Prior handouts discussed the calculation of the present value of benefits for single multiple decrement models, in particular for retirement plans. This Handout will describe methods of funding these types of plans which can be generalized for many aggregate financial risk models.

**Definitions and Terminology**

An **Actuarial Cost Method** is a procedure for determining the actuarial present value of pension plan benefits and expenses and to develop an actuarially equivalent value by time period (usually annually) which creates a mechanism for funding the plan. The portion of the actuarial present value of pension plan benefits which is allocated to a particular year by an actuarial cost method is called the **Normal Cost** of the plan for the year.

The **Present Value of All Benefits** can be thought of as a stream of Normal Cost’s from the inception of the plan to the end of the plan:

\[ \sum_{i=x}^{z} NC_{(y,i)} \]

- \( x \) = initial year of plan
- \( y \) = current year, also known as **Valuation Year**
- \( z \) = final year of plan
- \( PVB_y \) = Actuarial Present Value of Benefits in year \( y \).
- \( NC_{(y,i)} \) = Actuarial Present Value in year \( y \) of Normal Cost of Plan in year \( i \)

The **Normal Cost** is calculated on the Valuation Date, which is usually the first or last day of the plan year. In our examples, we will use Beginning of Year Valuation Dates.

The **Actuarial Accrued Liability** is the actuarial present value of benefits that is not provided by future Normal Costs:

\[ AAL_y = PVB_y - \sum_{i=y}^{z} NC_{(y,i)} \]

- \( AAL_y \) = Actuarial Accrued Liability in year \( y \).
The **Present Value of Future Normal Costs** is the value in the valuation year of all future Normal Costs yet to be allocated to the Actuarial Accrued Liability:

\[ PVFNC_y = \sum_{i=y}^\infty NC_{(y,i)} \]

\[ PVB_y = AAL_y + PVFNC_y \]

\[ PVFNC_y = \text{Present Value of Future Normal Costs in year } y \]

The **Actuarial Value of Assets** is the value (as determined by the actuary) of cash, investments and other property of the pension plan. The **Unfunded Actuarial Accrued Liability** is the excess of Actuarial Accrued Liability over the Actuarial Value of Assets in the valuation year. Note that this amount can be negative if assets exceed the Actuarial Accrued Liability

\[ AAL_y = UAAL_y + ASSETS_y \]

\[ PVB_y = UAAL_y + ASSETS_y + PVFNC_y \]

\[ PVB_y = \text{Actuarial Present Value of Benefits in year } y. \]

\[ UAAL_y = \text{Unfunded Actuarial Accrued Liability in year } y \]

\[ ASSETS_y = \text{Actuarial Value of Assets in year } y \]

\[ PVFNC_y = \text{Present Value of Future Normal Costs in year } y \]

The annual **Contribution** for the valuation year to the pension plan will be the sum of the Normal Cost and a payment to (or credit from) the Unfunded Actuarial Accrued Liability. For Life and Casualty Insurance Benefits, these annual or semi-annual are usually called **Premiums**.

**Pay-as-you-go Actuarial Cost Method**

Under this funding method, contributions are made to the plan only to pay plan expenses and to pay benefits. Under this method the Actuarial Accrued Liability and the Actuarial Value of Assets is always zero. Social Security was funded by this method until 1983 where a small reserve was built in to the system to pay for the baby boomers. Post-retirement health care benefits are usually funded by this method as well.

\[ AAL_y = 0 \]

\[ PVB_y = PVFNC_y \]

\[ NC_{(y,y)} = \text{value of year } y \text{ benefit payments and expenses} \]
Aggregate Actuarial Cost Method

Under this funding method, the Actuarial Accrued Liability is reset to the Actuarial Value of Assets each valuation year and the Normal Cost is calculated as a level percent of future salary or service. The Present Value of Future Value of Salary (or Service) is the actuarial value of all future value of salary or service for all active employees:

\[
AAL_y = ASSETS_y \\
UAAL_y = 0 \\
PVFNC_y = PVB_y - ASSETS_y \\
NC_{(y,y)} = S_y \cdot (PVFNC_y / PVFS_y)
\]

\[ S_y = \text{Salary (Service) in year } y \]
\[ PVFS_y = \text{Present Value of Future Salary (Service) in year } y. \]

Level Premium Actuarial Cost Method

Under this funding method, the Actuarial Accrued Liability is reset to the Actuarial Value of Assets each valuation year and the Normal Cost is calculated as an amortization of the Present Value of Future Normal Costs over a fixed period of n years:

\[
AAL_y = ASSETS_y \\
UAAL_y = 0 \\
PVFNC_y = PVB_y - ASSETS_y \\
NC_{(y, y)} = PVFNC_y / \ddot{a}_{\bar{n}}
\]

