A Physics
Lab Skills Manual

A compilation of laboratory techniques for use in the physics with calculus lab sequence

David Newton
De Anza College

TABLE OF CONTENTS

PREFACE v

1. LAB BOOK POLICY AND PROCEDURES 1
   1.1 The Minimum Amount of Material Required to Bring to Every Lab 1
   1.2 Preparing Your Lab Book 1
   1.3 Graphing in the Lab Book 2

2. LAB BOOK STRUCTURE 3

3. THE STRUCTURE OF A FORMAL LAB REPORT 5

4. SIGNIFICANT FIGURES 7

5. UNCERTAINTIES AND STATISTICS 10
   5.1 The Most Probable Range of a Quantity 10
   5.2 How to Find the MPV and the MPR with Statistical Methods 11
   5.3 Determining the Absolute Uncertainty from a Single Reading on an Incremental Scale 13
   5.4 The Absolute Uncertainty for a Digital Readout 14

6. TECHNIQUES OF COMPARING TWO VALUES TO ONE ANOTHER 15

7. UNCERTAINTY PROPAGATION 17
   7.1 The Two Basic Types of Uncertainties 17
   7.2 The Sources of Uncertainties in Measurements 17
   7.3 Finding the Total Uncertainty in a Measurement When *Both* Systematic and Random Uncertainties Exist 19
   7.4 Finding the Uncertainty in Calculations From Measurements 19
   7.5 The General Formula for Determining the Absolute Uncertainty in a Function of Several Variables 22

8. THE GAUSSIAN DISTRIBUTION 25

9. CONFIDENCE 28

10. REJECTING DATA 30

11. GRAPHING TECHNIQUES 33
11.1 The Basic Graph 33
11.2 Determining the Best Fit Line From Statistical Methods 33
11.3 Finding the Uncertainty in the Slope and y-intercept 35
11.4 Including Error Bars on a Graph and How to Use Them 37

12. COMPUTER USAGE 39

APPENDIX A A guide to the physics 4 lab sequence 40

APPENDIX B An optional evaluation form 41

APPENDIX C A guide to equipment care and lab cart handling 42

APPENDIX D A table of probabilities from the error function 43

BIBLIOGRAPHY 45

INDEX 47

PREFACE

The following articles cover a wide variety of techniques important in laboratory work. They range from the proper and useful keeping of a lab book to statistical methods in finding the best fit line to a set of data. Much of the content is based on conventional and accepted techniques practiced by research physicists and is intended to bring the student as closely as a lower division program can allow, to the activities of physicists in a research environment. This work does not offer any in-depth justification or mathematical proofs of the techniques used. The curious student who wants to know "why" is welcome to study more sophisticated works found in the bibliography.

Since lab practices are not entirely uniform, check with your lab instructor to see what points are emphasized over others. Some techniques require the use of partial differentiation which is often beyond the mathematical maturity of the student. The statistical methods are calculator intensive and the student who does not have a calculator with statistical functions will be badly handicapped.

Appendix C contains guidelines for the care and handling of lab equipment. Please read it carefully and be courteous to the other labs to keep the physics program running smoothly.

1. LAB BOOK POLICY AND PROCEDURES

1.1 The Minimum Amount of Material Required to Bring to Every Lab

1. A **lab note book** (Which may or may not be kept in the laboratory for the entire quarter. See you instructor.). The note book must be of the size: 10" X 7 7/8", be "quadrille ruled", and either one of two style numbers: 26-151 or 26-251 or 43-475.

2. A **ruler**. A #36 "C-thru" ruler is acceptable or anything else that is better. It is available in the college bookstore.

3. A **scientific calculator**. Trig and log functions are required. Statistical functions including linear regression are recommended.

4. A **pen and pencil**. A pencil is used to plot data points on a graph, otherwise a pen is used.

5. This **LAB SKILLS MANUAL** and the **4A LAB EXERCISES**.

1.2 Preparing Your Lab Book

Put your name and section number on the outside front cover. Save the first two pages for a table of contents. If you intend to write on both sides of a page, then number *both sides* of each page at the top right corner (or anywhere else, but be consistent); number *all* the pages in your lab book before the class begins. Add to the table of contents as the lab progresses.

Two rules to follow:

1. Write in the lab book with pen only. This eliminates the temptation to erase what you have written because you think you have made a mistake. *Do not erase anything you have written in your lab book*. *Do not use White Out*! If you think you've made a mistake, cross out with one line what you have just written (so it can still read) and re-write the correct version near or following it. The reason you don't want to erase *anything* in your book is that occasionally what you thought was a mistake turns out to be correct. You don't want to lose original information. Some people object saying this policy makes the lab book messy. A "messy" lab book is not a bad lab book. Of course, you must be able to read and locate data in your book, but a pristine lab book is usually contrived and artificial. Real lab books are not perfect and beautiful records of your lab work; they are written logs of what you have done *while you are doing it*! As such they may not always be beautiful looking. Do not, however, interpret these remarks to mean a sloppy book is desirable. Your book must be a clear, readable, chronological "diary" of tasks performed in the lab.

2. Absolutely no scratch paper is allowed. That includes any loose papers in your lab book. Loose or scratch paper of any kind will be thrown away by your instructor as soon as they are spotted and any work done on these will be lost. *This rule will be strictly enforced*! This ensures all original data, calculations and ideas are entered chronologically so that nothing is lost. The mentality that you take the original data on a piece of scrap paper and later transfer it to your book so it will look neat is wrong since it allows for mistakes to be made during that data transfer.

1.3 Graphing in the Lab Book

1. "Trend-analysis": When appropriate, plot a trend analysis graph *while the data is taken and recorded in a table*. This technique is useful for spotting blunders and lesser mistakes as soon as possible so that they can be corrected *while you are still taking the data*. This type of graph may be rough as a finished product, but its purpose is not for high-level accuracy. The size of this type of graph can vary but usually will be at least a half page.

2. "Quality": A hand-constructed graph, perhaps with computer-assistance, used for quantitative, graphical analysis. This graph should be a half page or larger in size.

3. "Computer": A graph suitable for reports and publications. This graph may be small in size and may be accompanied by the computer's statistical analysis. They must be securely taped into your lab book with scotch-style tape *all the way around the graph*. Don't use staples.

In principal, all the experiments are designed so that no work need be done outside of lab. At the discretion of your lab instructor, lab books may be removed from the laboratory so that extra work can be done outside of class.

2. LAB BOOK STRUCTURE

A lab book should have a discernable organization to it. The exact structure depends on the kind of experiment being performed. But all good lab books have one thing in common: the ability to yield information upon examination. Being able to retrieve information from a lab book is the mark of a successful book.

When beginning an experiment, you may be eager to start taking data immediately. Resist this temptation, as it leads to disorganized thinking and a disorganized lab book. First, organize your thoughts in your book. Design the experiment before taking measurements. Don't be surprised if, as the experiment progresses, the course of your procedure has to be altered after new insights are gained.

Underline section headings so they stand out to the reader. All sections of your lab book should be clearly titled and organized. A good lab book represents the flow of your work *as it actually unfolded during the lab*. Below, the standard sections of a typical lab book are listed. Use what fits your needs.

Purpose or Objective:
Every experiment tries to accomplish something. State your goal clearly. Proclaim your expectations.

Equipment List:
Record a list of the equipment used. Write down the serial number, model, and make of the equipment so you can get the same equipment if you need to repeat the experiment or take additional data.

Introduction:
Present relevant historical and background information. Motivate the necessity of the experiment.

Theory:
Derive pertinent equations and explain relevant concepts. Use diagrams as much as possible. Develop the basis for your uncertainty analysis here.

Procedure:
Discuss your plan for performing the experiment. Don't describe the content of your *entire* lab. Write down your experimental plan. Take measurements in a step-wise, linear fashion. This section describes in detail *your* procedure for performing the lab such that you or anyone else could re-create the experiment exactly as it was performed. If you decide to change your procedure as the lab progresses, make sure the change is labelled as a **procedure** and that you go back in your book to the original procedure section and clearly label page references to the change. Draw labeled diagrams to illustrate how the equipment is set up and how it is used.

Data:
Record all your measurements in a clear and readable fashion.
Use "table" form for easier reading.

Calculations:
Show one "sample calculation" in complete detail, then write the results only of other similar calculations in table form. Perform uncertainty calculations here.

Results:
Show the final relevant numerical results. Boxing important values will allow anyone quick access to your final calculations.

Graphs:
Draw your graphs while the data is being taken, point by point, so measuring errors and blunders can be easily detected ***before*** you stop taking data. For better quality graphs drawn after the data taking, use a computer if available. All quality hand-drawn graphs should occupy at least half of one full page. Create your data points with a sharp pencil. Never "connect the dots" on your graph one point to the next! Draw an "eye-ball" best fit line through the data points using a ruler if a linear relationship is expected. Remember that the "best fit" line through the data may not intersect even one data point.

Analysis: Compare your calculations to your expected results. Additional graphs may be helpful.

Discussion of Results: Discuss the significance of your results. Sometimes an idea that comes to you spontaneously in the lab is only recalled if jotted down somewhere in your lab book. Such an idea would be discussed in more detail in this section. Use complete sentences for clarity. Avoid discussing "human error" unless outright blunders were made. Use numbers with uncertainties. Discuss areas for improvement in technique, procedure and equipment. Discuss methods to eliminate or minimize systematic errors.

Conclusion: Address the experimental objective and state whether it was accomplished.

