Lesson 4.3.2
Exponential Growth and Decay

SACWAY
BRAINSTORMING ALGEBRA & STATISTICS

STUDENT NAME ______________________ DATE ____________

INTRODUCTION

Exponential Growth and Decay.

In our previous lesson, we experimented with the idea that a fixed-rate interest earning account could grow *continuously*. If such an account actually existed, the account balance would be given by

$$A = Pe^{rt}.$$

In this formula, P is the starting balance, r is the annual interest rate, and t is time in years.

In this lesson, we learn that *any* phenomenon that grows or decays exponentially can be modeled by a base e exponential model, just like the compound interest formula above.

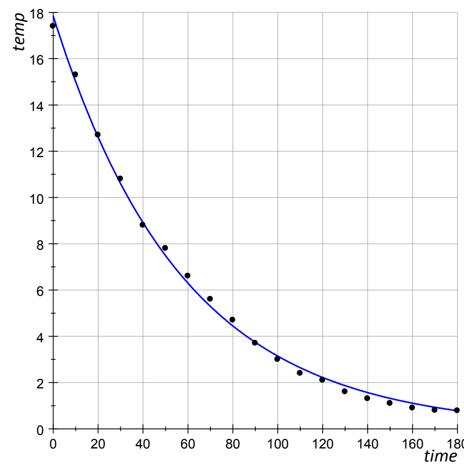

Figure 1: An exponential decay model.

A generic model which applies to every exponential growth or decay application is

$$A = A_0e^{kt}.$$

In this model, A represents an amount of something that grows or decays over time. The constants A_0 and k are analogous to P and r in the compound interest formula. A_0 is the initial amount (of something) at time $t = 0$, and k is the instantaneous growth/decay rate. In a growth model, k is positive, and in a decay model, k is negative.

Newton's Law of Cooling

As an object cools in a room whose temperature is held constant, its temperature decays exponentially as it approaches room temperature (this is Newton's Law of Cooling). Let A represent the object's temperature at time t, measured in degrees Celsius *above* room temperature (so $A = 0$ is room temperature). A_0 represents the object's initial temperature at time $t = 0$.

The following facts were determined by fitting an exponential model to real temperature values as pictured in Figure 1: An exponential decay model.. A temperature probe measured an initial temperature of 18° F (so $A_0 = 18$), above room temperature, and according to the model, it cooled at an instantaneous rate of 1.7% (so $k = -0.017$) per second.

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The exponential decay model for the temperature is

$$A = 18e^{-0.017t}.$$

TRY THESE

1. How many degrees above room temperature is the object in 30 seconds? Use the graph in Figure 1 to estimate the answer to this question.

2. Use the exponential decay formula above to answer Question 1 again. Do your answers agree (approximately)?

Suppose we need to know when the probe will reach 2° above room temperature.

3. Use the graph in Figure 1 to estimate the time when the probe's temperature is 2° above room temperature.

4. In the exponential decay formula, set the temperature, *A*, equal to 2° (degrees above room temperature), and solve for the time when this occurs. This is done by first dividing by the initial temperature (18°), applying the natural logarithm (*ln*) to each side, and using the fact that $\ln(e^u) = u$.

5. Does the time in Question 4 agree (approximately) with your estimate from Question 3?

NEXT STEPS

When taking medications internally, the amount of medication in the body breaks down over time, decaying exponentially. For example, the pain reliever *ibuprofen* has an hourly decay rate of about 35% (0.35).

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TRY THESE

Suppose a person takes an 800 mg tablet of ibuprofen. Use the fact that ibuprofen has an hourly decay rate of 35% to answer the following questions.

