Inferential Statistics and Probability
a Holistic Approach

Chapter 12

One Factor Analysis of Variance (ANOVA)

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ANOVA Definitions

- **Factor** – categorical variable that defines the populations.
- **Response** – variable that is being measured.
- **Levels** – the number of choices for the factor, represented by k
- **Replicates** – the sample size for each level, n_1, n_2, …, n_k.
- If $n_1 = n_2 = … = n_k$, then the design is **balanced.**

- **Ho:** There is no difference in the mean <response in context> due to the <factor in context>.
- **Ha:** There is a difference in the mean <response in context> due to the <factor in context>.

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Underlying Assumptions for ANOVA

- The *F* distribution is also used for testing the equality of more than two means using a technique called analysis of variance (ANOVA). ANOVA requires the following conditions:
  - The populations being sampled are normally distributed.
  - The populations have equal standard deviations.
  - The samples are randomly selected and are independent.

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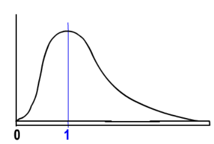
Characteristics of F-Distribution

- There is a "family" of *F* Distributions.
- Each member of the family is determined by two parameters: the numerator degrees of freedom and the denominator degrees of freedom.
- *F* cannot be negative, and it is a continuous distribution.
- The *F* distribution is positively skewed.
- Its values range from 0 *to* ∞. As $F \to \infty$ the curve approaches the X-axis.



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Analysis of Variance Procedure

- The Null Hypothesis: the population means are the same.
- The Alternative Hypothesis: at least one of the means is different.
- The Test Statistic: F=(between sample variance)/(within sample variance).
- Decision rule: For a given significance level α, reject the null hypothesis if *F* (computed) is greater than *F* (table) with numerator and denominator degrees of freedom.

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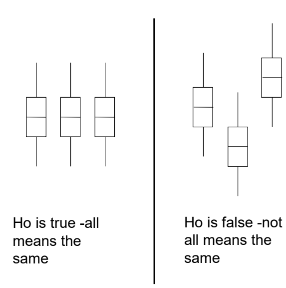
ANOVA – Null Hypothesis



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ANOVA NOTES

- If there are k populations being sampled (levels), then the df_{factor} = k-1
- If the sample size is n, then df_{error}= n-k
- The test statistic is computed by:F=[(SS_F)/(k-1)]/[(SS_E)/(n-k)].
- SS_F represents the factor (between) sum of squares.
- SS_E represents the error (within) sum of squares.
- Let T_C represent the column totals, n_c represent the number of observations in each column, and ΣX represent the sum of all the observations.
- These calculations are tedious, so technology is used to generate the **ANOVA table.**

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Formulas for ANOVA

$$SS_{Total} = \Sigma\left(X^2\right) - \frac{(\Sigma X)^2}{n}$$

$$SS_{Factor} = \Sigma\left(\frac{T_c^2}{n_c}\right) - \frac{(\Sigma X)^2}{n}$$

$$SS_{Error} = SS_{Total} - SS_{Factor}$$

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ANOVA Table

Source	SS	df	MS	F
Factor	SS_{Factor}	k-1	SS_F/df_F	MS_F/MS_E
Error	SS_{Error}	n-k	SS_E/df_E	
Total	SS_{Total}	n-1		

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EXAMPLE

- Party Pizza specializes in meals for students. Hsieh Li, President, recently developed a new tofu pizza.
- Before making it a part of the regular menu she decides to test it in several of her restaurants. She would like to know if there is a difference in the mean number of tofu pizzas sold per day at the Cupertino, San Jose, and Santa Clara pizzerias for sample of five days.
- At the .05 significance level can Hsieh Li conclude that there is a difference in the mean number of tofu pizzas sold per day at the three pizzerias?