3. THE STRUCTURE OF A FORMAL LAB REPORT

Typically in a research environment, a lab book is kept as a complete, written, ongoing record of that research. When that research is completed, the time has come for a paper to be written announcing the results. Hopefully, the findings are published in a journal. This section describes what is done *after* the experiment is completed. This section has nothing to do with writing in a lab book, but is concerned with the communication of lab findings to the public via the accepted method - a published report of the experiment. See your instructor for actual published reports in physics journals so you can see for yourself what the real thing looks like.

Writing an abstract: Not *every* report contains an abstract, but it is a common, if not required, feature of many publications.

An abstract is a brief summary or synopsis of an experiment.

An abstract is *not* a conclusion and an abstract will not contain the details of the experimental procedure nor equations or derivations. An abstract is written *after* the experiment is completed.

An abstract should contain no more than three or four complete sentences all in *one* paragraph. Let the first sentence state what you tried to do. The second sentence will describe what was actually done. The final one or two sentences will discuss the significance of the results.

When appropriate, the result is often expressed numerically. Typically, the significance of the result is related to how close the result was to the expected value (i.e., the theoretical prediction); in this context "how close" depends on the method of analysis and the uncertainties involved, but it should also be expressed numerically.

Writing an abstract is a particularly important skill. Abstracts are the "teasers" read by many other scientists to see if an experiment is interesting enough or relevant enough to warrant further investigation. By far, most people will only ever read the abstract of a publication. When investigating a subject to see what previous work has been done, it is a collection of abstracts that is first examined. A good academic library will contain numerous volumes devoted only to abstracts in physics, chemistry and many other fields of research.

If you are submitting a full report, the abstract is placed at the beginning of the report just after the title and authors. If you are submitting only an abstract, then on a single page it should contain:

1. The title of your experiment and the date it was performed.
2. The author's names.
3. The abstract.

More about a full report:

A successful report would allow the reader to duplicate the basic experiment; it is by this method that your results can be checked and confirmed.

Uncertainty analysis is discussed as relevant. All uncertainties should be explicitly justified; explain *why* quantity x has an uncertainty δx.

Graphs are a common and useful way to explain results; use them whenever possible. Graphs should be of high quality, preferably computer generated.

Of course, a summary or conclusion is the last part of your report. In the summary, you present the results of the experiment, explain the significance of your findings and outline possible improvements. Discuss areas for improvement in the measuring technique, procedure, and equipment. Discuss the possibilities for further research.

If references are made to previous experiments during your write-up, the complete title of the reference will be made in a bibliography. See your instructor or library for examples of professional physics publications and abstracts as an additional guide to writing them.

4. SIGNIFICANT FIGURES

A *significant figure* is a digit in a number representing a quantity, other than a leading or trailing zero introduced *only* to mark the decimal place position. The meaningful digit in a number furthest to the right is called the *least significant digit*.

One use of significant figures allows a quick and casual approximation of the final value of calculations. Also, looking at the amount of significant figures in a number gives an indication of the precision of that value. As an example, if you divide 2 by 3, your calculator will display 0.666666667 or as many sixes as the display will allow. It seems unreasonable that values containing only one significant digit (2 and 3) could yield a calculated answer nine digits long. This example violates our intuitive notion of how measured or known values yield final values in calculations. You can't get more precision than what you started with.

Use the following rule in writing final answers involving division or multiplication:

A final calculated value has the same number of significant figures as the <u>least</u> amount of significant figures found in any one factor of the calculation.

In the above example, since there is only one "sig fig" in each initial value, the final value cannot have more than one sig fig. The correct answer is: $2 \div 3 = 0.7$. See that the zero in front of the decimal point is used to mark the decimal place (this is considered good form); this zero is *not* a significant digit.

In addition and subtraction, use the significant figures to the right of the decimal place to guide your final answer.

Example: 234.45 + 52.432 = 286.88.

Here, 234.45 has only two digits to the right of the decimal place so the answer will have two digits to the right of the decimal place as well.

When using transcendental functions, the significant figure found in the argument of the function is equal to the significant figures stated in the result.

Example: ln(11.24) = 2.419; four sig figs in each.

Sometimes it is impossible to tell how many significant figures are stated in a vaguely written value.

Example: How many significant figures are in this value: 2100?

The answer is ambiguous. There are three possible answers: two, three, or four. The two zeroes after the 21 make the answer unclear. If the two zeroes are both not significant, they serve only to mark the position of the decimal place. In that case the answer would be: 2100 is two sig figs. But perhaps the zero right after the 21 *is* significant but the next zero is not. Then the correct answer would be three significant figures. Finally *both* zeroes may be significant in which case the correct answer would be four significant figures. In this last case some people would say "2100**.**" is the correct way of writing this value if both zeroes were significant. This convention is fine, but is not universally followed so be careful.

We have just seen that the existence of zeroes in a number may confuse the issue of how many sig figs the number contains. To avoid this ambiguity, use "scientific notation".

Stating numbers in scientific notation eliminates *all* ambiguities with regard to how many significant figures a number has.

In the previous example, 2.1×10^3 is a number with two sig figs, 2.10×10^3 has three sig figs, and 2.100×10^3 has four significant figures. Written in this fashion there is no problem determining exactly how many significant figures are stated in any value.

Another method besides scientific notation can be used to avoid ambiguities in stating a result. Place a line above or below the *least* significant digit. As an example $24{,}0\overline{0}0$ has four significant figures.

These methods should be used in the solution of homework problems. In formal lab work however, these "rules of thumb" for significant figures are not accurate enough. Instead, the significant figures of a final calculated result are determined by statistics and uncertainty propagation. In some of the following sections these methods are discussed.

Here are some more examples. In one or two complete sentences, you should be able to explain how the significant figures in each have been obtained.

13.0010 -- six sig figs.

0.006 -- one sig fig.

0.0060 -- two sig figs.

1402 -- four sig figs.

1200.00 -- six sig figs.

1200 -- ambiguous; could be two, three, or four sig figs.

1.20×10^4 -- three sig figs.

100. -- three sig figs.

Arithmetical examples:

11.21 x 1.23 = 13.8

Multiplication: 13.8 has three sig figs; this matches the *fewest* amount of sig figs used in the calculation: 1.23

(0.324)(2.2)/3.0012 = 0.24

Multiplication and division: 2.2 has the least amount of sig figs (two sig figs), so the answer has two sig figs.

8.1 + 9.3 - 4.5 + 6.5 = 19.4

All numbers have *one* significant digit to the right of the decimal place so the answer has *one* significant digit to the right of the decimal; note the answer has *three* sig figs while all the other numbers only have two.

5. UNCERTAINTIES AND STATISTICS

Any scientific number associated with measured values never has an *exact*, single value but instead a range of possible values. Sometimes a measured scientific value (say for example, the speed of light) is expressed as a single number with no uncertainty, in that case nevertheless there is still a convention that determines the interval of possible values. This interval of values is known as the *Most Probable Range* (MPR) of the measurement.

5.1 The Most Probable Range of a Quantity

There are two ways to write a scientific value which clearly states the MPR of the value: an "implicit" MPR and an "explicit" MPR.

In the implicit method, the way commonly used in textbooks, a measured quantity is printed as single value. We might say the speed of light is 3.00 X 10^8 m/s for example. To find the most probable range of this quantity use this rule:

If a measured quantity is expressed as a single value with no uncertainty, then the most probable range is given as that quantity plus or minus one half the unit of the last decimal place stated.

As an example of this technique say the recorded measurement of a length is 3.5 centimeters. This does *not* mean the length is *exactly* 3.5 cm, but that the length lies somewhere between 3.45 cm and 3.55 cm. So even though only one number was stated, an MPR is implied. Again for the speed of light, we see its uncertainty would be ± 0.005 m/s (Plus or minus half the last **unit** stated, 0.005 = ½ · 0.01. Remember that here, "unit" means the number one). So the most probable range for the speed of light would be:

(3.00 - 0.005) to (3.00 + 0.005)

or 2.095 to 3.005

(2.095 - 3.005) m/s

For another example of the implicit method, in Tipler's physics text, the value for the acceleration due to gravity at the earth's surface is stated in the front leaf cover as g = 9.81 m/s^2. Assuming this is a value obtained from measurements and *not* an exact value, see if you can apply the above information and write down the MPR for this printed value. The correct answer is given as a footnote at the bottom of this page, but don't look until after you've tried to get the right answer.[1]

As an example of the explicit method, to write 3.5 centimeters with an explicit most probable range of course we could just write out the range {e.g. MPR = (3.45 - 3.55) cm}, or as is more commonly done, we could write the value as a best or most probable value (MPV) plus or minus what is called the "absolute uncertainty" in the MPV, δ_AMPV. This method is the second way of stating the MPR of a measurement and is preferred. In the above example the format for this method would be as follows:

$$(L_{best} \pm \delta_A L) = (3.50 \pm 0.05)\ \text{cm}$$

Call this the "standard form" of the measurement.

See that the MPV is not written as 3.5 but as 3.50 so that the decimal places agree between the MPV and the absolute uncertainty (both are to the hundredths of a centimeter or two decimal places). This seems odd since it implies we gained precision, but in fact the precision was always to the hundredth place even though the number was originally written to just the tenths; such is the way of this technique.

