Inferential Statistics and Probability
a Holistic Approach

Chapter 4
Probability

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Probability

- Classical probability
  - Based on mathematical formulas
- Empirical probability
  - Based on the relative frequencies of historical data.
- Subjective probability
  - "one-shot" educated guess.

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Examples of Probability

- What is the probability of rolling a four on a 6-sided die?
- What percentage of De Anza students live in Cupertino?
- What is the chance that your favorite team will win the championship?

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Classical Probability

- Event
  - A result of an experiment, usually expressed as a letter (A, B,…)
- Outcome
  - A result of the experiment that cannot be broken down into smaller events
- Sample Space
  - The set of all possible outcomes
- Probability Event A Occurs - written as P(A)
  - Number of Outcomes in Event A / Number of Outcomes in Sample Space
- Example – flip two coins, find the probability of exactly 1 head.
  - Sample Space = {HH, HT, TH, TT}   A= {HT, TH}
  - P(A) = 2/4 = 0.50

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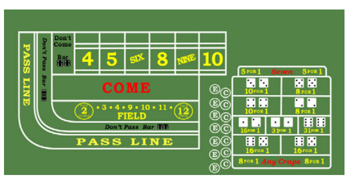
Example – Field Bet in Craps

Field Bet

•2 dice are rolled and totaled
•Player wins even money on 3, 4, 9, 10, 11,
•Player wins double on 2
•Player wins triple on 12
•Player loses bet on 5, 6, 7, 8



Who has the advantage in this game?

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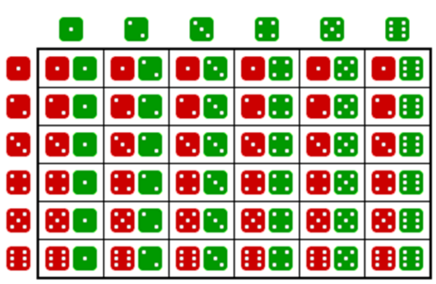
Sample Space
36 possible pairs of rolls



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More ways to make 5, 6, 7, 8

$$Sample\ Space = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

P(Win) = 16/36 = 0.444   P(Lose) = 20/36 = 0.556

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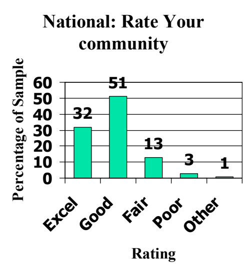
Empirical Probability

- Historical Data
- Relative Frequencies
- Example: What is the chance someone rates their community as good or better?
- **0.51 + 0.32 = 0.83**



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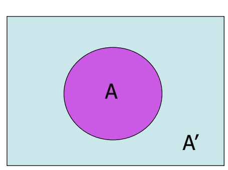
Rule of Complement

- Complement of an event
- The event does not occur
- A′ is the complement of A
- P(A) + P(A′) =1
- P(A) = 1 – P(A′)



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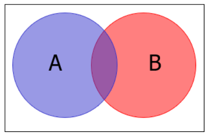
Joint Probability

- The UNION of two events A and B is that either A or B occur (or both). (All colored parts)
- The INTERSECTION of two events A and B is that both A and B will occur. (Purple Part only)
- Additive Rule:
  P(A or B) = P(A) + P(B) – P(A and B)



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Example

- In a group of students, 40% are taking Math, 20% are taking History.
- 10% of students are taking both Math and History.
- Find the Probability of a Student taking either Math or History or both.
- P(M or H) = 40% + 20% - 10% = 50%

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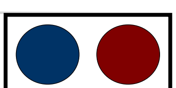
Mutually Exclusive



- Mutually Exclusive
- Both cannot occur
- If A and B are mutually exclusive, then
  - P(A or B) = P(A) + P(B)
- Example roll a die
  - A: Roll 2 or less B: Roll 5 or more
  - P(A)=2/6 P(B)=2/6
  - P(A or B) = P(A) + P(B) = 4/6

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Conditional Probability

- The probability of an event occuring GIVEN another event has already occurred.
- P(A|B) = P(A and B) / P(B)
- Example: Of all smart phone users in the US, 21% have an Apple iPhone and AT&T. 35% of all smart phone users have AT&T. Given a selected smart phone user has AT&T, find the probability the user also has an Apple iPhone.
- A=AT&T subscriber B=Apple iPhone
- P(A and B) = 0.21 P(A)=0.35
- P(B|A) = 0.21/0.35 = 0.60

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Marginal, Joint and Conditional Probability

- **Marginal Probability** means the probability of a single event occurring.
- **Joint Probability** means the probability of the union or intersection of multiple events occurring (and/or statements).
- **Conditional Probability** means the probability of an event occurring given that another event has already occurred.