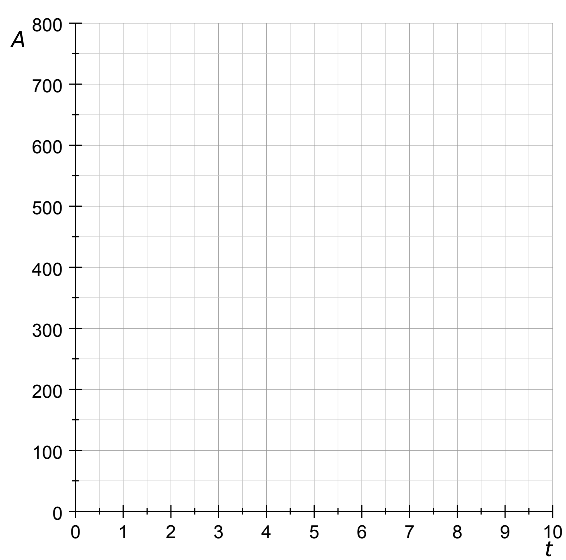
6. Give the formula which models the amount of ibuprofen in her body after *t* hours.

7. If a person takes an 800 mg tablet of ibuprofen, how much remains in her system after 2 hours?

8. Compute the amount of ibuprofen in her body for the times listed in the table below.

t **(Hours)**	***A*** **(Ibuprofen Amount)**
0	800
2	
4	
6	
8	
10	

9. Graph the exponential decay model using the points you computed above.



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10. Use the graph above to estimate the *half-life* of ibuprofen (the time when only half remains).

11. Using a process similar to that used in Question 4, solve the formula from Question 6 for the half-life of ibuprofen. Begin by setting the amount of ibuprofen, A, to half of its original amount.

12. Suppose the patient's doctor does not want her ibuprofen level to drop below 150 mg. Additionally, it should *never* be above 1000 mg. Use the graph above to determine how frequently she should take the medication.

13. Using a process similar to that used in Question 4, solve the formula from Question 6 for the time when her ibuprofen level reaches 150 mg. Does this agree with your answer from Question 12?

NEXT STEPS

Suppose that a population has initial size 1000 at time $t = 0$, and grows exponentially with a 20% (0.20) instantaneous annual growth rate.

TRY THESE

14. Give the formula which models the population size in t years.

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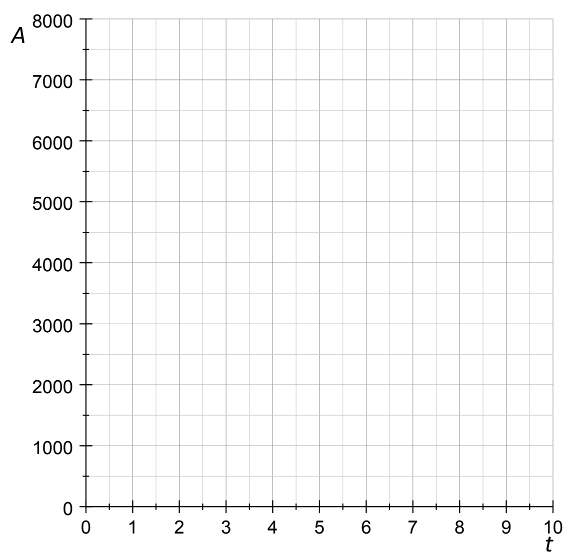
15. Use the formula to estimate the population size in ten years.

16. Compute the amount of ibuprofen in her body for the times listed in the table below.

t **(Hours)**	***A*** **(Population Size)**
0	1000
2	
4	
6	
8	
10	

17. Graph the exponential growth model using the points you computed above.



18. Use your graph above to estimate the time when the population size is 3000.

19. Using the formula from Question 14, solve for the time when the population time is 3000.

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20. Use your graph to estimate the population's *doubling time*.