5.2 How to Find the MPV and the

Average value:

$$\bar{X} = \frac{\sum_{i=1}^{N} x_i}{N}$$

Standard Deviation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} (\bar{X}-X_i)^2}{(N-1)}}$$

Standard Error:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}}$$

[1] 9.81 has its last place in the hundredths column. Half of 0.01 is 0.005. So the MPR is(9.805 - 9.815) m/s^2.

MPR with Statistical Methods

If a measurement can be repeated and is expected to yield the same result each time, then a statistical method for associating a most probable range with the measurement is used. The following method is valid only if at least five repeated measurements can be taken. Formally, there are other conditions that must be met to apply these methods, but we will ignore these for the present time.

From a given set of data values x_i, the best or most probable value is simply the *average value*, $\overline{x}$. The absolute uncertainty, the uncertainty in the mean value, is the so-called "standard deviation of the mean" or the "standard error". The conceptual meaning of these terms will be discussed in more detail below.
The arithmetical formulas for the values are given below:
The formulas stated above are in terms of the usual statistical symbols. To translate these symbols into the ones used in uncertainty analysis follow this guide:

The best value is the mean value: $L_{best} = \overline{x}$
The absolute uncertainty is the standard error: $\delta_A L_{best} = \sigma_{\overline{x}}$

Example: Say five values have been measured that *in principle* should all be the same and only random errors are assumed to exist in those measurements.

X_i	(cm)
X_1	4.4
X_2	4.3
X_3	4.4
X_4	4.2
X_5	4.5

From the formulas given above you should *explicitly* calculate the mean and standard error of this data set and confirm that its value is : (4.36 ± 0.05)cm. See that the MPR associated with this value is:

4.31 (4.36-0.05) cm to **4.41** (4.36+0.05) cm

Note that although the measurements were taken to only one decimal place (e.g., 4.**4**, 4.**3**, etc..), by using the statistical method *two* decimal places for the best value were obtained (4.**36**). We see that by repeating the measurement of a value that should be the same value in *principle* each time, the more times we repeat the measurement the greater the precision of our result. Remember, the uncertainty in $\overline{X}$ is *not* the standard deviation but the standard error. The standard deviation is the absolute uncertainty in any individual element of the data set. It is also conventional to write the uncertainty in the most probable value to one significant figure only.

The MPV is rounded to the same place as the absolute uncertainty.

In the above example, you might have calculated the standard error to be 0.05099 cm. However you will always round the absolute uncertainty to one significant figure- that is, 0.05. Then that value, 0.05, will determine that the best value is rounded to the hundredths place as well.

See that the uncertainty has the same units as the measured value. In the above example, both the mean and standard error have the units of centimeters.

5.3 Determining the Absolute Uncertainty from a Single Reading on an Incremental Scale

When taking a measurement from an instrument (analog or digital) there is a "rule of thumb", non-statistical method which enables the determination of the absolute uncertainty without the repetition of measurements. Although the statistical method referred to above is more reliable than this method, there are times when the rule of thumb is useful.

First, the term "**least count**" must be defined. Least count refers to the smallest increment of measure on the measuring device used. As an example, on a meter stick, the smallest increment of measure is typically a millimeter. So the least count would be one millimeter and the absolute uncertainty associated with a meter stick reading would be one half a millimeter.

The absolute uncertainty in a scale reading is equal to one half the least count:

$$\delta_a x = \frac{1}{2}\text{ (least count)}$$

The absolute uncertainty from an analog scale (e.g., a meter stick or pan balance) is equal to one half the least count of the scale. As an example, say a reading on a meter stick was 34.23 cm, interpolated to the hundredths place. The least count of the meter stick is 0.1 cm (or 1 millimeter). So the absolute uncertainty is one half this value or 0.05 cm. We would then write the measurement in standard form as: (34.23 ± 0.05)cm.

Instead of using plus or minus half the least count as the rule of thumb, some people prefer to use plus or minus *one tenth* the least count. You should check with your instructor for the details, however one thing should be kept in mind; whether you use one half or one tenth or something else, your choice reflects your own thinking about how accurate you can interpolate the increments you are reading. Your choice of one half or one tenth the least count as the absolute uncertainty may vary from one instrument to the next. If the least count increments are closely spaced as with a meter stick, you may think plus or minus half the least count is appropriate, but if you are reading an analog voltmeter with a wide spacing between each increment, plus or minus one tenth the least count may be appropriate.

Note the "decimal agreement" between the measurement and its absolute uncertainty; they are both rounded to the same decimal place. This requires the technique of "interpolation" or reading between the increments of a scale to push the precision of the instrument to its limits. Interpolation is a standard technique used when reading analog scales and is a skill honed by practice.

5.4 The Absolute Uncertainty for a Digital Readout

For a **digital readout**, the absolute uncertainty is <u>equal</u> to the least count (not one half the least count) or more specifically, the absolute uncertainty is equal to the unit of the last place of the readout. For example, if a digital balance displays 73.255 grams for the mass of an object, then $\delta_a m$ = 0.001 grams. So the measured mass of the body would be:

(73.255 ± 0.001) grams

You cannot interpolate a digital display.

6. TECHNIQUES OF COMPARING TWO VALUES TO ONE ANOTHER

The discrepancy test or the percent uncertainty test is a common method of comparing two numbers to see how close they are to one another.

To see the usefulness of this method imagine you want to compare the age of two people, 12 and 14. You could say they are two years apart and that is correct, but you could also say that two people aged 65 and 67 are also two years apart. Somehow being two years apart in age is a greater difference when you are 12 or 14 than when you are 65 or 67. This greater difference in a two year separation of age for younger people than older people is clearly understood when you apply a discrepancy test to each pair of ages.

Say two numbers that should be the same are to be compared: x and y. To compare them with a discrepancy test, subtract the two numbers and divide by one of them.

$$\frac{(x - y)}{y}$$

Which one of them you divide by is a matter of opinion. If you are comparing an experimental value to a known or theoretical value, then divide by the theoretical value. If you are comparing two values neither of which are known to be "correct" often you divide by the average value of the two numbers.

Although it yields no new numerical information, many people prefer to express the fraction as a percentage. To express a fraction as a percent value, multiply the fraction by the number ONE in the form of 100 over 100, or 100 per 100, or at last 100 per cent (cent meaning one hundred).

Do not confuse the discrepancy test with the discrepancy between two numbers. The discrepancy between two numbers refers to their subtracted difference with no division. As an example, say your measured value is 35.2 cm and the expected value is 37.5 cm; the discrepancy between them is 37.5 - 35.2 = 2.3 cm.

See that a discrepancy test between the ages discussed above is 15% for the 12 and 14 year olds [{(14-12)/13}·100% (why divide by 13?)] and 3% for a 65 and 67 age gap. So as you get older the *fractional* difference in your age and a friend's age decreases in value.

The question arises whether you should take the absolute value of the subtraction in the numerator or instead, sometimes have a negative value for the result. Opinions vary, but most often do not take the absolute value since the negative value *does* convey information about which number is larger, x or y. Taking the absolute value of the result dispenses with that information with nothing else gained as compensation.

The discrepancy test does not consider the absolute uncertainty of either value. Also, see that the discrepancy test value is always unitless. Usually you will round your discrepancy test value to one or perhaps two significant figures.

Another technique of comparing two numbers to one another is that of relating the discrepancy between the two numbers to how their most probable ranges overlap. As an example, say two measured quantities that in principle are the same, are stated in standard form as follows:

$$24.5 \pm 0.3 \text{ and } 21.3 \pm 0.5.$$

The experimenter's question is, "Are these two values in agreement or not?"

First, see that the discrepancy between the two values is: 24.5 - 21.3 = 3.2. Secondly, considering the spread of the most probable range of each value (24.2 - 24.8 and 20.8 - 21.8), it is clear that the discrepancy is far greater than the most probable ranges would allow to declare that the values are in agreement. Thus, by this method we conclude the two values are *not* in agreement.

In another case, say only one of two numbers is presented with an absolute uncertainty, and the other number is stated without an uncertainty, then the numbers would agree if the number without an uncertainty lies within, or is at least close to, the most probable range of the number presented with an absolute uncertainty.

Example: Say we have two numbers: 34.56 ± 0.08 and 34.61. The MPR for the first number is (34.48 - 34.64). Since the second number, 34.61, is within the MPR of the first, we conclude these two numbers are equal within the experimental uncertainty given. This subject will be discussed again in section 9.

7. UNCERTAINTY PROPAGATION

Many experimental values are not just read from an instrument but are the results of calculations involving measurements taken from instruments. In the case of a calculated result, its absolute uncertainty is determined using the method of uncertainty propagation. Absolute uncertainty has already been discussed, however to understand how uncertainties propagate in calculations, we must first introduce some new terms.

7.1 The Two Basic Types of Uncertainties

1. **Absolute uncertainty**. Let the absolute uncertainty in a quantity *x*, be represented by the symbol $\delta_a x$. The absolute uncertainty has the same units as the quantity it is associated with. The absolute uncertainty, $\delta_a x$, and the quantity x are traditionally written in the "standard" form: $x \pm \delta_a x$. Consequently, the absolute uncertainty allows the determination of a *most probable range* {i.e., $(x - \delta_a x)$ to $(x + \delta_a x)$} wherein the measurement *x* "most probably" lies. All this was previously discussed in section 5.

2. **Relative uncertainty.** The relative uncertainty in a quantity *x*, is represented by the symbol $\delta_r x$. The relative uncertainty is calculated from its definition:

$$\delta_r x \equiv (\delta_a x)/x.$$

See that the relative uncertainty is always dimensionless.