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Example

	Cupertino	San Jose	Santa Clara	Total
	13	10	18	
	12	12	16	
	14	13	17	
	12	11	17	
			17	
T	51	46	85	182
n	4	4	5	13
Means	12.75	11.5	17	14
Σ^2	653	534	1447	2634

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Example *continued*

$$SS_{Total} = 2634 - \frac{182^2}{13} = 86$$

$$SS_{Factor} = 2624.25 - \frac{182^2}{13} = 76.25$$

$$SS_{Error} = 86 - 76.25 = 9.75$$

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Example 4 *continued*

ANOVA TABLE

Source	SS	df	MS	F
Factor	76.25	2	38.125	39.10
Error	9.75	10	0.975	
Total	86.00	12		

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EXAMPLE 4 ***continued***

- **Design:** H_o: $\mu1=\mu2=\mu3$
  H_a: Not all the means are the same
- α=.05
- Model: One Factor ANOVA
- H_0 is rejected if F>4.10
- **Data:** Test statistic: F=[76.25/2]/[9.75/10]=39.1026
- H_0 is rejected.
- **Conclusion:** There is a difference in the mean number of pizzas sold at each pizzeria.

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One-way ANOVA: Cupertino, San Jose, Santa Clara

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Source  DF      SS      MS      F      P
Factor   2  76.250  38.125  39.10  0.000
Error   10   9.750   0.975
Total   12  86.000

S = 0.9874   R-Sq = 88.66%   R-Sq(adj) = 86.40%


                                Individual 95% CIs For Mean Based on
                                Pooled StDev
Level        N   Mean  StDev   --------+---------+---------+---------+-
Cupertino    4  12.750  0.957     (-----*----)
San Jose     4  11.500  1.291  (----*-----)
Santa Clara  5  17.000  0.707                          (----*----)
                               --------+---------+---------+---------+-
                                    12.0      14.0      16.0      18.0
```

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Post Hoc Comparison Test

- Used for pairwise comparison
- Designed so the **overall** signficance level is 5%.
- Use technology.
- Refer to **Tukey Test** Material in the textbook.

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Post Hoc Comparison Test

```
Grouping Information Using Tukey Method

              N    Mean  Grouping
Santa Clara  5  17.0000  A
Cupertino    4  12.7500    B
San Jose     4  11.5000    B

Means that do not share a letter are significantly different.
```

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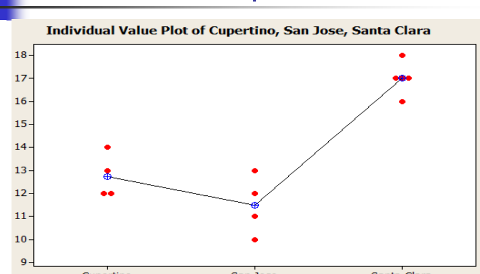
Post Hoc Comparison Test



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Example – Oranges & Orchards

- Valencia oranges were tested for juiciness at 4 different orchards. Eight oranges were sampled from each orchard, and the total ml of juice per 20 gms of orange was calculated.
- Test for a difference in juiciness due to orchards using alpha = .05
- Perform all the pairwise comparisons using Tukey's Test and an overall risk level of 5%.

Orchard A:	Orchard B:	Orchard C:	Orchard D:
11,13,12,14, 9,13,11,9	10,9,8,10, 11,12,7,8	13,15,14,11, 12,10,16,11	9,7,11,9, 9,11,10,8

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Example - Defintions

- Factor: Orchard (A, B, C or D)
- Response: Juiciness of orange
- Levels: $k = 4$
- Replicate: $n_A = n_B = n_C = n_D = 8$
- Design: Balanced
- Sample size: $n = 8 + 8 + 8 + 8 = 32$

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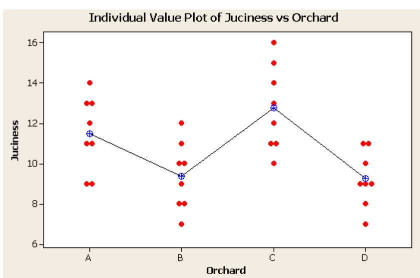
Example – Value Plot



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Example – Stats & ANOVA Table

Level	N	Mean	StDev
Orchard A	8	11.500	1.852
Orchard B	8	9.375	1.685
Orchard C	8	12.750	2.121
Orchard D	8	9.250	1.389

Source	DF	SS	MS	F	P
Factor	3	69.59	23.20	7.31	0.001
Error	28	88.88	3.17		
Total	31	158.47			

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Example – Tukey Test Grouping

Grouping Information Using Tukey Method

	N	Mean	Grouping
Orchard C	8	12.750	A
Orchard A	8	11.500	A B
Orchard B	8	9.375	B
Orchard D	8	9.250	B

Means that do not share a letter are significantly different.

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