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Contingency Tables

- Two data items can be displayed in a contingency table.
- Example: auto accident during year and DUI of driver.

	Accident	No Accident	Total
DUI	70	130	200
Non- DUI	30	770	800
Total	100	900	1000

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Marginal Probability

	Accident	No Accident	Total
DUI	70	130	200
Non- DUI	30	770	800
Total	100	900	1000

A = Accident

D = DUI

- Find the Probability a Driver had an Accident
- P(A) = 100/1000 = 0.10
- Find the Probability was not DUI
- P(D') = 1 – 200/1000 = 0.80

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Joint Probability

	Accident	No Accident	Total
DUI	70	130	200
Non- DUI	30	770	800
Total	100	900	1000

A = Accident

D = DUI

- Find the Probability a Driver had an Accident **and** was DUI
- P(A and D) = 70/1000 = 0.07
- Find the Probability a Driver had an Accident **or** was DUI
- P(A or D) = P(A) + P(D) – P(A and D) = (100+200-70)/1000 = 0.23

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Conditional Probability

	Accident	No Accident	Total
DUI	70	130	200
Non- DUI	30	770	800
Total	100	900	1000

A = Accident

D = DUI

- Find the Probability a DUI Driver had an Accident
- P(A|D) = 70/200 = 0.35
- Find the Probability a Driver who had an Accident is also DUI
- P(D|A) = 70/100 = 0.70

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Creating Contingency Tables

- You can create a hypothetical contingency table from reported cross tabulated data.
- First choose a convenient sample size (called a radix) like 10000.
- Then apply the reported marginal probabilities to the radix of one of the variables.
- Then apply the reported conditional probabilities to the total values of one of the other variable.
- Complete the table with arithmetic.

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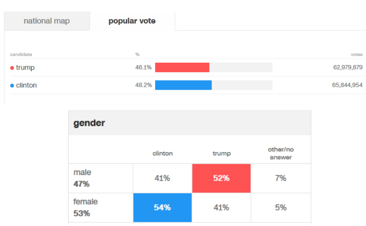
Example

Create a two-way table from the cross tabulation of gender from the 2016 election results (from CNN)



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Example

First select a radix (sample size) of 10000

	GENDER		
VOTED FOR	Female	Male	Total
Trump			
Clinton			
Other			
Total			10000

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Example

Then apply the marginal probabilities to the radix (53% female, 47% male)

	GENDER		
VOTED FOR	Female	Male	Total
Trump			
Clinton			
Other			
Total	5300	4700	10000

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Example

Then apply the cross tabulated percentages for each gender. Make sure the numbers add up.

	GENDER		
VOTED FOR	Female	Male	Total
Trump	2173	2444	
Clinton	2862	1927	
Other	265	329	
Total	5300	4700	10000

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Example

Finally, complete the table using arithmetic.

	GENDER		
VOTED FOR	Female	Male	Total
Trump	2173	2444	4617
Clinton	2862	1927	4789
Other	265	329	594
Total	5300	4700	10000

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Multiplicative Rule

- P(A and B) = P(A) x P(B|A)
- P(A and B) = P(B) x P(A|B)
- Example: A box contains 4 green balls and 3 red balls. Two balls are drawn. Find the probability of choosing two red balls.
- A=Red Ball on 1st draw   B=Red Ball on 2nd Draw
- P(A)=3/7   P(B|A)=2/6
- P(A and B) = (3/7)(2/6) = 1/7

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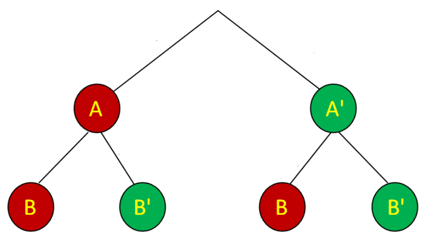
Multiplicative Rule – Tree Diagram



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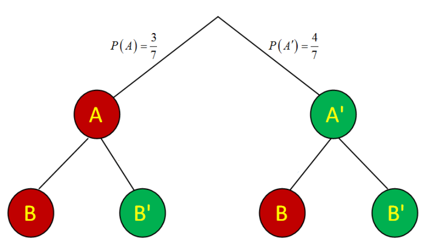
Multiplicative Rule – Tree Diagram



