Types of Random Variables

- Number of Claims
- Amount of Claim
- Total Claims
- Time Until Termination
- Economic

Time Value of Money

Why is money now worth more than money then?

- Time Preference – “I want it now!”
- Productivity of Capital – “I can make more than it costs to borrow.”

Some Economic Variables

- Interest rate
  - Pre and Post Retirement
  - Inflation
    - Medical, CPI
  - Rate of Return on Assets
    - Different Asset Classes
  - Salary Increases
    - Merit and Inflation Adjustments
  - Social Security Increases
    - Wagebase, COLA, Average National Earnings
Effective rate of interest

- \( i \) = interested credited at the end of one year.
- Simple Interest
- Compound Interest

Example 1

- An initial investment of $1000 earns 6% interest for 5 years. Find the future value under both methods of crediting interest.

Other Interest Terms

- Discount Rate \( (d) \) Interest credited (or taken) at the beginning of the year.
- Discount factor \( (v) \) Today's value of 1 unit one year from now.
- Relationship between \( i \), \( d \), and \( v \):
  - \( d = i / (1+i) \)
  - \( v = 1 / (1+i) \)

Present and Future value

- \( v^t \) = the PRESENT VALUE of 1 payable in \( t \) years.
- \( (1+i)^t \) = the FUTURE VALUE at time \( t \) of 1 unit today.
Example 2
- A zero coupon bond with a face value of $100,000 will be payable in 10.5 years. Find the present value of the bond assuming a 7% rate of interest.

Example 3
- A bank lends a borrower $1000 and immediately collects $80 of interest, leaving the borrower with $920.
- Then $d = \frac{80}{1000} = 8\%$ is the discount rate.
- The effective rate of interest is $\frac{.08}{1-.08} = .08696$

Nominal Rate of Interest
- Used when interest is charged more frequently than annually.
  - Monthly: Credit card, auto loans, mortgages
  - Daily: Savings accounts
- $i^{(m)}$ = an annual rate of interest credited mtthly in units of $1/m$

Example 4
- A credit card charges a nominal rate of interest of 12% compounded monthly. Determine the effective rate of interest.
Force of Interest

- Interest that is paid continuously

\[ \delta = \lim_{m \to \infty} i^{(m)} \]

\[ \delta = \ln(1 + i) \quad i = e^\delta - 1 \]

Example 5

- Find the force of interest when the effective rate of interest is 8%.
- Find the effective rate of interest when the force of interest is 12%.

Annuities paid annually

- Annuity-immediate
  - n equal payments at the end of each year
  - Present value \( \bar{a}_{n|i} \)
  - Future value \( \bar{s}_{n|i} \)
- Annuity-due
  - n equal payments at the beginning of each year
  - Present value \( \dot{a}_{n|i} \)
  - Future value \( \dot{s}_{n|i} \)

Example 6

- Find the present value of 10 equal payments of $1000 at the end of the year assuming an interest rate of 10%
Example 7

- Pension law requires that 401(k) Plan distributions made to a participant before age 59.5 are subject to a early distribution penalty of 10%.
- One option to avoid this penalty is to make “substantially equivalent” annual payments over the life expectancy of the participant.
- A participant currently age 54 has an account of $200,000 and a life expectancy of 25 years.
- Determine the annual amount payable at the beginning of the year that would satisfy this requirement assuming the interest rate is 9% per year.

Example 8

- An investor puts $2000 into savings at the end of each of 10 years. Find the value of the account after 10 years assuming a 7% annual rate of return.

Annuities paid more frequently

- Usually, annuities are paid more frequently than annual (mortgages, pensions, car loans)
- Present value assuming m payments
  - Annuity-immediate $a_{\overline{m}}^{(m)}$
  - Annuity-due $d_{\overline{m}}^{(m)}$

Example 9

- A pension plan is paying a 10 year certain benefit of $1000 monthly for 120 consecutive months. Find the present value of this benefit assuming benefits are paid at the beginning of the month and interest of 8% per year.
Annuities paid continuously

- The theoretical limit as periods per payment goes to infinity creates the continuous annuity

\[
\bar{a}_n = \lim_{m \to \infty} \bar{a}_{n}^{(m)} = \lim_{m \to \infty} a_{n}^{(m)}
\]

\[
= \int_{0}^{\infty} v^m dt = \frac{1 - v^n}{\delta} = \frac{i}{\delta} a_{n}
\]

Example 10

- Annual payments of $1000 are paid continuously over the next 5 years. Assuming an effective interest rate of 6% per year, determine the present value.

Annuity Functions in Excel

Knowing 3 of the following variables implies the 4th variable:

- PV or FV
- PV = Present Value
- FV = Future Value
- RATE = interest rate
- NPER = number of years
- PMT = annuity amount (shown as negative)

Life Insurance

- Payable at time of death
- n-year term insurance
- Whole life insurance
- Net single premium
- Actuarial present value
Example

- Assume the pdf of future lifetime is uniform where $f(t) = 1/100$. Find the net single premium of whole life policy issued at age 0 when the force of interest is 8%.

n-year pure Endowment

- A deferred lump sum benefit
- Payable at the end of deferment period if owner is alive.

Example

- An executive compensation plan pays a lump sum benefit of $100,000 to participants who do not terminate employment in the next 5 years.
- Find the present value of this benefit for a 50-year old executive using 7.5% interest.
- Assume that $5p_{50}$ for all causes is 75%.
- Benefit is paid immediately after the 5 year vesting requirement is made.

Life Annuities

- Analogous to a collection of endowments.
- Types of Life Annuities
  - Straight life annuity
  - n-year temporary annuity
  - Deferred life annuity
  - Unusual annuities
- Example in Excel