Although the relative uncertainty changes as the quantity x changes, the absolute uncertainty is typically a constant. As an example, say the measured quantity *x* has a value of 2.3 ± 0.1. Then the absolute uncertainty is 0.1, but the relative uncertainty is $0.1/2.3 = 0.04$. If the measured value *x* now changes to 5.5 ± 0.1, then the absolute uncertainty is still 0.1 but the relative uncertainty has a different value, $0.1/5.5 = 0.02$.

7.2 The Sources of Uncertainties in Measurements

1. **Random uncertainties**: After all known sources of error have been considered, it is still impossible to duplicate readings *exactly*. When observations are repeated, the readings will fluctuate slightly about some mean or average value. Note that random uncertainties refer to the fluctuations in measurements that are *equally probable* in being too large *or* too small.

Precision: If an experiment has small random uncertainties, it is said to have high precision. Numerically, the precision of a measurement is equal to the relative uncertainty of the measurement (or as is it sometimes called the "fractional uncertainty").

That is: precision = $\delta_r x$.

2. **Systematic uncertainties:** Any metallic measuring instrument such as calipers or a metal tape will give readings which are either too large or too small if used at a temperature other than the calibration temperature. An instrument which is not properly set to zero before making readings will give readings which are wrong. These are examples of systematic errors and usually can be eliminated or reduced to a minimum in most laboratory procedures. Note that for systematic uncertainties, the variations in the measurements are either always too big or are always too small.
We can now define accuracy.

Accuracy: **If an experiment has small systematic errors, it is said to have high accuracy.**

3. **Instrument uncertainties:** Any measuring instrument may have inherent errors as a result of manufacturing. Most of the apparatus used in the laboratory will have sufficient accuracy that these errors can be corrected or will be marked by the manufacturer so that appropriate corrections can be made. As an example the manufacturer of a measuring instrument might state the systematic error associated with the instrument as 3% of the scale reading; consult the instrument's manual for this information and treat it as a systematic uncertainty.

4. **Human errors**: These are errors introduced by the experimenter. Not correctly estimating the reading on a caliper, forgetting to level an apparatus, or failure to consider parallax (see below) in reading a meter are all possibilities in virtually any experiment. Errors introduced by personal bias in trying to make the readings fit some preconceived idea of the result can also occur. All human errors and their effects can be reduced to a minimum by careful, proper laboratory technique.

5. **Parallax error:** Mentioned just above, this type of error (traditionally called an error although really an uncertainty) can be random or systematic depending on the situation. Parallax refers to the variation in the reading of a measurement as the experimenter moves her head side to side or up and down. This uncertainty occurs more severely as the scale of the measuring instrument moves farther away from the object being measured. Getting the instrument as close as possible to the object being measured will reduce parallax error.

More on accuracy and precision:

A measurement could be quite precise but completely inaccurate. For an example, say a meter stick was used to measure the length of a body as being L_{meas} = 23.45 cm (the 0.05 being interpolated). Although the absolute uncertainty in the stick is equal to one half the least count of the stick (i.e., $\delta_a L$ = 0.05 cm, where the least count of the meter stick is one millimeter), by other methods the "true" value of the length of the body might be L_{true} = 22.50 cm. We would say the measurement had a precision of: $\delta_a L/L_{meas}$ = 0.05 cm / 23.45 cm = 0.002 or 0.2%. Although the level of precision small, generally considered to be good, the measurement is inaccurate because the true value (22.50 cm) is quite different from 23.45 cm; a systematic uncertainty must be present. The most probable range of the measured value {(23.45 - 0.05)cm to (23.45 + 0.05)cm or (23.40 to 23.50)cm} does not contain the true value. This measurement is precise but not accurate.

The source of a systematic uncertainty may be a worn edge on a meter stick (so it is wise to *never* use the end of a measuring device) or may be due to thermal expansion or contraction. In the above example would the stick be too long or too short? Whatever the case, systematic uncertainties can be minimized by ensuring the measuring devices are properly calibrated before using them.

If we can assume the fluctuations in a repeated measurement, which *in principle* should always yield the same number, is due to **random** uncertainties only, then the "standard deviation" is a measure of the "spread" of the data about the mean value. If the standard deviation is large, then the precision of the measurement is poor; if the standard deviation is small then the precision is high. We say the measurement is accurate if the most probable range, {Recall that MPR = $(x_{best} - \sigma_{mean})$ to $(x_{best} + \sigma_{mean})$} contains the true value.

7.3 Finding the Total Uncertainty in a Measurement When *Both* Systematic and Random Uncertainties Exist

The total uncertainty in a measurement is equal to the systematic and random uncertainties added in "quadrature" (see the next section for more on quadrature). That is:

$$\delta_{total} = [\ (\delta_{random})^2 + (\delta_{systematic})^2\ ]^{1/2}$$

The question becomes one of finding the systematic uncertainty in the instruments used in the experiment. Every measuring apparatus has some amount of systematic uncertainty. But unlike random uncertainties, systematic uncertainties cannot be reduced by repeating a measurement N times. There are many clever techniques to minimize the systematic uncertainties inherent in instruments and apparatus. For example when using an air track, it is a common technique to "run" the experiment in two opposite directions hoping the systematic error from the track will then cancel.

A common method of expressing the systematic uncertainty associated with a measuring instrument is to give the uncertainty in a percent form. For example, a pan balance may be rated with a systematic uncertainty of 1% and a stopwatch with 2%. In this case the total systematic uncertainty would be found by adding the two uncertainties in quadrature (although the justification for this is not strong). Consult with the instructor to see how much you should consider the systematic uncertainty in the apparatus of a particular experiment.

7.4 Finding the Uncertainty in Calculations From Measurements

When quantities are determined by combining several measurements, as in adding two measurements to find a third value or calculating average speed by dividing displacement with a time interval, there are rules to determine how many significant figures should be retained in the calculated result.

Finding the absolute uncertainty in an addition or subtraction:

When adding or subtracting several measured quantities, Q_1, Q_2, and Q_3, each with absolute uncertainties $\delta_a Q_1$, $\delta_a Q_2$, and $\delta_a Q_3$, the absolute uncertainty in the final result, $\delta_a Q$, is given by:

$$\delta_A Q = \sqrt{\delta_A^2 Q_1 + \delta_A^2 Q_2 + \delta_A^2 Q_3}$$

Notice that there is a "Pythagorean" sum involved in the definition. This is called "addition by quadrature" and is the correct method if it can be assumed that the uncertainties are independent of one another. Sometimes for simplicity, an algebraic sum is used instead, but this yields an unnecessarily larger final value.

Example: Say the width of a table top is measured by placing a meter stick on the table (*do not* use the end of the stick). To compute the width and its absolute uncertainty you must read the value of one end of the table on the stick, l_i, and the value of the other end of the table on the stick, l_f. The width of the table top is computed by subtracting l_i from l_f. And the uncertainty is computed by *adding* in quadrature the absolute uncertainty of each measurement. Note that even though a subtraction is involved, the uncertainties are always added. Let $l_f \pm \delta_a l_f = (85.38 \pm 0.05)$ cm and let $l_i \pm \delta_a l_i = (23.46 \pm 0.05)$ cm. Then $l_{width} = 85.38 - 23.46 = 61.92$ cm, and $\delta_a w = (0.05^2 + 0.05^2)^{1/2} = 0.07071 = 0.07$ cm. See that the absolute uncertainty is always rounded to one significant figure. So the final value for the width of the table is: (61.92 ± 0.07)cm. Notice the absolute uncertainty in the calculated result (0.07 cm) is larger than the uncertainty in the measurements (0.05 cm). *This* is what is meant by uncertainty propagation; the uncertainty usually propagates (i.e., increases) when we perform calculations.

Finding the absolute uncertainty when multiplying or dividing:

When multiplying or dividing several quantities each with their own relative uncertainties $\delta_r Q_1$, $\delta_r Q_2$, and $\delta_r Q_3$, the relative uncertainty, $\delta_r Q$, in the result is given by:

$$\delta_R Q = \sqrt{\delta_R^2 Q_1 + \delta_R^2 Q_2 + \delta_R^2 Q_3}$$

Example: Say the width of a table top has been measured to be (61.92 ± 0.07) cm and the length has been measured to be (235.23 ± 0.07) cm. Find the area of the table top with its absolute uncertainty.

This involves multiplication, so to find the absolute uncertainty in the area, we must first add the *relative* uncertainties by quadrature, then multiply this result by the calculated area. You should complete the algebra to prove:

$$\delta_a A = A\sqrt{\delta_r^2 l + \delta_r^2 w}$$

So in this example, $\delta_a A$ =

$$(61.92)(235.23)[0.000296^2 + 0.00113^2]^{1/2} = 17.01426 \text{ cm}^2 = \mathbf{20\ cm^2}.$$

Remember, the absolute uncertainty is rounded to one significant figure. Since the area is equal to 14565.4416 cm^2, we then round the area's value to match the place of the absolute uncertainty. The absolute uncertainty is rounded to the "tens" place so the area *must* be rounded to the tens place as well. The final result is:

(14,570 ± 20) cm^2 (how many sig figs are in the main value?)

The place to where the absolute uncertainty is rounded determines the place to where the calculated value is rounded.

In the numerical answer above, 14,570 has *four* significant figures. The precision of the value is 20/14,570 = 0.14% which is small. Having a small precision, we would expect the random uncertainty in the measurement to be small also.