21. Using the formula from Question 14, solve for the population's doubling time.

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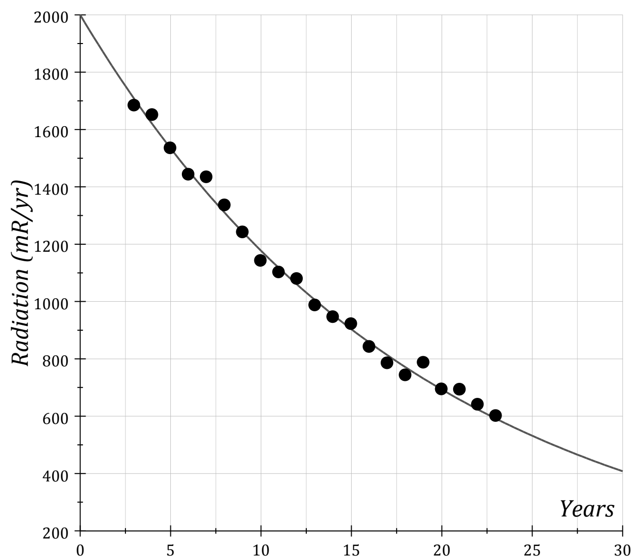
TAKE IT HOME

Previously, we examined the radioactivity levels in the Nevada Area 2 test site after 1985. We determined that an exponential model was an appropriate fit for the radiation level after *t* years.



After analysis, it is learned that the exponential model predicts an initial radiation level of 2000.2 mR/yr, with an instantaneous decay rate of 5.3%.

1. Use the form, $A = A_0 e^{kt}$, to give the formula for the exponential decay model for the Area 2 radiation level after *t* years.

2. Use the graph to estimate the ½ life of the radiation at Area 2.

3. Solve the exponential decay model in Question 1 for the ½ life (by setting the radiation level, *A*, to half of its original amount).

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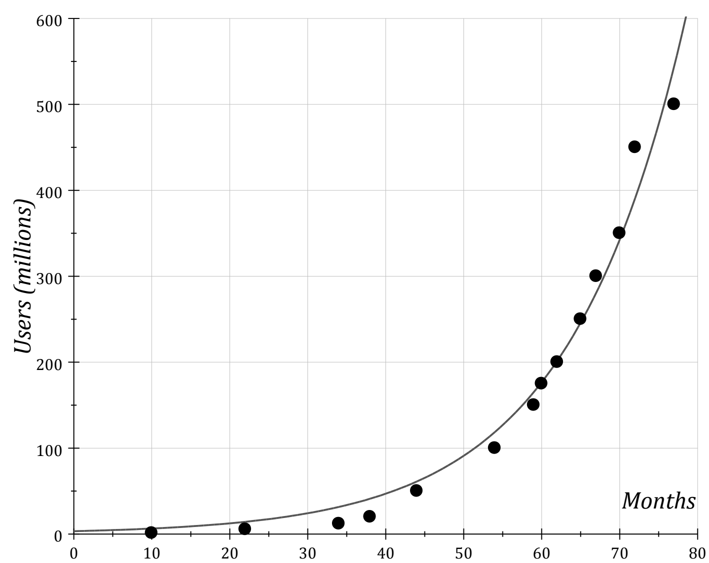
4. Use the formula from Question 1 to estimate the time when the radiation level was 500 mR/yr.

In February 2004, Facebook was born as a social networking tool. The company has been tracking its membership since December 2004. The graph below represents membership counts (in millions) at *t* months, beginning in December 2004 (through July 2010). An exponential model fits the data appropriately.



After analysis, it is learned that the exponential model predicts an initial membership of 3.32 million, and an instantaneous growth rate of 6.6%.

5. Use the form, $A = A_0 e^{kt}$, to give the formula for the exponential growth model for the number of Facebook members after *t* months.

6. Use the graph to estimate the *doubling time* of the Facebook membership. This might be easiest by estimating the time for the membership to grow from 100 to 200 million.

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7. Solve the exponential growth model in Question 5 for the doubling time (by setting the membership, A, to twice of its original amount).

8. Use the formula from Question 5 to estimate the time when the membership was 400 million.

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