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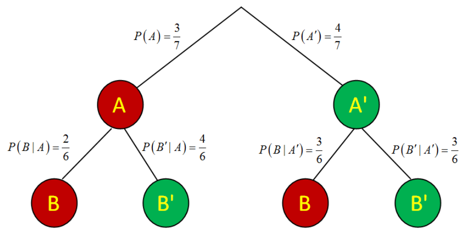
Multiplicative Rule – Tree Diagram



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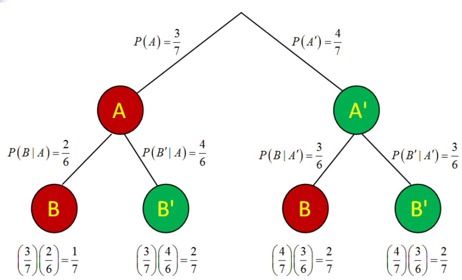
Multiplicative Rule – Tree Diagram



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Independence

- If A is not dependent on B, then they are INDEPENDENT events, and the following statements are true:
  - P(A|B)=P(A)
  - P(B|A)=P(B)
  - P(A and B) = P(A) x P(B)

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Example

	Accident	No Accident	Total
DUI	70	130	200
Non- DUI	30	770	800
Total	100	900	1000

A: Accident D:DUI Driver

P(A) = .10 P(A|D) = .35 (70/200)

Therefore A and D are **DEPENDENT** events as P(A) < P(A|D)

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Example

	Accident	No Accident	Total
Domestic Car	60	540	600
Import Car	40	360	400
Total	100	900	1000

A: Accident D:Domestic Car

P(A) = .10 P(A|D) = .10 (60/600)

Therefore A and D are **INDEPENDENT** events as P(A) = P(A|D)

Also P(A and D) = P(A)xP(D) = (.1)(.6) = .06

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Random Sample

- A random sample is where each member of the population has an equally likely chance of being chosen, and each member of the sample is INDEPENDENT of all other sampled data.

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Tree Diagram method

- Alternative Method of showing probability
- Example: Flip Three Coins
- Example: A Circuit has three switches. If at least two of the switches function, the Circuit will succeed. Each switch has a 10% failure rate if all are operating, and a 20% failure rate if one switch has already failed. Find the probability the circuit will succeed.

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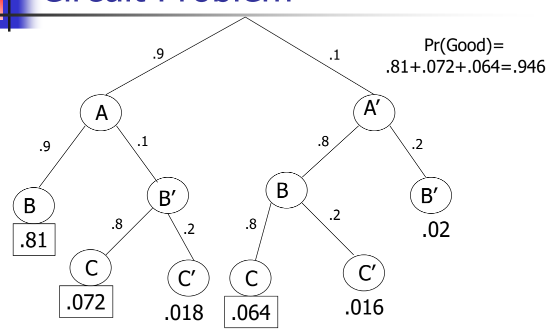
Circuit Problem



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Switching the Conditionality

- Often there are questions where you desire to change the conditionality from one variable to the other variable
- First construct a tree diagram.
- Second, create a Contingency Table using a convenient radix (sample size)
- From the Contingency table it is easy to calculate all conditional probabilities.

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Example

- 10% of prisoners in a Canadian prison are HIV positive.
- A test will correctly detect HIV 95% of the time, but will incorrectly "detect" HIV in non-infected prisoners 15% of the time (false positive).
- If a randomly selected prisoner tests positive, find the probability the prisoner is HIV+

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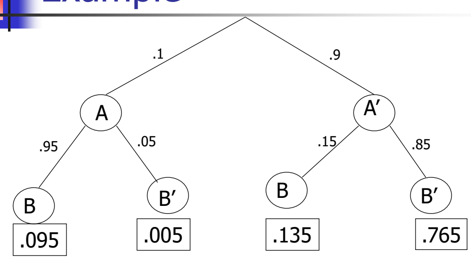
Example



A=Prisoner is HIV+
B=Test is Positive for HIV

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Example

	HIV+ A	HIV- A′	Total
Test+ B	950	1350	2300
Test- B′	50	7650	7700
Total	1000	9000	10000

$$P(A \mid B) = \frac{950}{2300} \approx .413$$

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