In order to determine the number of significant figures in your final calculation, you use the place to where your absolute uncertainty is rounded. The absolute uncertainty determines the significant figures in your final value and the absolute uncertainty is always rounded to one significant figure.

Finding the absolute uncertainty for quantities raised to a power:

When a quantity is raised to a power, the absolute uncertainty in the final value is obtained by multiplying the final value with the relative uncertainty in the quantity times that power to which the quantity is raised.

For example if a measurement has the value:

$L = (34.5 \pm 0.2)$ cm and we wish to find the absolute uncertainty in $L^{½}$, then:

$$\delta_A L^{½} = ½L^{½}(\delta_A L/L) = ½(5.8738\ cm^{½})(0.2/34.5) = 0.01703\ cm^{½} = 0.02\ cm^{½}$$

So the final value in standard form is: $(5.87 \pm 0.02)cm^{½}$

7.5 The General Formula for Determining the Absolute Uncertainty in a Function of Several Variables

Every calculation involves a formula. The formula can be thought of as a function where the calculated result is a function of perhaps several variables. As an example, in calculating an average speed, we know $v = \Delta x/\Delta t$. We would say the speed is a function of the two variables, Δx and Δt. There is a general method for finding the absolute uncertainty in a calculated result if we know the function used to get the result. The following information requires a familiarity with partial differentiation. Consult your instructor to see how much it will be emphasized.

$$\delta_A q = \sqrt{\left(\frac{\partial q}{\partial x}\,\delta_A x\right)^2 + \ldots + \left(\frac{\partial q}{\partial z}\,\delta_A z\right)^2}$$

The absolute uncertainty $\delta_A q$ when q is a function of several variables x,..., z each of which has an uncertainty of $\delta_A x, \ldots, \delta_A z$.

As an example, say a quantity *y* describes how far an object freely falls. This quantity then is a function of three variables: v_i, g, t, where these variables physically represent the initial velocity in the downward direction, the free fall acceleration, and the time of flight of the body. From kinematics: $y = v_i t + \frac{1}{2} g t^2$. So $y = y(v_i, t, g)$. Note that all quantities are directed downward.

In standard form let:

$v_i = (2.4 \pm 0.2)$ m/s
g = (9.80 ± 0.01) m/s^2
t = (3.45 ± 0.05) s

Question 1: Find $y \pm \delta_A y$ analytically (i.e., algebraically) and then numerically.

Question 2: Of the three given quantities, v_i, g, and t, which has the most important effect on *y*'s uncertainty and which has the least effect on *y*'s uncertainty?

Also, calculate $\delta_A y$ using all three uncertainty terms (this is to be done in question 1 anyway); then calculate $\delta_A y$ with only the largest of the three uncertainties. Compare the two values.

The solutions are presented on the next two pages. See if you can get the answers before looking at them.

Solution to question 1:

First find the partial derivatives, then substitute into the general formula.

$$\frac{\partial y}{\partial t} = (v_i + gt)$$

$$\frac{\partial y}{\partial g} = \frac{1}{2}t^2$$

$$\frac{\partial y}{\partial v_i} = t$$

so...

$$\delta_A y = \sqrt{[(v_i + gt)\delta_A t)]^2 + (\frac{1}{2}t^2\delta_A g)^2 + (t\delta_A v_i)^2}$$

Note that in this case the general formula has three uncertainty terms within the square root: $\delta_A t$, $\delta_A g$, and $\delta_A v_i$.

Now substitute the numbers.
$(\{[2.4+9.8(3.45)]0.05\}^2+\{(0.5)(3.45)^2(0.01\}^2+\{(3.45)(0.2)\}^2)^{½} =$

$1.938 \cong 2$ meters.

and we have,

y(t=3.45) = (2.4 m/s)(3.45 s) + (½(9.80 m/s^2)(3.45 s)^2 =

66.602 meters.

so ...

$y \pm \delta_A y = (67 \pm 2)$ meters

Solution to question 2:

The precision of v_i is 0.2/2.4 = 8%,: the precision of g is 0.01/9.80 = 0.1%, and the precision of t is 0.05/3.45 = 1%.

Considering that the precision of v_i is the greatest by a wide margin, one would guess that the effects due to its uncertainty probably dominate the others of g and t. To confirm this, substitute for the quadrature terms in the general formula.

For the $\delta_A t$ term: $\{[2.4+9.8(3.45)]0.05\}^2 = 3.28 \cong 3$
For the $\delta_A g$ term: $\{(0.5)(3.45)^2 (0.01\}^2 = 0.00354 \cong 0.004$
For the $\delta_A v_i$ term: $\{(3.45)(0.2)\}^2 = 0.476 \cong 0.5$

In this case, clearly the uncertainty in the time t, not the initial velocity as was first suspected, has the most significant effect on the calculation of the absolute uncertainty in *y*. In this case, the squared value of time in the defining equation ($y = v_i t + ½gt^2$) increases the sensitivity of the final uncertainty on time values.

By what factor must $\delta_A t$ be reduced so that the order of magnitude of its uncertainty term in the general formula is the same as that for the initial velocity term?

To compare the value of $\delta_A y$ with and without the two least significant variables (g and v_i), first consider the absolute uncertainty in *y* only dependent on the $\delta_A t$ term:

$\delta_A y = [2.4 + (9.80)(3.45)]0.05 = 1.81 \cong 2.$

Recall that $\delta_A y$ computed to one significant figure using all three terms in the general formula is the same value as above, namely 2.

The point of this discussion is that uncertainties do not play equally weighted roles in determining a final calculated uncertainty. One uncertainty term may dominate the others in the general formula. This being the case, the experimenter has little to gain in increasing the precision of the other less significant terms until all the uncertainty terms under consideration are of the same order of magnitude. Realizing this may save fruitless labor when seeking meaningfully to improve experimental results.

Don't improve the precision of a measurement that, as a consequence, does not <u>significantly</u> improve the precision of the final desired calculation. The precision of the largest

dominating term in the general formula must first be improved to match the precision of the other terms.

8. THE GAUSSIAN DISTRIBUTION

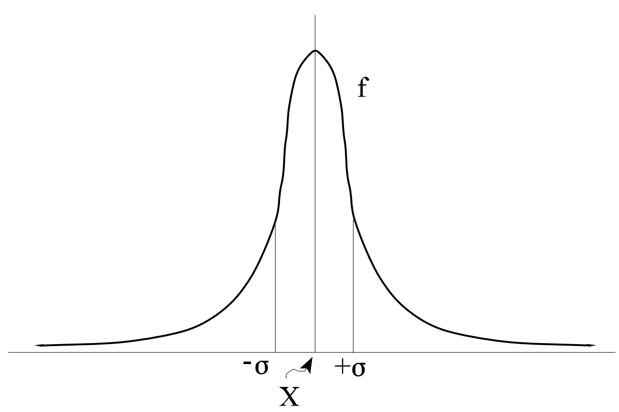
The statistical analysis of the previous section assumed that *all* the uncertainties involved were from the random variations associated with taking measurements. The primary mathematical tool used in describing the properties associated with random uncertainties is called the Gaussian distribution or simply a "Gaussian". This is the same function as the "normal" distribution and the "bell curve".

The purpose here is not to undergo an in-depth analysis, but to show how to employ this function for practical use. The function is an exponential:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-X)^2/2\sigma^2}$$

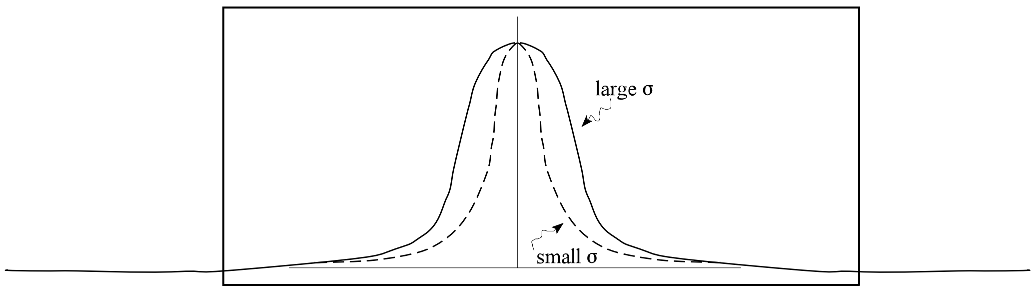
where σ is the standard deviation and X is the mean value.

From probability theory, the coefficient of the exponential is chosen such that the "area" under the Gaussian is equal to one. The function is then said to be "normalized".



A Gaussian distribution showing the standard deviation and mean value.

The standard deviation is one measure of how closely packed the data is about the mean value. In terms of measurement theory, this is equivalent to saying that a small standard deviation means the uncertainty in the most probable value, *X*, is small. Whether the standard deviation, σ, is small or large, the normalized Gaussian still has an area under the curve equal to one.



