Random Variable

- The value of the variable depends on an experiment, observation or measurement.
- The result is not known in advance.
- For the purposes of this class, the variable will be numeric.

Random Variables

- Discrete – Data that you Count
  - Defects on an assembly line
  - Reported Sick days
  - RM 7.0 earthquakes on San Andreas Fault
- Continuous – Data that you Measure
  - Temperature
  - Height
  - Time
Discrete Random Variable

- List Sample Space
- Assign probabilities $P(x)$ to each event $x$
- Use “relative frequencies”
- Must follow two rules
  - $P(x) \geq 0$
  - $\sum P(x) = 1$
- $P(x)$ is called a Probability Distribution Function or pdf for short.

Probability Distribution Example

- Students are asked 4 questions and the number of correct answers are determined.
- Assign probabilities to each event.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$P(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>0.4</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Probability Distribution Example

- Students are asked 4 questions and the number of correct answers are determined.
- Assign probabilities to each event.

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<td>0.4</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
</tr>
</tbody>
</table>
Mean and Variance of Discrete Random Variables

- Population mean $\mu$, is the expected value of $x$
  $$\mu = \Sigma [x \cdot P(x)]$$

- Population variance $\sigma^2$, is the expected value of $(x-\mu)^2$
  $$\sigma^2 = \Sigma [(x-\mu)^2 \cdot P(x)]$$

Example of Mean and Variance

<table>
<thead>
<tr>
<th>$x$</th>
<th>$P(x)$</th>
<th>$xP(x)$</th>
<th>$(x-\mu)^2P(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.1</td>
<td>0.0</td>
<td>0.625</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.225</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.4</td>
<td>0.050</td>
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<tr>
<td>3</td>
<td>0.4</td>
<td>1.2</td>
<td>0.100</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
<td>0.8</td>
<td>0.450</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1.0</strong></td>
<td><strong>2.5=\mu</strong></td>
<td><strong>1.450=\sigma^2</strong></td>
</tr>
</tbody>
</table>

Bernoulli Distribution

- Experiment is one trial
- 2 possible outcomes (Success, Failure)
- $p$=probability of success
- $q$=probability of failure
- $X$=number of successes (1 or 0)
- Also known as Indicator Variable
### Mean and Variance of Bernoulli

<table>
<thead>
<tr>
<th>$x$</th>
<th>$P(x)$</th>
<th>$xP(x)$</th>
<th>$(x-\mu)^2P(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$(1-p)$</td>
<td>0.0</td>
<td>$p^2(1-p)$</td>
</tr>
<tr>
<td>1</td>
<td>$p$</td>
<td>$p$</td>
<td>$p(1-p)^2$</td>
</tr>
<tr>
<td>Total</td>
<td>1.0</td>
<td>$p = \mu$</td>
<td>$p(1-p) = \sigma^2$</td>
</tr>
</tbody>
</table>

- $\mu = p$
- $\sigma^2 = p(1-p) = pq$

### Binomial Distribution

- $n$ identical trials
- Two possible outcomes (success/failure)
- Probability of success in a single trial is $p$
- Trials are mutually independent
- $X$ is the number of successes

Note: $X$ is a sum of $n$ independent identically distributed Bernoulli distributions

### Binomial Distribution

- $n$ independent Bernoulli trials
- Mean and Variance of Binomial Distribution is just sample size times mean and variance of Bernoulli Distribution

$$p(x) = C_x p^x (1-p)^{n-x}$$

$$\mu = E(X) = np$$

$$\sigma^2 = Var(X) = np(1-p)$$
Binomial Examples

- The number of defective parts in a fixed sample.
- The number of adults in a sample who support the war in Iraq.
- The number of correct answers if you guess on a multiple choice test.

Binomial Example

- 90% of intake valves manufactured are good (not defective). A sample of 10 is selected.
- Find the probability of exactly 8 good valves being chosen.
- Find the probability of 9 or more good valves being chosen.
- Find the probability of 8 or less good valves being chosen.

Using Technology

<table>
<thead>
<tr>
<th>X</th>
<th>p(X)</th>
<th>cumulative probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>1</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>2</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>3</td>
<td>0.0014</td>
<td>0.0014</td>
</tr>
<tr>
<td>4</td>
<td>0.0014</td>
<td>0.0028</td>
</tr>
<tr>
<td>5</td>
<td>0.0017</td>
<td>0.0045</td>
</tr>
<tr>
<td>6</td>
<td>0.0017</td>
<td>0.0062</td>
</tr>
<tr>
<td>7</td>
<td>0.0012</td>
<td>0.0074</td>
</tr>
<tr>
<td>8</td>
<td>0.0006</td>
<td>0.0080</td>
</tr>
<tr>
<td>9</td>
<td>0.0000</td>
<td>0.0080</td>
</tr>
<tr>
<td>10</td>
<td>0.0000</td>
<td>0.0080</td>
</tr>
</tbody>
</table>

Use Minitab or Excel to make a table of Binomial Probabilities.

- $P(X=8) = 0.19371$
- $P(X\leq8) = 0.26390$
- $P(X\geq9) = 1 - P(X\leq8) = 0.73610$
Poisson Distribution

- Occurrences per time period (rate)
- Rate (μ) is constant
- No limit on occurrences over time period

\[ P(x) = \frac{e^{-\mu} \mu^x}{x!} \]
\[ \mu = \mu \]
\[ \sigma = \sqrt{\mu} \]

Examples of Poisson

- Text messages in the next hour
- Earthquakes on a fault
- Customers at a restaurant
- Flaws in sheet metal produced
- Lotto winners

Note: A binomial distribution with a large n and small p is approximately Poisson with \( \mu \approx np \).

Poisson Example

- Earthquakes of Richter magnitude 3 or greater occur on a certain fault at a rate of twice every year.
- Find the probability of at least one earthquake of RM 3 or greater in the next year.

\[ P(X > 0) = 1 - P(0) \]
\[ = 1 - \frac{e^{-2} \cdot 2^0}{0!} \]
\[ = 1 - e^{-2} \approx .8647 \]
Poisson Example (cont)

- Earthquakes of Richter magnitude 3 or greater occur on a certain fault at a rate of twice every year.
- Find the probability of exactly 6 earthquakes of RM 3 or greater in the next 2 years.

\[ \mu = 2(2) = 4 \]
\[ P(X = 6) = \frac{e^{-4}4^6}{6!} = .1042 \]