Entry Age Actuarial Cost Method

Under this funding method, the Present Value of Accrued Benefits is calculated for each active participant at their entry date into the plan, and a Normal Cost is calculated for each year up to the Valuation Year as a level percent of pay or service. The Actuarial Accrued Liability is then equal to the Present Value on the Valuation Date of the sum of these past Normal Costs. The Actuarial Accrued Liability for inactive participants is simply the Present Value of Benefits. Any Unfunded Actuarial Accrued Liability is either recognized immediately or amortized over a fixed period:

\[
AAL_y = \sum_{j=\text{actives}} \sum_{i=x} PVB_{(j,i,y)} + \sum_{j=\text{inactives}} NC_{(j,i,y)} \\
UAAL_y = AAL_y - ASSETS_y \\
NC_y = \sum_{j=\text{actives}} NC_{(j,i,y)} \\
NC_{(j,i,y)} = \text{Present Value in Year } y \text{ of Normal Cost credited in year } i \text{ to active participant } j
\]

Contribution to Plan = NC_y + Payment to UAAL_y
Unit Credit Actuarial Cost Method

This funding method has gained in popularity since it is required by the Financial Accounting Standard Board (FASB) in disclosing pension obligations for corporate financial statements. Under this funding method, the Actuarial Accrued Liability is equal to actual value of the benefit accrued to valuation date, and may use current or final pay. The Normal Cost is the present value of the actual benefit accrual for the current year, that is the value of the increase in benefits due to one more year of service. The Actuarial Accrued Liability for inactive participants is simply the Present Value of Benefits. Any Unfunded Actuarial Accrued Liability is either recognized immediately or amortized over a fixed period:

\[
AAL_y = \sum_{j=\text{actives}} PV \text{ accrued benefits} + \sum_{j=\text{inactives}} PV B_{(j,y)}
\]

\[
UAAL_y = AAL_y - ASSETS_y
\]

\[
NC_y = \sum_{j=\text{actives}} NC_{(j,y,y)}
\]

\[
NC_{(j,y,y)} = \text{Normal Cost credited in year } y \text{ to active participant } j
\]

Contribution to Plan = NC\(_y\) + Payment to UAAL\(_y\)

Calculation of Present Value of Future Salary

When using funding methods that require calculations as level percentage of pay, it is necessary to define a salary scale assumption and to create annuity factors that take this into account. Let \(s_x\) = salary scale at age \(x\) as a percent of salary at last age in the table.

It is customary to have the last salary on the table, \(s_z\), be equal to one. Then, the PVFS factors can calculated using a recursive process, similar to those used for life annuities:

\[
PVFS_{z-1(\text{finalPay})} = s_z
\]

\[
PVFS_{x(\text{finalPay})} = s_x + v_p x PVFS_{x+1(\text{finalPay})}
\]

This factor could then multiplied by Salary at retirement to get PVFS, but it is more convenient to adjust the formula to us current pay.

\[
PVFS_x = PVFS_{x(\text{currentPay})} = PVFS_{x(\text{finalPay})} \left( \frac{s_x}{s_z} \right)
\]

\[
PV \text{ Future Salary} = \text{CurrentPay} \cdot PVFS_x
\]
Alternate method - Commutation Functions adjusted for Salary Scale

We can use the salary scale to adjust the commutation functions as follows

\[ sD_x = s_x \cdot D_x \]
\[ sN_x = \sum_{i=x}^{\infty} sD_x \]

The Present Value of Future Salary at age \( y \) until age \( z \) is a salary adjusted temporary annuity:

\[ PV \ Future \ Salary = Current \ Pay \cdot \frac{sN_y - sN_z}{sD_y} \]

**Homework:**

1. Find the annual contribution under the aggregate, unit credit and entry age cost methods for the following plan:

**Benefit Formula**
2.5% of Final Pay times years of Service payable at age 63

**Actuarial Assumptions**
Mortality Assumption: GAM94 Female
Interest Rate: 7.5%
Salary Scale: 4.5%
Turnover: 10% at age 20 reducing by .25% for each year (same as HW5)
Actuarial Value of Assets: $500,000
Retirement: Age 63

**Participants**

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<tr>
<th>Status</th>
<th>Retiree</th>
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<td>60000</td>
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<td>Mo Benefit at Retirement</td>
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</tbody>
</table>

2. For the above plan, conduct a cash flow analysis and show the present value of benefits are approximately the same.