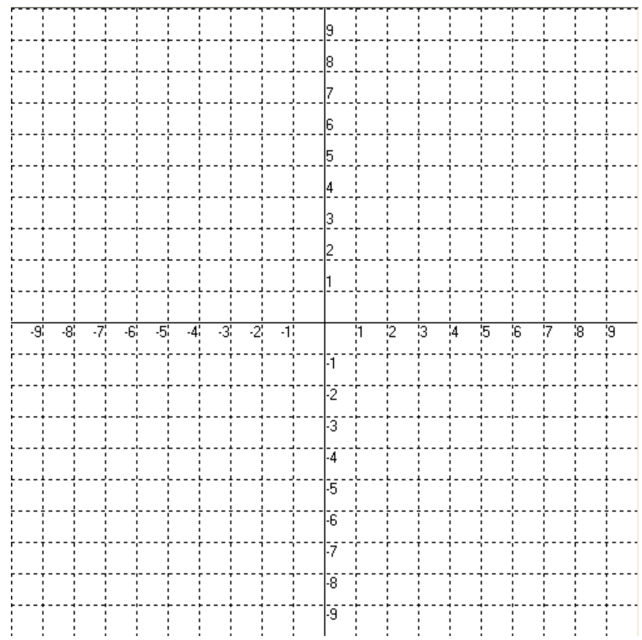
h. Determine the vertex of your revenue function and state its practical meaning in a complete sentence.

i. Write a short paragraph for your boss below, using complete sentences and correct grammar, what your conclusion is to the problem he asked you to solve.

Math 217 NAME ______________________________________

Take-It-Home: Introduction to Quadratic Functions

1. For the function $f(x) = -x^2 + 4x - 3$



   a. Does this graph of this quadratic function open up or does it open down?

   b. Find and graph the vertex. Is this point a minimum or a maximum for the function?

   c. Find and graph the vertical intercept.

   d. Find and graph the horizontal intercepts.

   e. Complete the graph of the quadratic function.

2. A person standing close to the top of a 120 feet tall building throws a baseball vertically upward. The quadratic function $s(t) = -16t^2 + 56t + 120$ models the ball's height above the ground, $s(t)$, in feet, t seconds after it was thrown.

   a. After how many seconds does the ball reach its maximum height? What is the maximum height?

   b. How many seconds does it take until the ball finally hits the ground?

   c. Find $s(0)$ and describe what this means.