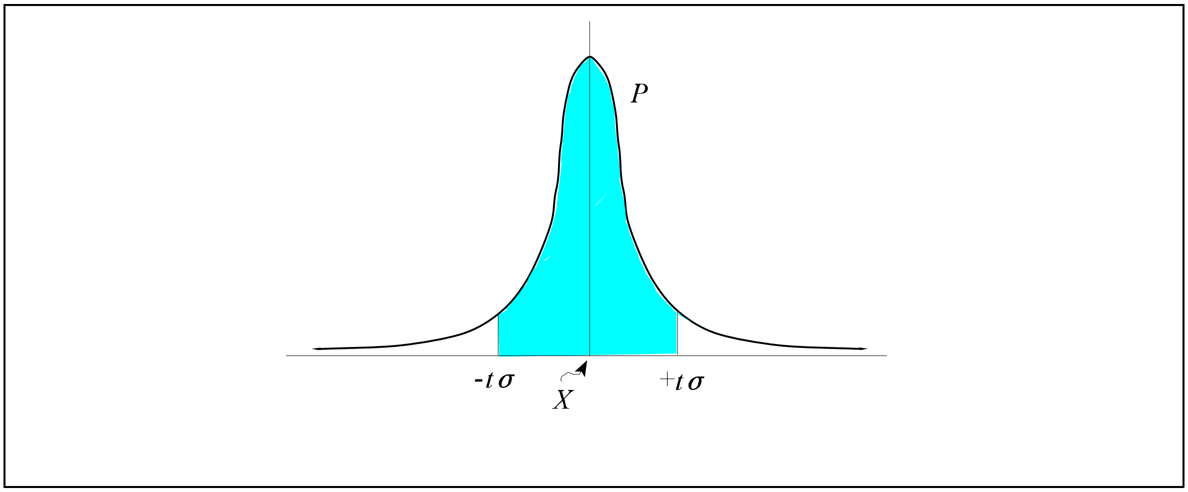
We are interested in using the Gaussian distribution to calculate the probability that some given measurement will lie within a certain range about the mean value. This range is typically expressed in terms of standard deviations. That is, we ask the question, "what is the probability of a measurement being within $t\sigma$ of X?" Where t is any real number but often, for simplicity, is just an integer. For example, if $t = 1$, then we are concerned with the probability that a particular measurement lies within *one* standard deviation of the mean.

If we make the substitution: $(x - X)/\sigma = z$ then it can be shown that the probability P that a measurement lies within $t\sigma$ of the mean value is:

$$P(\text{within } t\sigma) = \frac{1}{\sqrt{2\pi}} \int_{-t}^{t} e^{-z^2/2}\, dz$$

This function has the special name, the *error function*, and is denoted as erf(t). As is typical, for a value t the probability P is looked up in a table since the integral cannot be evaluated analytically. That table is found in appendix D. Again, it is not the purpose here to prove or derive these findings. Consult the bibliography for more in-depth studies.

Graphically, what we are doing is shown below:



The shaded area under the Gaussian represents the probability that a measurement lies with tσ of the mean value *X*. This shaded area is always *less* than one.

From the table found in appendix D, verify that the probability a measurement lies within one (t = 1) standard deviation is 0.68 or 68%. The probability that a measurement lies within two standard deviations (t = 2) is 0.954 or 95.4% and for 3σ, P = 99.7%. See that as t becomes large the probability quite rapidly approaches one.

It is sometimes useful to know the probability a measurement lies *outside* the range of ± tσ. Then:

$$P(\text{outside } t\sigma) = 1 - P(\text{within } t\sigma).$$

This form is useful when discussing "confidence" as seen in the next section.

9. CONFIDENCE

It is useful to state one's findings by associating a numerical "confidence" with them. Typical statements might be: "We are ninety-five percent confident of our result." or "This result has a seventy percent level of confidence." The information from the last section provides a straightforward way of interpreting one's results with a confidence estimate. This method is useful in the common experimental situation of comparing a calculated result to an expected value where they should be the same. The techniques in this section represent other ways of comparing two numbers that should be the same in addition to the previously discussed procedures found in chapter 6.

If a calculated value is equal to its expected value, then their discrepancy would be zero. Of course, with experiments rarely is this the case. In reality, the calculated value will differ somewhat from the expected value. When we take the difference between the calculated and expected value, that discrepancy has its *own* uncertainty by the methods of uncertainty propagation discussed earlier. We know intuitively, if the subtraction yields a small number (remember it should be zero), then our agreement is good, and if the subtraction gives a large number, our result may not be enviable. We can express the discrepancy in terms of how many standard deviations are associated with it. For good agreement, we want a *large* probability, reflecting the idea that the calculated and expected results are very probably the same. If the probability is a small number, this reflects the improbability of the two numbers actually being the same. So we use the probability outside a $\pm t\sigma$ range about the mean.
We would use:

$$P(\text{outside } t\sigma) = 1 - P(\text{within } t\sigma)$$

By convention, a probability of 5% or less is unacceptable.

As an example, say the difference between a calculated and expected value is two standard deviations. After looking in appendix D, we see that there is only a 4.46% chance the difference would be this great (or greater). Rounding to 4%, this is less than 5% so the agreement is not reasonable.

Example: Consider an experiment where g was calculated with an uncertainty to be:

$$g = (9.82 \pm 0.04)\text{m/s}^2.$$

The expected value of g is 9.81 m/s^2. Does this calculated value agree with the expected value or not?

Solution:

Here we will consider the 0.04 to be the standard deviation. The discrepancy between the calculated and the expected is : 9.82 - 9.81 = 0.01

The number of standard deviations associated with this discrepancy is: 0.01/0.04 = 0.25.

The discrepancy is 1/4 of a standard deviation. Using appendix D, with t = 0.25≅ 0.3, then 100 - 23.58 = 76.42. That is, there is a 76% chance that these two numbers agree. We would say in this example we are 76% confident of the agreement between our calculated value and the expected or known value. The calculated value is acceptable.

Example: A student wishes to check two results, each with its own uncertainty where, in principle, the results should be equal. The two results are: 33 ± 2 and 39 ± 3.

Do these two results agree?

Solution:

Assume the two uncertainties are standard deviations. We find the discrepancy to be 6 (39-33 = 6). Since this is a subtraction, the standard deviation we will use is the addition in quadrature of 2 and 3 (i.e., $\sqrt{2^2 + 3^2} = 3.6 \cong 4$). We see the deviation between the two values is 6/4 = 1.5 standard deviations. After conferring with the appendix, we associate a 13% probability with this discrepancy. Not great agreement, but larger than 5%, so we conclude the two numbers are equal with a 13% confidence level.

In this last example, using the methods introduced in chapter 6, we get a discrepancy test result of 15% between the two values and we see that their two most probable ranges just barely do not overlap. Both of these results are compatible with the marginal but acceptable confidence level of the example.

10. REJECTING DATA

There comes a time for every experimenter when, after examining a set of data, the question is asked, "Is this measurement correct?" It is difficult to know when rejecting data is appropriate. Sometimes discoveries are made from data that is different than expected. Therefore, do not blithely reject data because it does not meet an expectation; seriously consider what you are doing and seek sound justification for doing it. This section presents a method for justifying the rejection of data, it is called Chauvenet's criterion.

Say that a given set of measurements, all measuring a value that in principle should be the same, yields a results as follows:

4.5, 4.7, 4.4, 4.8, 2.5 (in centimeters)

Here, the 2.5 measurement is suspicious. Hopefully the apparatus is still set up and the measurement can be repeated several times for confirmation. But as well there is a statistical method for determining whether a number appearing to be different from the others can be rejected.

From the numbers above, the mean and standard deviation are:

$$\bar{x} = 4.18 \quad \sigma = 0.95$$

We see that the suspect measurement is 1.8 standard deviations from the mean[2]. From the methods of a previous section and using appendix D, we would expect about seven measurements out of one hundred to be away from the mean by this much (t = 1.8 and 100% - 93% = 7% = 7/100). But only *five* measurements were actually taken, not one hundred. So how many measurements out of five would we expect to be off by 1.8 standard deviations? Using a simple ratio we can find the answer.

$$\frac{7}{100} = \frac{x}{5}$$

$$x = \frac{(5 \cdot 7)}{100}$$

$$x = 0.35$$

$$x = 0.4$$

Out of five measurements we would expect 0.4 of them to be away from the mean by 1.8 standard deviations. Now let's see what Chauvenet's criterion says.

[2] To see this, divide the deviation by the standard deviation: (4.18-2.5)/0.95 = 1.8.

Chauvenet's criterion states that if the *expected* number of measurements at least as off as the suspect measurement, is less than 0.5, then the suspect measurement may be considered for rejection. The choice of 0.5 is arbitrary but is used as a convention unless special reasons dictate otherwise.

Applying Chauvenet's criterion to the above example, for only five measurements we would expect 0.4 of them to be away from the mean by 1.8 standard deviations. Since 0.4 is less than the Chauvenet value of 0.5, then the 2.5 cm measurement may be considered for rejection.

Another way of saying it is that given a set of data, you may expect to have several measurements out of a set of data a certain distance from the mean, or you may expect to have only one measurement a certain distance from the mean, but if you expect to have less than *half* a measurement in your data a certain distance from the mean, then you may consider the measurement a candidate for rejection.

If the number of measurements in the above example were fifteen instead of five, then with the same mean and standard deviation values as above, the expected number of measurements off by 1.8 standard deviations would be 1 and the 2.5 cm measurement would not be considered anomalous, in fact we would *expect* one of our measurements to be off by at least this much.

In a large data sample, if the measurements follow a Gaussian distribution then we expect *some* amount of measurements to be away from the mean value by one or more standard deviations.

Another method of eliminating suspicious data is to discard all data that is three (or some other value) or more standard deviations away from the mean value. This method is particularly simple, but bear in mind that for a large number of measurements some amount of data would be expected to deviate from the mean by three or more standard deviations and then should *not* be discarded.

