4C Lab: Bernoulli's Equation

Goal: To examine Bernoulli's equation experimentally using an inverted soda bottle.

Equipment List:

- Pop bottle with 2 different caps
- Rod and two clamps from the Clement-Desormes apparatus
- Timer
- Meter stick
- Containers for catching water
- Beakers/graduated cylinders
- Vernier calipers
- Paper towels

Pre-lab exercise: derive equation (6)

Background/Theory:

Bernoulli's equation arises from examining the work energy equation for a system of a steady, non-viscous, irrotational fluid.

$$P + \frac{1}{2}\rho v^2 + \rho g h = constant \qquad (1)$$

Or $$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \qquad (2)$$

Where *P* is the pressure of the fluid, ρ is the density, *v* is the velocity and *h* is the height. The equation of continuity:

$$A_1 v_1 = A_2 v_2 \qquad (3)$$

Relates the mass of fluid Δm crossing area *A* in time Δt yielding a constant volume flow rate $I_v = Av$. For our system of a pop bottle open to the atmosphere at the top and bottom and taking $h_2 = 0$, equation (2) reduces to:

$$\frac{1}{2}\rho v_1^2 + \rho g h_1 = \frac{1}{2}\rho v_2^2 \qquad (4)$$

Combining equations (3) and (4) yields:

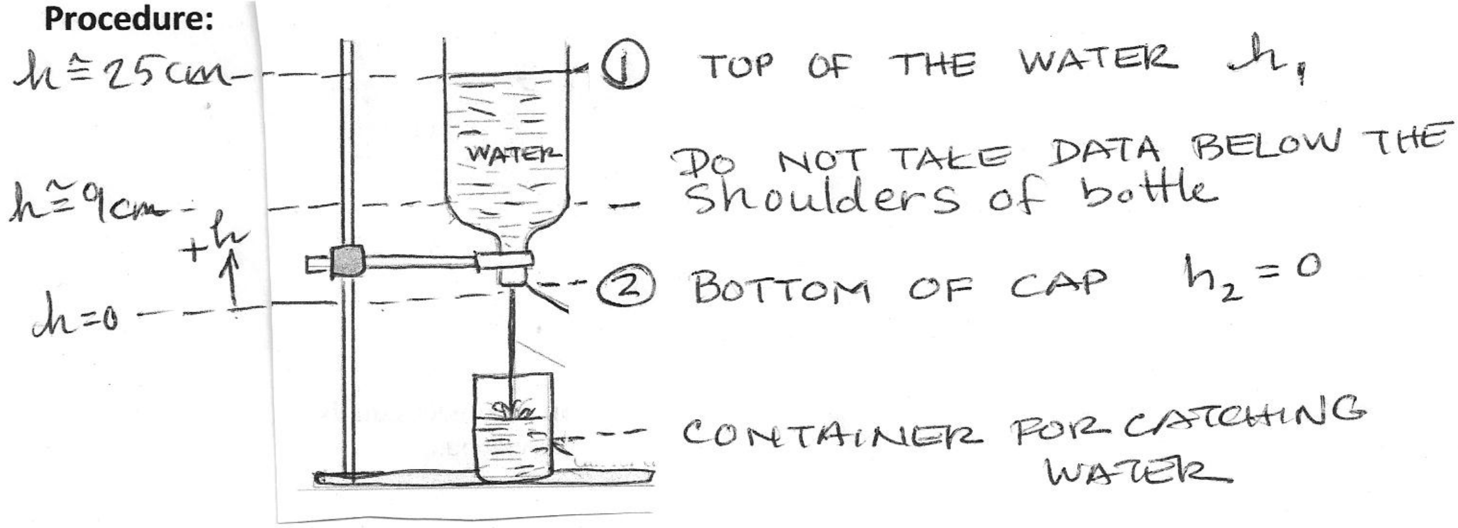
$$v_1^2 = {2gh_1}\Big/\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right] \cong {2gh_1}\Big/\left(\frac{A_1}{A_2}\right)^2 \qquad (5)$$

Integrate (h_1 is a variable and v_1 is related to h_1) to obtain:

$$h^{1/2} = h_o^{\,1/2} - \left(\frac{A_2}{A_1}\right)\sqrt{\frac{g}{2}}\,t \qquad (6)$$

(The minus sign arises because the height is a decreasing function.) Where h_o is the height of the fluid at time t = 0. NOTE: The height is measured from the cap bottom, where h_2 = 0!!

Procedure:



1. Set up the equipment as shown. The containers are not stable, so do a bit of engineering to make sure your table is not flooded. Attach a meter stick to measure the water level. REMEMBER! $h_2 = 0$ in our derivation. If you put the zero end of the meterstick on the table surface, be sure to record the height h_2 so you can subtract that from your all your *h* values.
2. Choose a cap and measure the diameter with the vernier caliper. Calculate A_2.
3. Find A_1 either with diameter measurement of the pop bottle or volume of water divided by height. You need only find A_1 once.
4. Fill the bottle with water and stop the water from falling with your finger. You take data in the region where h is 9 cm to about h = 24 cm, where the diameter is constant. By removing your finger and letting the water fall into the container and then stopping the opening again to record the height and time, you take pairs of data in *h* and *t*. Take approximately 10 data points, adjusting your intervals as necessary.
5. Repeat for a second, different aperture cap.

Analysis:

1. Graph your results of *h* as it relates to *t* so that the slope is equal to $(A_2/A_1)(g/2)^{1/2}$. Linearize your data --- **do not graph *h* versus *t*.** Compute the value of the slope using the linear fit in Excel.
2. Find A_2 from the slope for each data set and compare with a discrepancy test.