Example:

Given the following set of measurements assumed to follow a Gaussian distribution.

3.4, 3.5, 2.9, 2.7, 3.4, 3.7, 3.2, 3.1, 5.2, 2.8, 3.3, 3.1 (all in seconds)

Use Chauvenet's criterion to determine if the 5.2 measurement should be considered for rejection.

Solution:

First, find the mean and standard deviation for the given set of data.

$$\overline{x} = 3.36 \quad \sigma = 0.65$$

Next, find the deviation of the suspected measurement and find how many standard deviations the deviation is away from the mean value.

$$5.2 - 3.36 = 1.84$$
$$1.84 / 0.65 = 2.83$$

So the suspected measurement is 2.8 standard deviations away from the mean value. Using the table in appendix D, the probability that a measurement will be within $t\sigma$ (here, $t\sigma$ = 2.833 · 0.65 = 1.84) from the mean is 99.49%. In using the table remember, here $t = 2.8$.

In two hundred measurements then, we would expect only one of them to be 2.8 standard deviations or more from the mean value. Therefore in only twelve total measurements, we would expect only 0.06 measurements that far from the mean value. The value 0.06 is far less than the 0.5 from Chauvenet, so on this basis the measurement 5.2 may be considered for rejection.

11. GRAPHING TECHNIQUES

Taking data in an experiment is one thing, making sense of the data is another. Graphing is a standard way of visually interpreting numerical data and gaining insight into one's data.

11.1 The Basic Graph

Your graph should have a title describing what it represents without having to refer to the lab record. It should be scaled so that all the data points cover at least half the page. If the origin needs to be included in your graph then you have to scale the axes to include it as well. Both the vertical and horizontal axis of your graph should be clearly labelled and the increments of the axes should be legible. Make sure the physical quantities on each axis are labelled with the appropriate units.

If the data points on your graph look like they form a line or you suspect from theory that your data should be linear, then draw an "eye-balled" best-fit line through the data points using your ruler. The best-fit line need not pass through any data points. In determining the slope of the line, use the data points to draw the best-fit line, but **do not use the data points to determine the slope.** Use two other points on your drawn line that are far apart from each other, even outside the data range. The data points determine the line and the line determines the slope. Remember, the slope has the same units as the "rise over the run". Check with your instructor to see if calculations done on your graph page are allowed.

"Linearizing" data:

If you know from your theoretical analysis that the relationship between an independent and dependent variable is non-linear, thereby not resulting in a straight-line graph, it is possible to specially scale one or both of the axes such that a straight line would result. For example, if in theory two variables are related by a power of 2 such as:

$$y = \mathrm{k}\, x^2$$

Then graphing y versus x will yield a parabola but graphing y versus x^2 results in a straight line graph with a slope of k.

11.2 Determining the Best Fit Line From Statistical Methods

For a rough graph, "eye-balling" a best fit straight line through the data may be good enough. You may require, however, a more precise knowledge of the slope and y-intercept. You may also want a quantitative way of determining how well your data fits a straight line. A statistical method exists for determining these important parameters. The results are stated without proof.

The so called "least squares" fit to a straight line, or linear regression, presumes you have N data pairs, (x_i, y_i) that have a linear relationship, y = *m*x + *b*, where *m* is the slope of the line and *b* is the y-intercept. This being so, it can be shown statistically that the "best" slope, *m*, and the "best" y-intercept, *b* are given by:

$$m = \frac{\sum_{i=1}^{n} x_i y_i - \frac{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n}}{\sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}}$$

$$b = \frac{\sum_{i=1}^{n} y_i}{n} - m \frac{\sum_{i=1}^{n} x_i}{n}$$

The measure of how close your data is to a straight line is called the "coefficient of determination", r^2, where its value will lie between 0 and 1, with a value of 1 indicating a perfect fit. r^2 is given by:

$$r^2 = \frac{\left[\sum xy - \frac{\sum x \sum y}{n}\right]^2}{\left[\sum x^2 - \frac{(\sum x)^2}{n}\right]\left[\sum y^2 - \frac{(\sum y)^2}{n}\right]}$$

Many calculators have these functions built in and all that is required of the user is an input of the data.

As an example, say three pair of data points have been measured and it is suspected that a linear relationship exists between them. For ease of calculation only three data pairs are presented here, but five is usually the minimum required.

X	Y
2.2	10
5.6	29
12.5	55

Use the above formulas and your calculator to show that:

$$m_{best} = 4.3: \ b_{best} = 2.36: \ r = 0.99.$$

Since r is so close to 1, this indicates the relationship is quite linear.

11.3 Finding the Uncertainty in the Slope and y-intercept

The experimenter is always concerned with uncertainties. Often the slope of a line or its y intercept will allow the immediate calculation of a desired experimental value. Finding the uncertainty in the slope or the y-intercept therefore is desirable. The following procedure is not for the faint hearted as it involves some tedious calculations, but with a hand calculator (especially a programmable one) the job is not too bad.

First, we will assume that the uncertainty in the y values of the data are large compared to the uncertainties in the x values, so that we may ignore the uncertainty in the x values. This is not usually a problem since the uncertainty in one of the two measurements, *x* or *y* is, in fact, larger than the other. Also, it is assumed the uncertainty in *y* is a constant, not changing as *y* changes.

So we define an uncertainty in the y value and call it σ_y.

$$\sigma_y^2 = \frac{1}{N-2} \sum (y_i - b - mx_i)^2$$

Note that since we expect all the *y* values to different from one another (this *is* a suspected linear relation between *x* and *y*), averaging the *y* values and then finding the standard deviation of the mean to find the uncertainty in y would be inappropriate since that method is only valid when the *y* values are *supposed* to be the same.

So that no single equation gets too big, we also define the special value Δ as follows:

$$\Delta = N(\Sigma x_i^2) - (\Sigma x_i)^2$$

We can now define the uncertainties in the slope *m* and the y intercept *b*.

$$\sigma_b^2 = \frac{\sigma_y^2 \sum x_i^2}{\Delta}$$

$$\sigma_m^2 = N\sigma_y^2/\Delta$$

To continue the same example from above, lets now find the uncertainties in the slope and y intercept.

First, separately calculate the values of σ_y and Δ.

You should verify them.

$\sigma^2_y = 11$

and

$\Delta = 165$

So the absolute uncertainty in the y - intercept is:

$\delta_b = 3.58 \cong 4$

And the absolute uncertainty in the slope *m* is:

$\delta_m = 0.447 \cong 0.5$

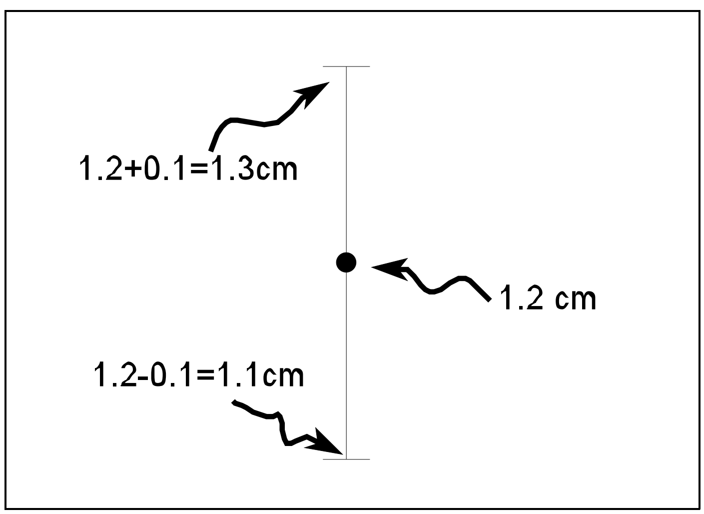
Notice the peculiar result that the uncertainty in the y-intercept is very large compared to the value of the y-intercept itself ($2.36 \pm 4 \rightarrow 2 \pm 4$!!), but that the uncertainty in the slope is reasonable when compared to the slope's value (4.3 ± 0.5). This can be understood in noting that typically in linear regression, the y-intercept is an *extrapolated* value, that is, a value obtained that is beyond and outside the data range itself. This introduces the sensitivity of the y-intercept to the angle of the slope. That is, a slight change in the slope can introduce a large change in the y-intercept. Therefore we see that the uncertainty in the y-intercept is large. For this reason, it is best to design the analysis of the data so that the result is *not* obtained by the extrapolation to a y-intercept but from the value of the slope.

As well, note that this entire process, finding the best slope and y-intercept and their uncertainties, could be done entirely without drawing a graph. Of course, a graph should still be drawn because the visual information from a graph can give the experimenter insights into the data that no other method can yield.

11.4 Including Error Bars on a Graph and How to Use Them

Graphs involving measured values typically include "error bars". The error bars graphically represent the absolute uncertainty of a measurement and allow a rough calculation of the uncertainty in the slope that is simpler than the statistical method discussed earlier.

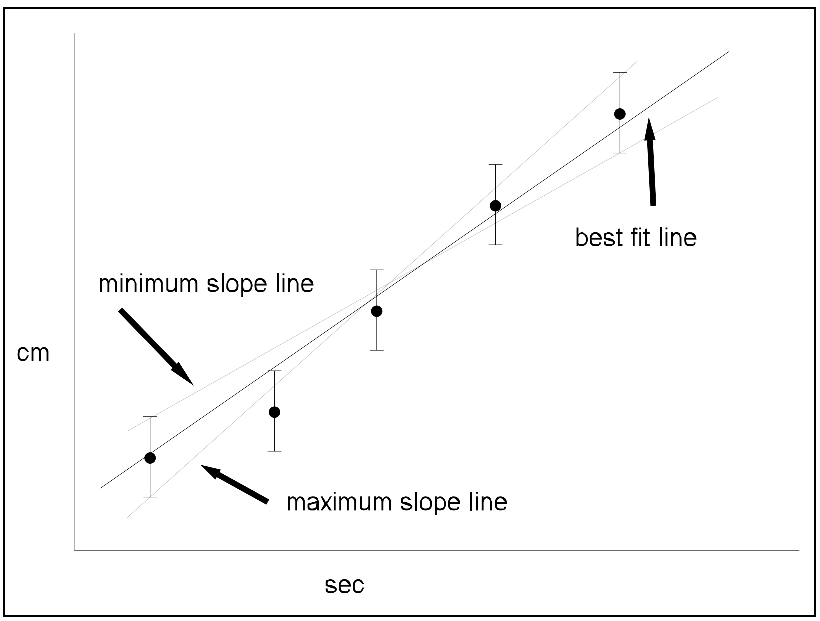
In the diagram below, the data point is plotted in the usual fashion; as shown, the absolute uncertainty is drawn above the data point and then below the data point representing the "plus and minus" aspect of the absolute uncertainty (i.e., measurement = value ± absolute uncertainty).



An error bar graphically expresses the absolute uncertainty of the measurement. This measurement is (1.2 ± 0.1)cm.

After all the data points with error bars are drawn, the "best fit" line is drawn as usual. With error bars, the new feature is that two more lines can also be drawn that represent the maximum possible slope and the minimum possible slope as shown in the diagram to the right. This allows the calculation of the absolute uncertainty in the slope. Admittedly, this is a subjective process; the more analytical process was discussed earlier.

As an example of this technique, say the graph shown below has a measured slope of the best fit line of 3.4 cm/s (note that the slope has the units of rise/run). If the maximum slope line has a measured slope value of 3.9 cm/s and the minimum slope line has a slope of 2.9 cm/s, then the final slope value may be written with an absolute uncertainty as (3.4 ± 0.5) cm/s. See that the minimum and maximum slopes determine the "most probable range" of the slope's value.



The two dotted lines show a maximum and minimum slope that can be used to construct the uncertainty in the slope. The solid line is the best fit line.

For another example of the uncertainties in the slope, say that from theory it is known that the slope m of a graph is related to a quantity to be found Q by the equation: $Q = 2.5\, m$. If the slope is known with an absolute uncertainty as $m = 23.5 \pm 0.8$ then Q with an absolute uncertainty would be equal to

59 ± 2.

Solution:

Q = (2.5)(23.5) = 58.75 : the uncertainty in Q is (2.5)(0.8) = 2.0 or just 2 to one significant figure (where 2 is rounded to the units place), so 58.75 is also rounded to the units place.

12. COMPUTER USAGE

The computer is an invaluable tool found in all modern labs. The use of the computer includes automated data gathering and data analysis on a statistical and graphical basis.

At De Anza, using the computer for data acquisition is not as yet employed. In the professional lab, computerized data acquisition is common, but for the beginning lab student, it is a better learning experience to measure data without a computer first, and then learn to use a computer to do the job later. Computers can be used to analyze data in a "spreadsheet context" and to draw graphs.

As students, it is important that the computer *not* be used as a "crutch" but as a tool to enhance your analysis. Using the computer as a crutch means drawing a graph with the computer when you couldn't draw the graph yourself or doing calculations with a computer that you don't understand or couldn't perform yourself. Use the following rule: if you *can* do it yourself, it's okay to use the computer; if you can't do it yourself then learn how first, *then* you can use the computer.

Two Mackintosh computers are available from the physics storeroom. "Cricket Graph" is an excellent program for drawing graphs. See your instructor for the details on its use.

The use of programmable calculators in the lab is encouraged. Many tedious, repetitive lab calculations can be made less so with these mini-computers.

APPENDIX A

A guide to the physics 4 lab sequence.

Lab	equipment used	Uncertainty	Other
4A	meter sticks vernier calipers counter timers air tracks pan balance digital balance	analog scales and digital scales uncertainties and statistics linear regression	lab book technique graphing writing abstracts formal reports
4B	VOM, DMM, power supplies function generator oscilloscope circuits	systematic uncertainties in electronic equipment linear regression	introduction to unguided research
4C	microwave generators optics sonometers	uncertainty propagation from partial differentiation	unguided research and exploration. group reports
4D	Interferometers E/M measurements	all of the above	

APPENDIX B
An optional evaluation form.

This form is an example of what your instructor may use to evaluate your lab book. If used, you will find a copy of this form in the lab exercise book for your specific lab.

Physics Lab Record Evaluation

Name________________________ Course___________ Section____

Lab Recording Skills	1	2	3	4	5	6
1. Establish Experimental Objective Devise Tentative Plan						
2. Draw Detailed Diagrams and Discuss Theory						
3. Collect and Organize Data						
4. Thoroughly Analyze Data Sample Calculations						
5. Make Good Graphs with Title Label Axes, Proper Scales & Size						
6. Discuss Results Error Analysis						
7. Compare Results with Theory Draw Valid Conclusions						
8. Compose a Readable, Sequential, Detailed, Direct-Recorded Record						

APPENDIX C

A guide to equipment care and lab cart handling

Part of lab training involves learning how to treat equipment with respect and how to use it properly. What follows is a list of the *minimum* that is expected from the student in handling lab equipment.

The golden rule for returning equipment to storage carts:

The neatness and order of equipment returned to the cart is *at least as good* as it was found at the beginning of your lab.

The storage of equipment on the lab carts:

1. Cables are color and length sorted and hanging from the side of the cart.
2. All electrical cords are neatly coiled and placed in the correct storage box if kept separately from the equipment it drives.
3. Weight sets are correctly ordered.
4. Equipment is not stacked on top of other equipment unless absolutely necessary.
5. All equipment of a similar type is kept in the same place on the cart if possible.
6. Alligator clips or any accessory clips are removed from banana plugs and stored in their own box on the lab cart.

All non-working equipment should be tagged and returned to the instructor, not to the cart.

Instructor's responsibilities:

1. The tech receives a complete written equipment list at least one week in advance of the first lab day.
2. See that students carry out their responsibilities.
3. Equipment taken from shelves is delivered to a "dumb" site to be re-shelved by the tech or student-aides and is not put back on the shelves in the wrong place.
4. All mal-functioning equipment tagged by the students is delivered to the physics tech.

APPENDIX D

The percentage probability, *P*, that a measurement *x* will lie within *t* standard deviations, σ, on either side of the mean value, *X*.

$$P(\text{within } t\sigma) = \frac{1}{\sqrt{2\pi}} \int_{-t}^{t} e^{-z^2/2}\, dz \qquad \{\text{where } z = (x-X)/\sigma\}$$

t	P(%)	*t*	P(%)
0.0	0.00	1.7	91.09
0.1	7.97	1.8	92.81
0.2	15.85	1.9	94.26
0.3	23.58	2.0	95.45
0.4	31.08	2.1	96.43
0.5	38.29	2.2	97.22
0.6	45.15	2.3	97.86
0.7	51.61	2.4	98.36
0.8	57.63	2.5	98.76
0.9	63.19	2.6	99.07
1.0	68.27	2.7	99.31
1.1	72.87	2.8	99.49
1.2	76.99	2.9	99.63
1.3	80.64	3.0	99.73
1.4	83.85	3.5	99.95
1.5	86.64	4.0	99.994
1.6	89.04	5.0	99.99994

BIBLIOGRAPHY

Yardley Beers, *Introduction to the Theory of Error*, (Addison Wesley, 1957)

John R. Taylor, *An Introduction to Error Analysis*, (University Science Books, 1982)

Marsh White, *Experimental College Physics*, (McGraw-Hill Book Company, 1940)

INDEX

Absolute uncertainty, 11-14, 16-18, 20-22, 24, 36-38
Abstract, 5
Accuracy and precision, 18
Accuracy, 2, 18
Addition by quadrature, 20
Average value, 11, 15, 17
Chauvenet's criterion, 30, 31
Coefficient of determination, 34
Confidence, 27-29
Discrepancy, 15, 16, 28, 29
Discrepancy test, 15, 16, 29
Error bars, 37
Extrapolation, 37
Fractional uncertainty, 17
Gaussian, 25-27, 31
Graphing, 2, 33, 40
Interpolate, 13, 14, 18
Least count, 13, 14, 18
Least squares, 34
Linearizing, 33
Linear regression, 1, 34, 36, 40
Most probable range, 10, 11, 16-19, 38
Most probable value, 11, 12, 26
Parallax, 18
Precision, 7, 11, 12, 14, 17-19, 21, 24
Quadrature, 19-21, 24, 29
Random uncertainties, 17, 19, 25
Relative uncertainty, 17, 20, 21
Significant figure, 7, 12, 13, 20, 21, 24, 38
Slope, 33-38
Standard deviation of the mean, 11, 26, 35
Standard deviation, 11, 12, 19, 25-27, 29-32, 35
Standard error, 11-13
Standard form, 11, 13, 16, 21, 22
Systematic uncertainties, 18, 19, 40
Total uncertainty, 19
Uncertainties in the slope, 36, 38
Uncertainty propagation, 8, 17, 20, 28, 40